

Nonlinear Micro Income Processes with Macro Shocks^{*}

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[\[Link to supplemental appendix\]](#)

Abstract

We propose a nonlinear framework that exploits both macro and micro data to study the dynamic transmission of aggregate and idiosyncratic shocks to household income. We develop identification arguments and estimation methods that combine macro time series and time series of panels. Using U.S. aggregate data and the PSID, we find that business-cycle fluctuations modulate the persistence of heterogeneous individual histories and the risk faced by households. We document how aggregate and idiosyncratic shocks propagate over time for households in different macro and micro states. Lastly, we quantify the welfare cost of aggregate and idiosyncratic risk.

Keywords: Income process, business cycle, persistence, exposure to aggregate shocks.

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1 Introduction

In this paper, we propose a nonlinear framework to study the dynamic transmission of aggregate and idiosyncratic shocks to income by leveraging both macro and micro data. Our approach makes it possible to empirically examine how business-cycle fluctuations modulate the persistence of heterogeneous individual histories and the risk faced by households. We also consider questions such as how aggregate and idiosyncratic shocks propagate over time for units in different macro and micro states, and how these shocks contribute to the cost of business-cycle risk. Answering these questions is important. They are essential to documenting the dynamics of income inequality over the business cycle. Furthermore, how the incomes of heterogeneous agents respond to macro and micro shocks is key for consumer and firm behavior, and for the design of optimal monetary and fiscal policies (Bhandari, Evans, Golosov, and Sargent, 2021).

The literature on income risk has uncovered significant nonlinearities in the dynamics of individual incomes (Arellano, Blundell, and Bonhomme, 2017; Guvenen, Karahan, Özkan, and Song, 2021) and in their variation over the business cycle (Guvenen, Ozkan, and Song, 2014). Moreover, a growing recent literature investigates the heterogeneous effects of monetary policy shocks on individual-level outcomes (Holm, Paul, and Tischbirek, 2021; Andersen, Johannesen, Jørgensen, and Peydró, 2023; Amberg, Jansson, Klein, and Rogantini Picco, 2022). Yet, a methodology for modeling the interaction between micro and macro shocks capable of integrating non-linearities in the life-cycle and business-cycle dynamics of income is still lacking. This is our main contribution.

We consider a nonlinear Markovian micro income process with a macro state variable of the following form:

$$\begin{aligned}\eta_{it} &= Q_{\eta}(\eta_{i,t-1}, Z_t, Z_{t-1}, u_{it}), \\ Z_t &= Q_Z(Z_{t-1}, V_t),\end{aligned}$$

where u_{it} and V_t are micro and macro shocks, and η_{it} and Z_t are potentially unobserved. A measurement system connects these two latent variables to observed micro and macro data, specifically, a flexible persistent-transitory model for the micro states and a dynamic factor model for the macro states. Our triangular formulation has the potential to allow for feedback from the micro to the macro level, as Z_t can incorporate distributional characteristics of the micro data. The assumption underlying the triangular structure is

atomicity, meaning that no single individual unit influences the aggregate state.

Based on our income process we will highlight two key quantities. The first one is the elasticity of individual persistent earnings to the aggregate business-cycle state Z_t :

$$\beta_{it} = \frac{\partial \eta_{it}}{\partial Z_t}.$$

In our setup, β_{it} is a measure of a household’s exposure to shocks to the aggregate state that is heterogeneous along both the income distribution and business-cycle conditions. In addition, β_{it} may vary with the idiosyncratic shock u_{it} , and so the impact of an aggregate shock may differ depending on idiosyncratic events (such as a job loss or a promotion).¹

The second quantity is income persistence:

$$\rho_{it} = \frac{\partial \eta_{it}}{\partial \eta_{i,t-1}}.$$

Here, ρ_{it} is a measure of nonlinear persistence (Arellano et al., 2017, ABB) that may vary depending on the position in the income distribution and the idiosyncratic shocks hitting the household. Moreover, unlike in ABB, our model allows for the aggregate state to affect income and, thus, for the shape of persistence to be different in good or bad times.

Documenting how β_{it} s and ρ_{it} s vary across income histories and over the business cycle allows us to paint a rich picture of the interaction between individual and aggregate income dynamics. Our model also allows us to flexibly measure how features of the income process such as (conditional) dispersion and skewness depend on business-cycle conditions. Since the model completely specifies the law of motion of individual income η_{it} and the aggregate state Z_t , it can be used for impulse response analysis and to quantify the cost of both micro and macro sources of income risk, as we illustrate empirically.

We study the nonparametric identification of this model bringing together macroeconomic and microeconomic techniques. The micro side builds on ABB. The macro side relies on identification results for factor models estimated from time series aggregates (Stock and Watson, 2016). To combine the two parts, we create a time series of short panels, such that each panel is representative of the economy in that period. Thus, identification

¹Our measure is different from, but related to, others in the growing empirical literature on the heterogeneous effects of recessions. For example, Guvenen, Schulhofer-Wohl, Song, and Yogo (2017) document variation in individual income exposure to aggregate income—referred to as “worker betas”—across age and income groups, while Patterson (2023) examines how these heterogeneous exposures relate to differences in marginal propensities to consume across demographic groups.

relies on three dimensions of our combined dataset: large cross-sections, short individual panels, and long time series spanning sufficient cyclical fluctuations.

We propose an approach to estimation and inference that uses a flexible parametric version of the model and can be implemented with stable simulation-based algorithms. This approach was first introduced in [Arellano and Bonhomme \(2016\)](#) and was adapted to a setup with time-varying latent variables in ABB. Here it is further extended to a long time series of short panels involving both micro and macro latent variables. Our stochastic EM algorithm iterates between draws of the latent variables from their posterior distributions evaluated at current parameter values, and updates of the parameters from quantile regressions based on those draws. Furthermore, the method of inference we develop is robust to forms of cross-sectional and unit-level dependence in micro shocks u_{it} that arise from unobserved common factors and realistic sampling designs.

We also develop a methodology for impulse response function analysis in our nonlinear context where we want to measure the importance of macro and micro shocks and their interactions. We start by considering an experiment in which we directly perturb a state variable at some point in time. We then compare the trajectory of the system following the perturbation with a baseline trajectory in the absence of perturbation. Since we wish to obtain comparability of impulse responses across households with different processes, we need to consider ways of introducing comparable perturbations. We do so via a set of *rules* that map perturbations to a common system of units, and we show that different perturbation experiments can be associated with different formulations of local shocks.

We take our model to quarterly macro time series data for the U.S. and a time series of panels that we construct from the Panel Study of Income Dynamics (PSID) spanning the period 1970-2019, thus covering seven recessions. PSID waves are annual up to 1997 and biennial afterwards. For consistency of the microdata, we then form sequences of biennial subpanels of four waves each covering all the years available, in the spirit of [Storesletten, Telmer, and Yaron \(2004\)](#). Compared to standard long panel approaches, this *time series of panels* approach has the advantage of mitigating concerns over the representativeness of the data. In this paper, our primary focus is on disposable household income net of taxes and transfers. However, for comparability with other studies, we also present estimates based on male earnings and household earnings before taxes and transfers.²

²The last results are limited to units with positive earnings. The share of households with zero earnings stays below 2% for most of the period, rising slightly during the Great Recession. For men, zero earners remain under 6% until the Great Recession, when the rate nears 8%. See [Figure 2](#) and its discussion.

Empirical results. Our analysis yields a number of novel empirical insights. To begin, our results illustrate the asymmetric impact of the business cycle on income persistence. Specifically, income persistence ρ_{it} increases for low-income households and decreases for high-income households during recessions. That is, during a downturn, it is harder for low-income earners to leave the low-income state, whereas for high-income earners remaining high-income becomes more difficult. The cyclical variation we find in the persistence of past income histories coexists with the ample variability along income and micro-shock distributions uncovered by ABB.

Our results also highlight the presence of heterogeneous exposures to aggregate shocks. The coefficients β_{it} tend to be higher when associated with bad idiosyncratic shocks. Moreover, we find that β_{it} s are countercyclical: they are higher in recessions and lower in expansions. This is a key finding because the cyclical behavior of income elasticities to the macro state—particularly the self-amplifying nature of negative aggregate shocks—has major implications for the cost of aggregate income risk, as we argue below.

In addition, we document two main facts about income skewness. First, we find that left skewness is countercyclical, consistent with the findings in [Guvenen et al. \(2014\)](#). Second, we find that skewness decreases with income at any point in time, consistent with the findings in ABB, but differentially so depending on the phase of the business cycle. This *tale of two skewnesses* is a clear reflection of the nonlinear transmission of micro and macro shocks. In this and other dimensions, business-cycle variability is most pronounced for male earnings and, to a lesser extent, for household earnings and disposable income, in that order—that is, from the income measure with the least insurance to the one with the most—but the patterns are qualitatively similar for all three measures.

Concerning impulse response functions, we find a large direct impact of macro shocks but with generally short-lived effects. These impacts, however, are highly heterogeneous across income measures (larger for male earnings, smaller for disposable income) and along the persistent income distribution (larger at the bottom, smaller in the middle), and they interact with idiosyncratic uncertainty by compounding the negative consequences of bad micro shocks. In contrast, micro impulse responses decay slowly, with different degrees of persistence depending on the initial level of income. In fact, as we show below, there is a tight link between impulse responses and β_{it} s and ρ_{it} s.

Finally, we use our framework to quantify the cost of both macro and micro sources of risk. For macro risk, we compute the (per cent) compensating variation that equalizes the expected lifetime utility of income with and without macro shocks (as in [Lucas, 1987](#),

2003), where (rational) expectations are based on our estimated income process. In this exercise, the cyclical behavior of the nonlinear income exposure to macro shocks β_{it} is a key determinant of the cost of business cycles. In the presence of empirically plausible amplification effects, the yearly cost of macro risk can be as high as 5 percent of income, whereas it is negligible under a linear transmission of macro shocks. This is a novel channel through which macro uncertainty can lead to welfare losses at the household level, distinct from curvature in the utility function. Although as expected the cost of micro risk is higher, for many units (mainly young and low-income) macro shocks explain a nontrivial fraction of the total cost of risk. It is worth emphasizing that an income process linear in aggregate shocks would miss all of the rich cyclical patterns documented here. Thus, a core lesson from our paper is the importance of accounting for nonlinearities in β_{it} when the goal is to study the consequences of aggregate fluctuations.

Selected literature. Our paper contributes to several strands of the literature. First, we build on the vast literature on income dynamics, both with and without business cycles, e.g., [Gottschalk and Moffitt \(1994, 2009\)](#), [Meghir and Pistaferri \(2004\)](#), [Blundell, Pistaferri, and Preston \(2008\)](#), [Browning, Ejrnæs, and Álvarez \(2010\)](#), [Altonji, Smith, and Vindangos \(2013\)](#); see [Blundell, Bollinger, Hokayem, and Ziliak \(2024\)](#) for a comprehensive review. Within that body of work, our paper is most closely related to [Storesletten et al. \(2004\)](#), [Guvenen et al. \(2014\)](#), [Arellano et al. \(2017\)](#), [Guvenen, McKay, and Ryan \(2023\)](#), [Halvorsen, Holter, Ozkan, and Storesletten \(2024\)](#), [Guvenen, Pistaferri, and Violante \(2022\)](#), and the multi-country GRID project. Relative to this work, we are the first to develop a framework capable of integrating aggregate shocks and rich nonlinear dynamics at the micro level.

Second, we contribute to the literature on estimating heterogeneous agents models using micro data ([Arellano and Bonhomme, 2017](#); [Liu and Plagborg-Møller, 2023](#); [Fernández-Villaverde, Hurtado, and Nuño, 2023](#)). Compared to them, we offer a principled approach to building nonlinear reduced forms for structural models when agents face potentially latent macro-level uncertainty. More generally, we add to an early literature on the combination of household survey data with time series data ([Tobin, 1950](#); [Chetty, 1968](#); [Maddala, 1971](#)) and to recent work on the econometrics of models with aggregate shocks ([Hahn, Kuersteiner, and Mazzocco, 2020](#); [Almuzara and Sancibrián, 2024](#)) by developing novel tools for identification analysis and estimation in a time series of panels framework.

An important reference is [Chang, Chen, and Schorfheide \(2024\)](#), who propose functional vector autoregression methods to combine macro aggregates with repeated cross-

sections. In a setup with both macro and micro data, the effects of macro shocks at the unit level and their impact on cross-sectional distributions are distinct but related empirical objects. While we focus on the former, [Chang et al. \(2024\)](#) aim at the latter. From this perspective, our papers are complementary. Another recent related reference is [Sargent and Selvakumar \(2025\)](#), who propose a dynamic mode decomposition method to study distributional dynamics of income and consumption. Compared to both papers, a distinctive feature of our work is the use of panel data to analyze household dynamics.

Lastly, our empirical results speak to a vast macro literature on inequality and aggregate fluctuations (e.g., [Krusell and Smith, 1998](#); [Krueger, Mitman, and Perri, 2016](#); [Ahn, Kaplan, Moll, Winberry, and Wolf, 2018](#); [Bhandari et al., 2021](#)), and to work that seeks to quantify the welfare cost of business cycles ([Lucas, 1987, 2003](#); [Storesletten, Telmer, and Yaron, 2001](#); [Otrok, 2001](#); [Barlevy, 2004](#); [Galí, Gertler, and López-Salido, 2007](#); [Krebs, 2007](#)). In particular, we document new empirical patterns about the interaction between cyclical variation and nonlinearities in the income processes that have the potential to amplify the welfare consequences of recessions and expansions.

Outline. The paper is organized as follows. Section 2 outlines our framework of analysis and the key objects of empirical interest. Section 3 introduces the statistical population by means of a time series of panels and studies identification. Section 4 details our estimation strategy. In Section 5, we present empirical results on nonlinear persistence, exposure to aggregate shocks, and skewness over the business cycle. Section 6 develops methodology for nonlinear impulse response functions of macro and micro shocks, while Section 7 focuses on quantifying the idiosyncratic and aggregate components of income risk. Section 8 concludes. Additional material can be found in the Supplemental Appendix.

2 Framework

In this section we describe our framework of analysis that combines time series aggregates with longitudinal micro-level survey data. While our interest is in the impact of macro shocks on nonlinear income risk, our approach provides a basis to build rich nonlinear reduced forms for heterogeneous agents models with both macro and micro uncertainty.

2.1 Model

We model log income y_{it} of household i at time t as the sum of a persistent component η_{it} and a transitory component ε_{it} ,

$$y_{it} = \eta_{it} + \varepsilon_{it}.$$

While the researcher observes log income y_{it} , the two components η_{it} and ε_{it} are latent.³

Following [Arellano et al. \(2017, ABB\)](#), the persistent component is a flexible first-order Markov process, the transitory component is serially independent (using biennial observations), and both processes may depend on covariates x_{it} such as age. However, unlike ABB, our focus is on understanding how aggregate conditions impact income trajectories. For this purpose, we introduce a time series aggregate Z_t , a macro state variable that affects η_{it} and ε_{it} , and that we will infer from aggregate time series data.

Persistent component. We specify the persistent component as

$$\eta_{it} = Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}, u_{it}). \quad (1)$$

In (1), Q_η is strictly increasing in its last argument and the shock u_{it} is i.i.d. uniform on $(0, 1)$, independent of $(\eta_{i,t-1}, Z_t, Z_{t-1}, x_{it})$. Hence, for all $\tau \in (0, 1)$, $Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}, \tau)$ is the conditional τ -quantile of η_{it} . This model allows for a general nonlinear relationship between the persistent income component η_{it} and its various determinants: lagged income $\eta_{i,t-1}$, the aggregate factor Z_t , covariates x_{it} , and idiosyncratic shocks u_{it} .

To aid interpretation, it is useful to consider the following special case of Equation (1):

$$\eta_{it} = \rho\eta_{i,t-1} + \gamma\eta_{i,t-1}Z_t + \delta Z_t + \underbrace{g(Z_t, u_{it})}_{=\zeta_{it}}, \quad (1')$$

where for simplicity we have omitted the dependence on Z_{t-1} . The quantity $\zeta_{it} = g(Z_t, u_{it})$ is a *composite income shock*, measured in log income units, which exhibits aggregate variation due to the presence of Z_t and idiosyncratic variation driven by u_{it} .

In model (1'), the impact of a marginal change in the aggregate state Z_t on the persistent

³We build on a long tradition that uses panel data to decompose income into persistent and transitory components ([Hause, 1977](#); [Lillard and Willis, 1978](#); [MaCurdy, 1982](#)); see [Arellano \(2014\)](#) for a survey.

component of income (the household's aggregate exposure β_{it}) can be decomposed as

$$\frac{\partial \eta_{it}}{\partial Z_t} = \underbrace{\gamma \eta_{i,t-1} + \delta}_{\text{income heterogeneity}} + \underbrace{\frac{\partial g(Z_t, u_{it})}{\partial Z_t}}_{\text{shock distribution}}.$$

The first term shows that the impact of an aggregate shock can vary along the income distribution, a point highlighted in an extensive literature in macroeconomics; the second shows that aggregate shocks may alter the distribution of income shocks, e.g., by affecting their variance (as in [Storesletten et al., 2004](#)) or skewness (as in [Guvenen et al., 2014](#)).⁴

Compared to (1'), which permits interactions between Z_t and past income $\eta_{i,t-1}$ and between Z_t and the shock u_{it} , model (1) allows for a third type of interaction between past income $\eta_{i,t-1}$ and the shock u_{it} . Model (1) is therefore better able to capture nonlinearities in income persistence, including the observation from ABB that different shocks to the persistent component may be associated with different degrees of persistence. This is in addition to the rich life-cycle patterns and the unrestricted relationship between macro and micro responses that model (1) can generate. The latter plays a key role in empirically capturing the shape of individual impulse responses to aggregate shocks.

Transitory component and initial condition. We specify the transitory component as

$$\varepsilon_{it} = Q_{\varepsilon,t}(x_{it}, v_{it}), \quad (2)$$

with $Q_{\varepsilon,t}$ strictly increasing in its last argument, and v_{it} an i.i.d. shock, uniform on $(0, 1)$ and independent of x_{it} and u_{it} . The function $Q_{\varepsilon,t}$ may vary over time in unrestricted ways reflecting general aggregate effects (coming from Z_t or other factors) on transitory income shocks. Our specification can also accommodate non-Gaussianity in the density of transitory shocks. In practice, ε_{it} will potentially be a mix of substantive transitory shocks and measurement error, and without further assumptions our approach will not allow us to distinguish between the two. For this reason, in our empirical analysis we will mostly focus on interpreting the properties of the persistent component.

⁴For a special case consider $g(Z_t, u_{it}) = \sigma(Z_t)q(u_{it})$, where $\frac{\partial g(Z_t, u_{it})}{\partial Z_t} = \frac{\partial \sigma(Z_t)}{\partial Z_t} q(u_{it})$ reflects the impact of Z_t on the variance of the composite shock distribution $\sigma^2(Z_t)$.

Lastly, we specify the initial condition of the persistent income process as

$$\eta_{i,t_0} = Q_{\text{init},t_0}(x_{i,t_0}, v_{i,t_0}),$$

with Q_{init,t_0} strictly increasing in its last argument, and v_{i,t_0} uniformly distributed on $(0, 1)$, and independent of x_{i,t_0} , u_{it} and v_{it} . With a similar rationale as for $Q_{\varepsilon,t}$, we let Q_{init,t_0} depend flexibly on the initial time period t_0 (which may differ across individuals).

Since we permit general time-variation in both the transitory component and the initial condition, other factors beyond the business-cycle state Z_t can influence the evolution of income in our model. Thus, as $Q_{\varepsilon,t}$ and Q_{init,t_0} reflect a mix of business-cycle and other factors, we will interpret them as rich time-varying “controls”, our chief goal being instead to document the dynamics of the persistent component captured by Q_η .

Macro state variable. The unobservable aggregate state Z_t is estimated from observables W_t by relying on a linear dynamic factor structure:

$$\begin{aligned} W_t &= \Lambda Z_t + e_t, \\ Z_t &= \Phi Z_{t-1} + \sigma_V V_t = Q_Z(Z_{t-1}, V_t), \end{aligned} \tag{3}$$

with V_t an i.i.d. standard normal vector of shocks independent of e_t at all lags and leads, and with the elements of e_t specified as mutually independent Gaussian autoregressive processes. In this formulation, the aggregate state Z_t will typically display persistence, reflecting the dynamic impact of current and past aggregate shocks $\{V_{t-\ell}\}_{\ell \geq 0}$.⁵

In our empirical analysis, W_t consists of variables that are informative about aggregate fluctuations (e.g., GDP and the unemployment rate) while Z_t is a parsimonious summary indicator of the state of the business cycle.⁶ One interpretation of Z_t draws on [Angeletos, Collard, and Dellas \(2020\)](#) who find that a single serially uncorrelated process, what they call the *main business cycle* (MBC) shock, accounts for the largest share of the unpredictable variation in each of the variables W_t at business cycle frequencies. This MBC shock is not a specific structural shock (e.g., TFP or demand), but possibly a mix of shocks that produce

⁵Since Z_t can be a vector, the restriction to a first-order process in (3) is without loss of generality—any $\text{VAR}(p)$ can be cast as a $\text{VAR}(1)$ in companion form. Our setting also accommodates richer processes with heteroskedasticity and nonlinearities, provided Q_Z can be identified from the time series W_t (see Section 3).

⁶The idea of extracting a low-dimensional summary of the covariation of multiple economic time series is at the core of the concept of business cycle ([Burns and Mitchell, 1946](#)). The use of linear factor models for that purpose also has a long history in economics; see [Stock and Watson \(2016\)](#) for a survey.

consistent impulse-response patterns across cyclical variables. In that light, we can view the shock V_t in (3) as an approximation to the MBC shock, and the business cycle state Z_t as a persistent process that captures the cumulative effect of MBC shocks over time.⁷

Macro-micro feedback. We can view Equations (1) and (3) as a nonlinear VAR(1) model for $\{\eta_{it}\}_i$ and Z_t , where i indexes the relevant time- t cross-sectional population—the precise notion is discussed in Section 3. What remains is to specify the link between the macro and micro sides. We impose two restrictions formalized in Assumption 3 below.

First, conditional on Z_t , the aggregate data W_t carry no extra information about shocks u_{it} to the dynamics of η_{it} . This is analogous to Liu and Plagborg-Møller (2023, Assumption 1) and implies that Z_t is a sufficient statistic of the macro data for micro-level persistent dynamics. Second, we impose that individual past income shocks do not affect Z_t . It is important to highlight here that our model does not rule out feedback from the micro to the macro side. We can accommodate such feedback provided it is channeled through aggregate summaries of the micro state distribution. In that case, one could augment W_t with moments from the time- t earnings distribution and redefine Z_t accordingly.⁸ Instead, what we require is that no single unit in the sample has aggregate effects, that is, that units are *atomistic*. In Supplemental Appendix A, we use a stylized example to illustrate how the equilibrium conditions from standard heterogeneous agents models with aggregate shocks can be cast in the form of (1) and (3), and how atomicity is to be interpreted.

To summarize, our framework can capture rich aggregate dynamics and macro-micro interactions. The limit is statistical rather than conceptual: as shown in Section 4, estimation precision depends on the time series length T , and a high-dimensional Z_t can quickly exhaust the degrees of freedom offered by a short time series.

2.2 Objects of empirical interest

A primary goal of the empirical analysis is to quantify the effect of aggregate shocks on nonlinear income processes. Our framework allows us to recover several quantities of

⁷One difference is that Angeletos et al. (2020) recover the MBC shock by maximizing the forecast error variance decomposition of W_t over a band of business-cycle frequencies, whereas we recover our shock V_t (and the state Z_t) by first applying a band-pass filter to W_t (tailored to business-cycle frequencies) and then using dynamic factor techniques; see Section 5.1. Empirically, IRFs of W_t to V_t are quantitatively similar to IRFs to the MBC shock, albeit slightly less persistent. See Supplemental Appendix F.3 for a comparison.

⁸It is worth highlighting that our framework can also accommodate sectoral and regional variation by including sector-level or region-level aggregate factors in Z_t and measurements in W_t .

interest. A first quantity is the following measure of *nonlinear persistence* which extends the ABB persistence to a setup with aggregate shocks:

$$\rho(u_{it}, \eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}) = \frac{\partial Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}, u_{it})}{\partial \eta_{i,t-1}}. \quad (4)$$

That is, persistence is measured by how a change in the micro state $\eta_{i,t-1}$ affects its next-period value. In a linear autoregressive model, $\rho(u_{it}, \eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}) = \rho$ is constant and equal to the autoregressive root of the process. In contrast, in our nonlinear process, this measure is state-dependent: persistence may vary with the past position in the income distribution $\eta_{i,t-1}$, with aggregate conditions Z_t and Z_{t-1} , and with covariates (e.g., age) x_{it} .

A key feature of the measure $\rho(u_{it}, \eta_{i,t-1}, Z_t, Z_{t-1}, x_{it})$ is that it captures how persistence varies with the micro shock u_{it} , and how u_{it} interacts with the remaining determinants of persistent income. In micro panels, a robust finding (first documented in ABB) is that persistence decreases for good-shocks/low- η and bad-shocks/high- η combinations. This reflects the fact that a good shock arriving in a low-income state has sometimes the power to erase a bad income history; the unlucky reverse holds for high- η households reached by a bad u . This feature is absent from linear income processes. But the novel framework of this paper adds an extra layer. In (4), the entire shape of ρ as a function of u_{it} and $\eta_{i,t-1}$ —the intensity with which past income histories are wiped out by big shocks—can change with the aggregate state of the economy Z_t . Our empirical analysis in Section 5 presents evidence of business cycle variation in nonlinear persistence.

A second key quantity is the persistent income *nonlinear exposure to aggregate shocks*:

$$\beta(u_{it}, \eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}) = \frac{\partial Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}, u_{it})}{\partial Z_t}. \quad (5)$$

This measure captures how the exposure of a household to the aggregate state varies with the determinants of persistent income η_{it} . In particular, the aggregate exposure may vary along the persistent income distribution $\eta_{i,t-1}$, over the life-cycle x_{it} , or as a function of the idiosyncratic shock the household is hit with u_{it} . It may also display cyclical patterns as it depends on the aggregate states Z_t and Z_{t-1} . It is worth noting that this measure is not available in models of income risk that exclude macro shocks, and in most specifications that do include aggregate uncertainty, it is typically restricted to be constant, thus ruling out cyclical exposures and interactions with micro-level states and shocks.

A third quantity of interest is the following measure of *conditional skewness*:

$$\begin{aligned} \text{sk}(\eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}) &= \left[Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}, 0.9) - Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}, 0.1) \right]^{-1} \\ &\times \left[Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}, 0.9) + Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}, 0.1) - 2Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, x_{it}, 0.5) \right]. \end{aligned} \quad (6)$$

This is the Kelley skewness of the predictive distribution of η_{it} . In micro panels, skewness tends to be decreasing in past $\eta_{i,t-1}$, that is, income risk is tilted to the upside for the low- η and to the downside for the high- η . As for persistence, the novelty of our paper is to allow us to trace how skewness changes with the aggregate state of the economy Z_t (and Z_{t-1}). How skewness varies with Z_t is a paramount empirical question. [Güvenen et al. \(2014\)](#), for example, find that recessions have a sizable negative impact on the skewness of one-year income growth. Our framework enhances previous analyses in various directions. First, it allows us to quantify the cyclical patterns of persistent income risk directly, without the need to proxy it by income changes that complicate imputing the effect of specific macro shocks. Second, it allows us to explore the role of the household's position in the income distribution and in the life cycle. Thus, our approach provides a unified treatment of the two types of income skewness that have been highlighted in the literature: across the income distribution (indexed by $\eta_{i,t-1}$) and over the business cycle (indexed by Z_t).⁹

Beyond exploring the cyclical behavior of nonlinear income processes, our setup allows us to recover impulse response functions (IRF) to both macro shocks (i.e., a shock to Z_t such as a recession or boom) and micro shocks (i.e., a shock u_{it} such as a promotion or demotion), as we discuss in Section 6. Finally, our framework allows us to quantify the contribution of macro and micro shocks to the cost of income risk, the topic of Section 7.

3 Statistical population and identification

Here we describe the statistical population concept for our framework which is based on what we refer to as a *time series of panels*. Specifying the data generating process involves two challenges: allowing for potentially omitted aggregate shocks and accounting for the fact that the cross-sectional population under observation changes over time. This section focuses on identification and we defer estimation and inference until Section 4.

⁹ Similarly, we can define measures of conditional dispersion and kurtosis. We report estimates of these quantities in Supplemental Appendix E.

3.1 Time series of panels

We begin by introducing the notion of time series of panels, a data structure whose infinite-sample counterpart is the statistical population of our setting. The researcher observes a time series of aggregate indicators

$$W_t, \quad t = 1, \dots, T + S,$$

which we use to measure the aggregate factor Z_t and a sequence of panel datasets

$$Y_t, \quad t = 1, \dots, T,$$

where each $Y_t = \{y_{i,t+s} : i \in \mathcal{I}_t, 0 \leq s \leq S - 1\}$ contains individual observations on units in a sample \mathcal{I}_t of size N_t , representative of the population of interest at time t . The length of each panel, S , aims at preserving the representativeness of the samples \mathcal{I}_t while ensuring identification of the parameters of the model.¹⁰ Our approach relies on large cross-sections (large N_t), short subpanels (small and fixed S) and a long time series spanning sufficient aggregate variation coexisting with an evolving population of micro units (large T).

An advantage of our time series of panels approach, compared to standard long panel approaches, is that it mitigates concerns over the representativeness of the data when attrition depends on income shocks or covariates (such as age). For example, a long panel of households spanning the years 1970 to 2019 (as in our empirical analysis) will over-represent more stable units with higher incomes compared to an actually representative sample in any given year. A precursor of the time series of panels is the analysis of cyclical income risk in the U.S. by [Storesletten et al. \(2004\)](#).¹¹

We are now ready to describe the statistical population. For conciseness we use $\{A_t\}$ as shorthand for the time series process $\{A_t : -\infty < t < \infty\}$.

Assumption 1 (Macro states). *There is a macro stochastic process $\{Z_t, e_t, G_t, \omega_t\}$ such that*

¹⁰This structure nests several important cases. First, a rotating panel scheme, where a share $1/S$ of individuals is refreshed every period, is covered by our setup, since an individual may appear in several subpanels; i.e., $i \in \mathcal{I}_t$ neither precludes nor implies $i \in \mathcal{I}_\tau$ for $\tau \neq t$. Second, repeated cross-sectional data corresponds to the case $S = 1$ with T equal to the number of cross-sections. Third, a single long panel corresponds to $T = 1$ and S equal to the number of time periods—alternatively, a long panel can always be split into an overlapping sequence of shorter panels.

¹¹An additional advantage of this approach is that it allows us to handle covariates that would otherwise be collinear with time, e.g., age. Moreover, it may be expensive (sometimes impossible) to keep track of the same units (e.g., companies) for very long periods.

- (a) $\{Z_t\}$ is stationary and satisfies (3) with V_t i.i.d. over t and standard normal;
- (b) the entries of $\{e_t\}$ and $\{G_t\}$ are mutually independent and independent of $\{Z_t\}$, with e_t stationary and Gaussian, and with G_t i.i.d. over t ; and
- (c) $\{\omega_t\}$ is a process of potentially arbitrary dimension that encompasses Z_t , e_t and G_t , as well as any other time series factors that may affect the evolution of the micro data (e.g., through ε_{it}).

Assumption 2 (Micro processes with macro states). *There is a cross-section of micro stochastic processes $\{\eta_{it}, \varepsilon_{it}, x_{it}\}$ such that*

- (a) $\{\eta_{it}, \varepsilon_{it}, x_{it}\}$ is i.i.d. over i given $\{\omega_t\}$; and
- (b) $\{\eta_{it}, \varepsilon_{it}\}$ given $\{\omega_t, x_{it}\}$ satisfies (1) and (2) with u_{it} and v_{it} mutually independent, i.i.d. over t , and uniformly distributed on $(0, 1)$.

Assumption 3 (Atomicity). *Conditional on x_{it} and G_t , for each unit i , the persistent micro shock u_{it} is independent of the macro shocks V_t and the errors e_t at all lags and leads.*

In Assumptions 1, 2 and 3, the process G_t represents potentially unobserved or omitted aggregate variables, such as shocks orthogonal to the business cycle or survey-specific factors. We assume that G_t is independent over time, and that Z_t (the aggregate of interest) is unaffected by G_t , while assuming that W_t is uninformative about G_t . This extends to our nonlinear setup the standard shock orthogonality idea from linear macroeconomic models (e.g., Ramey, 2016). The process ω_t contains Z_t , e_t , G_t , and any other aggregate factor affecting ε_{it} and the initial η_{it} .

The remaining parts of Assumptions 1 and 2 impose (1), (2) and (3), while Assumption 3 requires that no single unit has influence on the aggregate. Nonetheless, as discussed in Section 2, our framework permits feedback from the micro to the macro side of the model, as summaries of micro distributions may be included in Z_t .

3.2 Identification

The identification problem is to determine the unknown functions Q_η and Q_Z from the probability distribution of observables. As anticipated, our argument applies to $N_t, T \rightarrow \infty$ with S fixed. In our setup, the distribution of observables is the distribution P^S that assigns probabilities to events involving finite segments of the stochastic process $\{\xi_t^S\}$

where $\xi_t^S = (\{W_{t+s-1}\}_{s=0}^S, F_t^S)$, for F_t^S the cumulative distribution function (CDF) of $\{y_{i,t+s}\}_{s=0}^{S-1}$ given $\{x_{i,t+s}\}_{s=0}^{S-1}$ at time t , that is,

$$F_t^S(\mathbf{y}|\mathbf{x}) = P\left((y_{it}, \dots, y_{i,t+S-1}) \leq \mathbf{y} \mid (x_{it}, \dots, x_{i,t+S-1}) = \mathbf{x}, \omega_t \right).$$

F_t^S is a random function that describes cross-sectional probabilities of $\{y_{i,t+s}, x_{i,t+s}\}_{s=0}^{S-1}$. The randomness in this function comes from the time series process $\{\omega_t\}$ of potentially arbitrary dimension and unknown to the researcher introduced in Assumption 1(c).

We next outline our identification analysis which brings together techniques from the macro- and microeconomic literatures, linked by the time series of panels.

Identification of the macro process. Knowing P^S implies knowing the distribution of the stochastic process $\{W_t\}$ and, in particular, the autocovariance function $\ell \mapsto \text{Cov}(W_t, W_{t-\ell})$. In our linear Gaussian environment, this suffices to identify Q_Z provided there are enough measurements and a factor normalization holds. Proofs are in Supplemental Appendix B.

Proposition 1. *Under Assumption 1, if (a) $(\dim(W_t) - \dim(Z_t))^2 > \dim(W_t) + \dim(Z_t)$ and (b) the upper block of Λ is the identity matrix, then Q_Z is identified.*

This follows from standard results for linear state-space models. We note that condition (b) could be replaced by alternative factor normalizations (Stock and Watson, 2016). Although in more general nonlinear and non-Gaussian settings a different technique will be needed, it is often possible to learn about Q_Z using external time series data alone. On the other hand, when Z_t is directly observed (as in, e.g., Storesletten et al., 2004 where Z_t is the NBER recession indicator), this step is not needed as Q_Z is automatically identified.

Identification of the micro process. For a given t , knowledge of F_t^S allows the researcher to pin down the latent variable distributions that condition on the unknown macro shocks coincidental with the time- t panel. This is a microeconomic identification problem.

Proposition 2. *Suppose $S \geq 4$ and almost surely over realized paths of $\{\omega_t\}$, for each t ,*

- (a) *the density of $\{y_{i,t+s}, \eta_{i,t+s}\}_{s=0}^{S-1}$ given $\{x_{i,t+s}\}_{s=0}^{S-1}$ and ω_t is bounded away from zero and infinity and so are all corresponding marginals and conditionals, and*
- (b) *with $f_{\eta_r|y_r, x, t}$ the density of η_{ir} given y_{ir} , $\{x_{i,t+s}\}_{s=0}^{S-1}$ and ω_t , the families*

$$\mathcal{F}_-^{r,t}(\mathbf{x}) \equiv \left\{ f_{\eta_r|y_{r-1}, x, t}(\cdot | \mathbf{y}, \mathbf{x}) : \mathbf{y} \in \mathbb{R} \right\} \text{ and } \mathcal{F}_+^{r,t}(\mathbf{x}) \equiv \left\{ f_{\eta_r|y_{r+1}, x, t}(\cdot | \mathbf{y}, \mathbf{x}) : \mathbf{y} \in \mathbb{R} \right\}$$

are complete for $t < r < t + S - 1$ and for all x .

Let \bar{P}^S be the probability distribution of the vector/function-valued stochastic process $\{\bar{\xi}_t^S\}$ where $\bar{\xi}_t^S = (\{W_{t+s-1}\}_{s=0}^S, \{F_{\eta,t+s,t}\}_{s=1}^{S-2})$ with

$$F_{\eta,r,t}(\bar{\eta}|\eta, x) = P(\eta_{ir} \leq \bar{\eta} | \eta_{i,r-1} = \eta, x_{ir} = x, \omega_t).$$

Then, under Assumption 2, \bar{P}^S is identified.

Under the conditions of Proposition 2, given a path for the aggregate shocks, there is a known injective mapping from the joint CDF of observables F_t^S to the latent variable CDFs $\{F_{\eta,t+s,t}\}_{s=1}^{S-2}$. This is established as in ABB using the spectral decomposition techniques of Hu and Schennach (2008) and Wilhelm (2015). The *completeness* assumption stated in part (b), a nonparametric analog of the rank conditions often used in identification arguments, is instrumental for this result. Proposition 2 then simply asserts that the researcher can determine from P^S the joint distribution \bar{P}^S of those CDFs and the macro data $\{W_{t+s-1}\}_{s=0}^S$.

A by-product of this operation is the identification of the transitory component CDF, $F_{\varepsilon,t+s,t}(\varepsilon|x) = P(\varepsilon_{i,t+s} \leq \varepsilon | x_{i,t+s} = x, \omega_t)$ for $1 \leq s \leq S - 2$. Under the cross-panel consistency requirement $F_{\varepsilon,t,t} = F_{\varepsilon,t,t-1}$, the CDF $F_{\text{init},t}(\eta|x) = P(\eta_{it} \leq \eta | x_{it} = x, \omega_t)$ is also identified. This is a mild condition asking the transitory income distribution of the populations of two consecutive panels to agree *within* a given period, and it is compatible with unrestricted nonstationarity *across* periods.

It is important to note that Proposition 2 gives sufficient conditions for identification, but it may still be possible to recover the object of interest under a different set of assumptions and a different value of S . There are many other models of micro-level dynamics for which a large enough S enables identification of time-specific latent variable CDFs using micro data alone.¹² The challenge is linking them to the macro-level states of interest.

Proposition 3. *Let Assumptions 1, 2, 3, and the conditions of Propositions 1 and 2 hold. With $f_{z_r|W_t^S}$ the density of (Z_r, Z_{r-1}) given $W_t^S = (W_t, \dots, W_{t+S-1})$ suppose that the family*

$$\mathcal{F}_{z_r|W_t^S} \equiv \left\{ f_{z_r|W_t^S}(\cdot|W) : W \in \mathbb{R}^{\dim(W_t)S} \right\}$$

¹²For example, if given macro shocks, y_{it} follows a canonical income process where η_{it} is a random walk, one needs $S \geq 3$. If, instead, y_{it} features heterogeneity in transitory shocks (as in Chamberlain and Hirano, 1999 or Almuzara, 2020), one needs $S \geq 5$. This highlights a key advantage of time series of panels over time series of cross-sections: Observing the same unit over a number of periods makes it possible to separate permanent from transitory components directly by exploiting information on individual transitions.

is complete for some $t < r < t + S - 1$. Then, Q_η is identified.

Our identification analysis clarifies the role played by each element in our framework: large cross-sections help recover the time- t short-panel distribution F_t^S ; the subpanel length S balances the ability to identify latent variable distributions $\{F_{\eta,r,t}\}$ with the credibility of sample representativeness; finally, long time series of both external macro data and micro data allow the researcher to link the time- t cross-sections to the latent aggregate of interest while averaging out the impact of omitted macro factors.

4 Empirical specification and estimation

In this section we specify a flexible class of parametric models for our framework and outline a suitable estimation technique. Our empirical strategy relies on series expansions of the functions Q_η , $Q_{\varepsilon,t}$ and $Q_{\text{init},t}$ to create nonlinear descriptions of income processes. We build on the stochastic pseudo-likelihood EM method of [Arellano and Bonhomme \(2016\)](#) and its extension to time-varying latent variables in ABB, further extending it to a long time series of short panels involving both macro and micro latent variables.

Empirical specification. For the conditional quantile function of η_{it} we set

$$Q_\eta(\eta, \tilde{Z}, Z, x, u) = \psi(\eta, x)' \Theta(u) \varphi(\tilde{Z}, Z), \quad (7)$$

where ψ, φ are vectors of known basis functions (such as orthogonal polynomials) and Θ is a matrix of linear splines with nodes $(\bar{u}_1, \dots, \bar{u}_L)$ determined by the parameter vector θ . The vector θ shapes (through Q_η) the measures of persistence, aggregate exposure, dispersion and skewness we introduced in Section 2, and the IRFs and risk decompositions we will introduce in Sections 6 and 7. The task is to estimate θ .

For the transitory component and for the base-period η_{it} we set

$$Q_{\varepsilon,t}(x, u) = \psi_\varepsilon(x)' \Delta_{\varepsilon,t}(u), \quad (8)$$

$$Q_{\text{init},t}(x, u) = \psi_{\text{init}}(x)' \Delta_{\text{init},t}(u), \quad (9)$$

where $\psi_\varepsilon, \psi_{\text{init}}$ are vectors of known basis functions and, for each t , $\Delta_{\varepsilon,t}, \Delta_{\text{init},t}$ are vectors of linear splines with nodes $(\bar{u}_1, \dots, \bar{u}_L)$ determined by the parameter vectors $\delta_{\varepsilon,t}, \delta_{\text{init},t}$. As stated in Section 2, the quantile functions $Q_{\varepsilon,t}$ and $Q_{\text{init},t}$ deliver the distribution of ε_{it} and of

η_{it} at the beginning of each subpanel conditioned on time effects. Specifying $Q_{\varepsilon,t}$ and $Q_{\text{init},t}$ as flexible functions of time allows us to absorb long-term trends and other unmodeled sources of non-stationarity in the data.¹³

An arbitrarily good approximation to the true Q_η , $Q_{\varepsilon,t}$ and $Q_{\text{init},t}$ can be obtained by a suitable choice of basis functions for sufficiently regular classes as in nonparametric series methods. Instead, we adopt a flexible parametric perspective in which θ , $\delta_{\varepsilon,t}$ and $\delta_{\text{init},t}$ are finite-dimensional. While the identification argument of Section 3 covers nonparametric models, the precision with which the objects of interest can be estimated is limited by the time series length T , or more concretely, by the scarcity of recessions and booms in the sample. In that context, our approach balances flexibility with statistical power.

Moments. Our model implies complete-data moments involving $\bar{y}_{it}^S = \{y_{i,t+s}, x_{i,t+s}\}_{s=0}^{S-1}$, $\bar{\eta}_{it}^S = \{\eta_{i,t+s}\}_{s=0}^{S-1}$ and $\bar{Z}_t^S = \{Z_{t+s}\}_{s=0}^{S-1}$ that pin down the true θ and $\delta_t = (\{\delta_{\varepsilon,t+s}\}_{s=0}^{S-1}, \delta_{\text{init},t})$,

$$E\left[m_\theta\left(\theta_0; \bar{y}_{it}^S, \bar{\eta}_{it}^S, \bar{Z}_t^S\right)\right] = 0, \quad E\left[m_\delta\left(\delta_{0t}; \bar{y}_{it}^S, \bar{\eta}_{it}^S\right)\right] = 0. \quad (10)$$

These are moments from quantile and exponential regressions (see Supplemental Appendix C.1), and they are infeasible as they depend on macro and micro latent variables.

Pseudo posteriors. To operationalize our approach, we need to transform the infeasible moments (10) into feasible moments that depend only on observable data. We do so in two steps by sequentially integrating out the unobserved latent variables against a convenient choice of pseudo posterior distributions. Specifically, in a first step we integrate η_{it} out with respect to its (unit-level) posterior density given the micro data y_{it} and x_{it} and the aggregate state Z_t . In a second step, we integrate Z_t out with respect to its aggregate posterior density given W_t . Proceeding in this way has the advantage of clarifying the role of micro and macro components for estimation, in analogy to our identification analysis. It also leads to a tractable numerical implementation that draws on known algorithms from microeconomic and macroeconomic traditions.

Let f be a generic probability density function (PDF). In a first step, we define the (still

¹³Following ABB we model tails as conditionally exponential, such as

$$Q_\eta(\eta, \tilde{Z}, Z, x, u) = \begin{cases} Q_\eta(\eta, \tilde{Z}, Z, x, \bar{u}_1) - \exp\left(\psi_{\text{lo}}(\eta, \tilde{Z}, Z, x)' \theta_{\text{lo}}\right) \ln\left(\frac{\bar{u}_1}{u}\right) & \text{if } u < \bar{u}_1 \\ Q_\eta(\eta, \tilde{Z}, Z, x, \bar{u}_L) + \exp\left(\psi_{\text{up}}(\eta, \tilde{Z}, Z, x)' \theta_{\text{up}}\right) \ln\left(\frac{1-\bar{u}_L}{1-u}\right) & \text{if } u > \bar{u}_L \end{cases}$$

where $\psi_{\text{lo}}, \psi_{\text{up}}$ are vectors of known basis functions and $\theta_{\text{lo}}, \theta_{\text{up}}$ are unknown parameters included in θ .

infeasible) partial-data moments, for hypothetical values $\theta, \theta', \delta_t, \delta_t'$,

$$\begin{aligned}\bar{m}_\theta(\theta; \theta', \delta_t', \bar{y}_{it}^S, \bar{Z}_t^S) &= \int m_\theta(\theta; \bar{y}_{it}^S, \bar{\eta}^S, \bar{Z}_t^S) f(\bar{\eta}^S | \bar{y}_{it}^S, \bar{Z}_t^S, \theta', \delta_t') d\bar{\eta}^S, \\ \bar{m}_\delta(\delta_t; \theta', \delta_t', \bar{y}_{it}^S, \bar{Z}_t^S) &= \int m_\delta(\delta_t; \bar{y}_{it}^S, \bar{\eta}^S) f(\bar{\eta}^S | \bar{y}_{it}^S, \bar{Z}_t^S, \theta', \delta_t') d\bar{\eta}^S.\end{aligned}$$

The conditional density $f(\bar{\eta}^S | \bar{y}_{it}^S, \bar{Z}_t^S, \theta, \delta_t)$ is the individual micro-level posterior from ABB but conditioning on aggregate states and time effects. It is fully determined via Bayes rule through our parametric model, and it can be efficiently sampled from using Sequential Monte Carlo techniques (Arellano, Blundell, Bonhomme, and Light, 2023, Section 4.3).

By (10) and the law of iterated expectations,

$$E\left[\bar{m}_\theta(\theta_0; \theta_0, \delta_{0t}, \bar{y}_{it}^S, \bar{Z}_t^S)\right] = 0, \quad E\left[\bar{m}_\delta(\delta_{0t}; \theta_0, \delta_{0t}, \bar{y}_{it}^S, \bar{Z}_t^S)\right] = 0, \quad (11)$$

which indicates the partial-data moments (11) provide valid restrictions on θ and δ_t .

Let λ be a vector containing the macro parameters. Write $\bar{Y}_t^S = \{\bar{y}_{it}^S\}_{i \in \mathcal{I}_t}$ and $\bar{W} = \{W_t\}_{t=1}^{T+S}$. In a second step, we define the aggregated observed-data moments,

$$\begin{aligned}\bar{M}_\theta(\theta; \theta', \delta_t', \lambda', \bar{Y}_t^S, \bar{W}) &= \int \left[\frac{1}{N_t} \sum_{i \in \mathcal{I}_t} \bar{m}_\theta(\theta; \theta', \delta_t', \bar{y}_{it}^S, \bar{Z}_t^S) \right] f(\bar{Z}^S | \bar{W}, \lambda') d\bar{Z}^S, \\ \bar{M}_\delta(\delta_t; \theta', \delta_t', \lambda', \bar{Y}_t^S, \bar{W}_t) &= \int \left[\frac{1}{N_t} \sum_{i \in \mathcal{I}_t} \bar{m}_\delta(\delta_t; \theta', \delta_t', \bar{y}_{it}^S, \bar{Z}_t^S) \right] f(\bar{Z}^S | \bar{W}, \lambda') d\bar{Z}^S.\end{aligned}$$

The conditional density $f(\bar{Z}^S | \bar{W}, \lambda)$ is the smoothing posterior from model (3), and it can be efficiently sampled from using the Kalman filter. Again, by (11) and iterated expectations,

$$E\left[\bar{M}_\theta(\theta_0; \theta_0, \delta_{0t}, \lambda_0, \bar{Y}_t^S, \bar{W})\right] = 0, \quad E\left[\bar{M}_\delta(\delta_{0t}; \theta_0, \delta_{0t}, \lambda_0, \bar{Y}_t^S, \bar{W})\right] = 0. \quad (12)$$

Functions \bar{M}_θ and \bar{M}_δ give restrictions on θ and δ_t (given λ) that only depend on observables.¹⁴ We rely on the sample counterpart of the moments in (12) for estimation.

¹⁴This is in the spirit of the unbiased likelihood estimate used by Liu and Plagborg-Møller (2023, Section 3.1) in a full-information Bayesian setup. Our approach is based on pseudo likelihood functions but allows for panel data, micro-level latent variables and potentially omitted aggregate shocks.

Stochastic EM implementation. The main challenge in exploiting (12) for estimation is to integrate the primitive moments against the pseudo posteriors of η_{it} and Z_t . We follow [Arellano and Bonhomme \(2016\)](#) and adopt a simulation-based approach. Let $\widehat{\lambda}$ be macro parameter estimates obtained from the macro data \overline{W} alone (e.g., maximum likelihood). We rely on the following stochastic EM algorithm to estimate θ and $\{\delta_t\}_{t=1}^T$, which iterates between simulation smoothing of macro and micro latent variables and running quantile and exponential regressions.

Algorithm 1. Initialize parameters $\widehat{\theta}^{(0)}$ and $\{\widehat{\delta}_t^{(0)}\}_{t=1}^T$. For $j = 1, \dots, J$, iterate between the following:

1) Pseudo-Stochastic E step:

(i) draw $\overline{Z}(j) = \{Z_t(j)\}_{t=0}^{T+S}$ from the macro posterior $f(\overline{Z} | \overline{W}, \widehat{\lambda})$,

(ii) independently over units i and subpanels t , draw $\overline{\eta}_{it}^S(j) = \{\eta_{i,t+s}(j)\}_{s=0}^{S-1}$ from the micro posterior $f(\overline{\eta}_{it}^S | \overline{y}_{it}^S, \overline{Z}^S(j), \widehat{\theta}^{(j-1)}, \widehat{\delta}_t^{(j-1)})$.

2) Pseudo M step:

(i) update the parameters to $\widehat{\theta}^{(j)}$ and $\{\widehat{\delta}_t^{(j)}\}_{t=1}^T$ by quantile and exponential regressions treating $\left\{ \left\{ \eta_{i,t+s}(j), y_{i,t+s}, x_{i,t+s}, Z_{t+s}(j) \right\}_{s=0}^{S-1} \right\}_{i \in \mathcal{I}_t}^T$ as data, based on (12).

For some $\mu \in (0, 1)$, set $\widehat{\theta} = (\mu J)^{-1} \sum_{j=(1-\mu)J}^J \widehat{\theta}^{(j)}$ and $\widehat{\delta}_t = \sum_{j=(1-\mu)J}^J \widehat{\delta}_t^{(j)}$.

We present algorithms to perform steps 1(i) and 1(ii) in Supplemental Appendix C.2. Our approach departs from full-information likelihood estimation in two directions. First, we use complete-data moments from quantile and exponential regressions instead of solving the score equations from the complete-data likelihood. This carries a significant computational simplification as quantile and exponential regressions are fast and stable to run, compared to the mostly intractable score equations of the model. Second, rather than smoothing latent variables $\{\{\overline{\eta}_{it}^S\}_{i \in \mathcal{I}_t}, Z_t\}_{t=1}^T$ using their full joint posterior given $\{\overline{Y}_t^S\}_{t=1}^T$ and \overline{W} , we use only certain slices of the posterior (akin to composite-likelihood methods): e.g., we integrate unit- i micro latent variables $\overline{\eta}_{it}^S$ conditioning on unit- i data \overline{y}_{it}^S rather than the full microdata $\{\overline{Y}_t^S\}_{t=1}^T$. This comes potentially at a cost in terms of asymptotic efficiency but it has the advantage that the micro-level pseudo posteriors do not require us to model the cross-sectional dependence induced by the omitted aggregate factors G_t .

Even though our focus is on income processes, it is important to highlight that this method offers a flexible, general-purpose way to estimate rich models of micro-level latent variables subject to macro shocks, and can be applied far more generally.

Large-sample properties. We next summarize statistical properties of $\widehat{\theta}$ and functions of $\widehat{\theta}$ which cover the objects in Section 2.2, and the IRFs and risk calculations we discuss later. See Supplemental Appendix C.3 for further discussion. Our asymptotic analysis assumes $N_t, T \rightarrow \infty$ with S fixed, and $T/N_t \rightarrow 0$. That is, the number of individuals in each subpanel and the number of subpanels is large while the length of each subpanel is small, reflecting our identification argument. We also assume that the time-series dimension is small relative to the cross-sectional dimension, as in our empirical setting.

Given the macro parameter estimate $\widehat{\lambda}$, Algorithm 1 delivers $\widehat{\theta}$ and the sequence $\{\widehat{\delta}_t\}_{t=1}^T$. In our empirical analysis, we are interested in objects that can be expressed as a smooth function $\gamma(\theta)$ of θ , for which it is natural to use the plug-in estimator $\widehat{\gamma} = \gamma(\widehat{\theta})$. Recall that θ_0 denotes the true value of θ and let $\gamma_0 = \gamma(\theta_0)$. Our main result is as follows (see Supplemental Appendix C.3):

Proposition 4. *Let Assumptions 1, 2, 3 hold with (7), (8), (9). As $N_t, T \rightarrow \infty$ with $T/N_t \rightarrow 0$,*

$$\sqrt{T}(\widehat{\theta} - \theta_0) \xrightarrow{d} N(0, \Sigma_{\theta_0}) \text{ and } \sqrt{T}(\widehat{\gamma} - \gamma_0) \xrightarrow{d} N(0, J_{\theta_0} \Sigma_{\theta_0} J_{\theta_0}'),$$

where Σ_{θ_0} is symmetric and positive semi-definite, and J_{θ_0} is the Jacobian of γ evaluated at θ_0 .

Proposition 4 implies that the precision of the estimator is determined by T , not by N_t . The main reason is that identification of θ relies on time series variation to average out the effect of the unobserved factors G_t .¹⁵ Instead, the role of cross-sectional variation is to help control the error coming from the estimation of the time-varying parameters δ_t .

Empirical implementation. For Q_η in (7), we model ψ as a third-order polynomial in η combined with a second-order polynomial in x , and we construct φ as a restricted second-order polynomial in (\widetilde{Z}, Z) : we exclude the linear term from interactions between η and x , and include the quadratic term only in the intercept. We set ψ_{lo} and ψ_{up} to second-order polynomials in η, x, \widetilde{Z} and Z without interactions. Regarding $Q_{\varepsilon,t}$ in (8) and $Q_{init,t}$ in (9) we allow their intercepts and tail coefficients to change over time. We also include in $Q_{init,t}$ a time-invariant second-order polynomial in x . The rank-space grid size is set to $L = 11$. For estimation, we run Algorithm 1 for 500 iterations, averaging the last 100 of them.

Inference. To gauge the statistical precision of our estimates below we report confidence intervals computed by a parametric bootstrap technique that exploits our model structure

¹⁵The convergence rate would be the same if Z_t were observable (e.g., the NBER recession indicator).

while accounting for (i) cross-sectional dependence induced by omitted aggregate factors G_t and (ii) unit-level dependence induced by the sampling design of the PSID. The specifics are discussed in Supplemental Appendix C.4.

5 Macro shocks and nonlinear income processes

We now present our empirical analysis of the effects of macro shocks on the nonlinear income process. We discuss the macro data and business cycle state in Section 5.1, followed by a description of our main source of micro data, the PSID, in Section 5.2. We then turn to the quantification of aggregate effects on measures of nonlinear persistence (Section 5.3), exposure to aggregate shocks (Section 5.4), and conditional skewness (Section 5.5).

5.1 Macro data and the aggregate state

We extract the aggregate state Z_t from a vector of macro observables W_t informative about the business cycle consisting of GDP, consumption, investment, the unemployment rate and hours worked. The data are quarterly and cover 1960Q1–2019Q4. GDP, consumption, investment and hours are measured in log per-capita terms while the unemployment rate is multiplied by -1 to make it procyclical as the other entries of W_t . All series are detrended by removing a two-sided 40-quarter low-pass filter from them.¹⁶

For estimation, we normalize the loading on GDP to one. Therefore, the business cycle process Z_t is measured as a log deviation from its per-capita growth trend. We specify Z_t and each entry of e_t as independent AR(2) processes and estimate the macro parameters λ by Gibbs sampling using a flat prior. Supplemental Appendix C.2 discusses the details. Figure 1 shows the time series evolution of W_t and estimates of Z_t .

Three takeaways from the figure are as follows. First, and not surprisingly, the variables included in W_t show strong comovements. As a result, filtering out Z_t is statistically easy, as evidenced by the tight 90% probability bands drawn around the estimate. Second, there is substantial cyclical variation in Z_t , and this remains true when we consider temporally aggregated versions of it. For example, Figure 1 marks with a diamond the estimates of Z_t for the fourth quarter of each year. They clearly retain a meaningful share of the aggregate variation we see at the quarterly frequency. This is important because income data at the

¹⁶The data come from Federal Reserve Economic Data (FRED). To construct W_t we took from [Angeletos et al. \(2020\)](#) the subset of variables most informative about the MBC shock, although our results are robust to alternative specifications that expand W_t with other macro indicators.

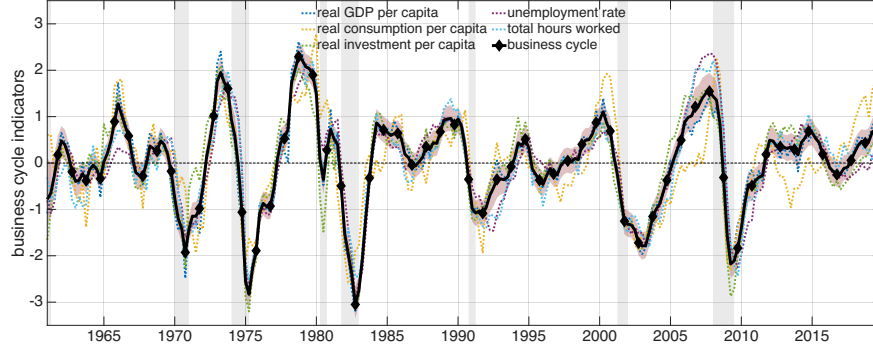


FIGURE 1. Business cycle indicators and estimates of Z_t

Note: We plot real GDP, consumption and investment per capita, the unemployment rate and hours worked alongside our estimate of the business cycle factor Z_t . We standardize them to mean zero and unit variance. Red shaded areas are 90% pointwise probability bands and gray areas indicate NBER-dated recessions.

micro level refer to annual income. Third, there is evidence of some asymmetry between recessions and expansions, particularly in the last 40 years: Recessions are deep and occur suddenly, while expansions are mild and unfold gradually. It would be possible to enrich model (3) to accommodate that feature, although this is unlikely to impact our estimates of Z_t given their precision. Nonetheless, when reporting summaries that condition on the aggregate state in this section we will adopt the notion of a typical recession, $\tilde{Z}_r = -2\sigma_Z$, a steady state, $\tilde{Z}_{ss} = 0$, and a typical expansion, $\tilde{Z}_e = \sigma_Z$, where $\sigma_Z^2 = \text{Var}(Z_t)$. This choice does not affect the results on impulse responses and risk calculations in Sections 6 and 7.

In this context, a central object is the IRF of the macro state variable Z_t to the shock V_t . We will return to it when discussing IRFs in Section 6.2.¹⁷

5.2 Micro data and cyclical patterns in the PSID

As we outlined in Section 3, the micro data for our study takes the form of a time series of panels. We draw to this end from the long history of panel data on income and earnings available in the Panel Study of Income Dynamics (PSID).

Established in 1968, the PSID initially surveyed a nationally representative set of U.S. households. Ever since, it has followed those families and, as their children come of age and form independent households, incorporated those new units into the panel. With the periodic addition of refresher and immigrant samples, the PSID design aims at capturing

¹⁷We also discuss IRFs of W_t to V_t and compare them to the IRFs estimated in Angeletos et al. (2020) for the MBC shock in Supplemental Appendix F.3.

the process of household formation and dissolution in the U.S. economy over time. It also accords with our notion of time series of panels in Section 3 as it allows us to form a long sequence of short subpanels, each reflecting representativeness at a point in time, which taken together span a rich history of aggregate fluctuations.

Interviews were conducted annually between 1968 and 1997, and biennially after 1999. Whether annual or biennial, the year- k interview (which typically occurs between March and November) asks the household to report annual income for year $k - 1$. We make use of all available waves beginning in 1970 and ending in 2019: we exclude 1968 and 1969 as some income and demographic variables were not collected in those waves, and we stop at 2019 before the COVID-19 pandemic. We then form a time series of panels where each subpanel has $S = 4$ biennial waves, although, crucially, we exploit all years. We use biennial panels because we are constrained by the change in data collection frequency after 1999. This also has the advantage of making the assumption of serially uncorrelated ε_{it} more plausible. But nonetheless, prior to 1999, the panels we construct have base years which cover both even and odd years, so we make use of all available annual observations. Given the biennial nature of the subpanels, $t - 1$ below is understood as two years before t , and with a slight abuse of notation we denote by Z_t the value of the macro state in the fourth quarter of t . This is relevant to interpret the income process and summaries.

In our data selection, a household is so that the representative person is male, married and aged 25 to 60, and so that income is positive.¹⁸ We consider three different measures of income: male earnings, household earnings, and disposable income. Male earnings are the labor income of the household head. Household earnings add in the labor income of the spouse. Disposable income adds transfers and subtracts federal taxes. We take transfer income as reported in the PSID, which includes welfare payments, social security income, and unemployment benefits. We compute taxes with the tax functions estimated by [Borella, De Nardi, Pak, Russo, and Yang \(2023\)](#). Lastly, we deflate income to 2016 dollars using the consumer price index (CPI-U).¹⁹

We construct y_{it} by residualizing log income against the following covariates interacted with a quadratic time trend: education, race, family size, number of children, an indicator for the presence of dependents, and state of residence. Except for family size and number of children, the other variables are treated as categorical. We then add back to all units the

¹⁸In addition to the main sample representative of the U.S. population, the PSID maintains a low-income sample known as the Survey of Economic Opportunities. Our analysis makes use only of the former.

¹⁹Beyond partialling out state-of-residence dummies, we do not explicitly account for differences in state taxes but one possibility is to use the tax functions in [Fleck, Heathcote, Storesletten, and Violante \(2025\)](#).

same constant representing a household with typical covariates.²⁰ We also exclude age from this operation which, as anticipated, enters Q_η and Q_{init,t_0} in the form of x_{it} .²¹

Descriptives. Figure 2 offers a first look at the micro data using repeated cross-sections ($S = 1$). The period covers seven NBER recessions (the two downturns of the 80s merged into a single bar). Panel (a) shows the fraction of zeros that would be dropped to compute y_{it} . It shows that most of the time less than 2% of the sample reports zero annual household earnings (blue). This applies to recessions too, except for the Great Recession in 2007 when the fraction briefly exceeded 2%. The story is different for male earnings, for which zeros constitute a larger fraction and there seems to be a steeper upward trend over time.

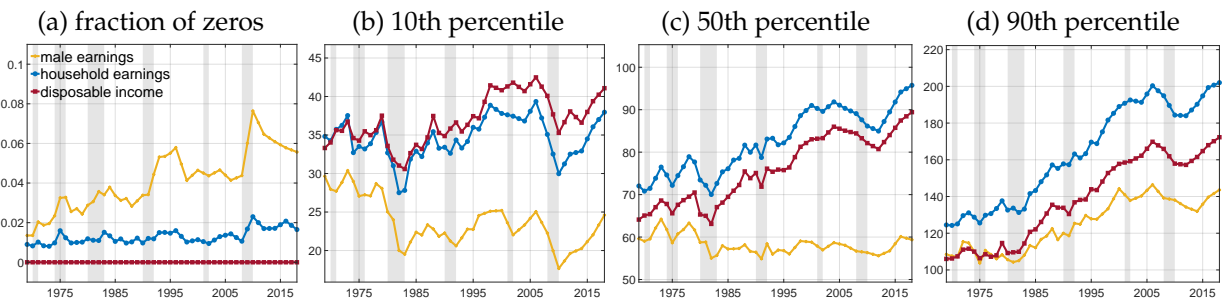


FIGURE 2. Distributional characteristics of income measures over time

Note: We show the fraction of zeros and the 10th, 50th and 90th percentiles (conditional on positive income) for our income measures in thousands of 2016 dollars. Shaded areas are NBER recessions.

Given the demographic criteria in our data selection, practically no unit reports zero disposable income for a whole year. Since this is also the relevant measure for studying the effects of the business cycle on consumption and welfare, disposable income will be our primary focus below. We report results for male and household earnings for comparability, with the caveat that doing so abstracts from the extensive margin. To the extent that this margin is cyclical and recessions imply larger drops in labor income than what we observe, our results for male and household earnings should be taken as a lower bound on the impact of negative macro shocks along those margins.

Panels (b) to (d) in Figure 2 illustrate the evolution of low, middle and high incomes over time. One highlight is that there are divergent secular trends in the income distribution,

²⁰This is a white college graduate with family size four, two children, no dependents, who resides in the state of New York. This level shift has no impact on the dynamics of η_{it} or any of the summaries of interest.

²¹We estimated a version of the model with x_{it} as an argument of $Q_{\epsilon,t}$ and found little effect of x_{it} . Given this, we report results based on the more parsimonious specification.

with fast growth at the top and stagnation (or even decline) at the bottom. These trends have been extensively studied in the literature, usually linking them to changes in female labor force participation, structural transformation, reforms to the tax and social security systems, and other phenomena.²² But importantly for us, these trends coexist with sizable cyclical variation. A quick glance reveals that incomes fall in recessions in ways that are heterogeneous over the income distribution (more at the bottom than in the middle), across income measures (more for male earnings than for disposable income) and depending on the severity of the downturn (the double-dip recession of the 80s and the Great Recession being the worse). Figure 2 is also indicative of the redistributive role of taxes and transfers, as going from household earnings to disposable income pushes low incomes up and higher incomes down, and of the insurance role of both spousal income and taxation which tend to mitigate income losses during recessions.

Model fit. We fit our model to the macro and micro data with the empirical specification of Section 4, where we also explain how estimation and inference are implemented. In Supplemental Appendix D we show that our model matches the time series evolution of quantiles and moments of observed income levels and growth rates remarkably well.

5.3 Nonlinear persistence: ρ tilts with the aggregate state

We now turn to quantifying empirically the effect of aggregate shocks on different aspects of the nonlinear income process. We begin by discussing the persistence of past income histories and how it changes over the business cycle. To that end, we use the measure of nonlinear persistence $\rho(u, \eta, Z_t, Z_{t-1}, x)$ defined in Equation (4) of Section 2.

The top panels of Figure 3 show our estimates of $\rho(u, \eta, Z_t, Z_{t-1}, x)$ for the three measures of income we consider. We plot persistence by quantiles of the current shock $u = u_{it}$ and past persistent income $\eta = \eta_{i,t-1}$ where age $x = x_{it}$ is averaged out, $Z_{t-1} = \tilde{Z}_{ss}$ is held at its steady state value and Z_t takes on different values representing a typical recession \tilde{Z}_r (red), the steady state \tilde{Z}_{ss} (yellow) or a typical mild expansion \tilde{Z}_e (blue). On the bottom panels, we report the recession minus the expansion persistence. If the recession persistence is above (below) its expansion counterpart according to a one-sided pointwise test at the 5% significance level, we indicate that with magenta (cyan).

²²In our framework of analysis, part of these low-frequency changes are controlled for by netting out the effect of covariates interacted with time trends in the construction of y_{it} , while another part will be absorbed by the time-varying functions Q_{init,t_0} and $Q_{\epsilon,t}$.

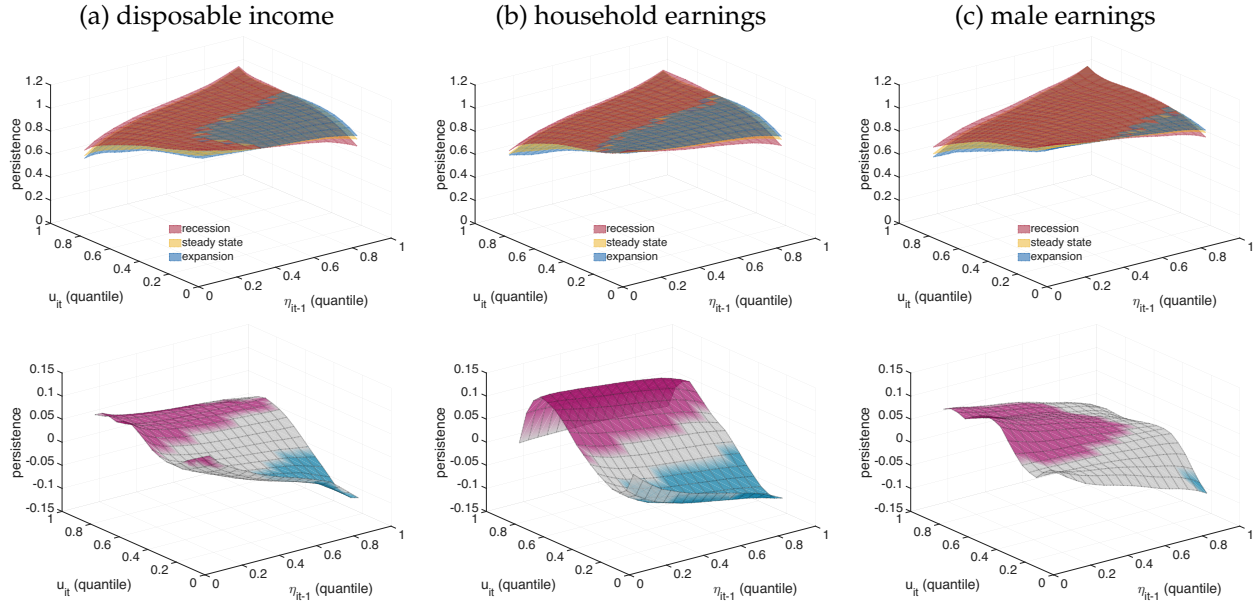


FIGURE 3. Nonlinear persistence.

Note: The top panels report the persistence measure $\rho(u, \eta, Z_t, Z_{t-1}, x)$ by quantile of the shock $u = u_{it}$ and past persistent income $\eta = \eta_{i,t-1}$. Here, age $x = x_{it}$ is averaged out, $Z_{t-1} = \tilde{Z}_{ss}$ and Z_t is a recession \tilde{Z}_r , the steady state \tilde{Z}_{ss} or an expansion \tilde{Z}_e (see Section 5.1). The bottom panels show the gap in persistence between recession and expansion, $\rho(u, \eta, \tilde{Z}_r, \tilde{Z}_{ss}, x) - \rho(u, \eta, \tilde{Z}_e, \tilde{Z}_{ss}, x)$. The difference is painted magenta (cyan) if it is statistically positive (negative) at the 5% significance level (one-sided test).

We highlight two takeaways from Figure 3. First, it confirms the nonlinear persistence pattern uncovered in ABB over a much longer history and across income measures. For example, in the steady state surface for disposable income, the average persistence is 0.92. It falls to 0.85 for a unit in the 90th η -percentile hit with a bad shock $u = 0.1$ and to 0.60 for a unit in the 10th η -percentile hit with a good shock $u = 0.9$. As already discussed, since ρ is a measure of how closely related current and past incomes are, this captures the fact that a big shock of a given direction may sometimes erase an entire income history.

The second takeaway is that the ABB persistence pattern is itself macro state-dependent, as aggregate shocks *tilt* the estimated persistence surface. Comparing recessions and expansions, ρ increases for low- η and decreases for high- η units (particularly those affected by bad shocks) during a downturn. In words, a low- η household has it more difficult to leave the low-income state in the midst of a contraction, while a high- η household finds it harder to remain high-income. These effects are large both statistically and economically. For disposable income, ρ decreases by 0.09 for the 90th η -percentile with $u = 0.1$ and increases by 0.07 for the 10th η -percentile with $u = 0.9$ as we move from $Z_t = \tilde{Z}_e$ to $Z_t = \tilde{Z}_r$.

These patterns are quite similar across income measures. The increase in persistence at low incomes during recessions tends to be slightly bigger for our earnings measures than for disposable income, possibly suggestive of the insurance role of taxes and transfers in attenuating the impact of negative aggregate shocks for the left tail of the income distribution—although the comparison abstracts from extensive margin effects.

5.4 Exposure to aggregate shocks: β is countercyclical

A second key aspect of the nonlinear income process is the persistent income exposure to macro shocks, measured by the nonlinear aggregate exposure coefficient $\beta(u, \eta, Z_t, Z_{t-1}, x)$ given in Equation (5) of Section 2. Figure 4 presents our estimates.

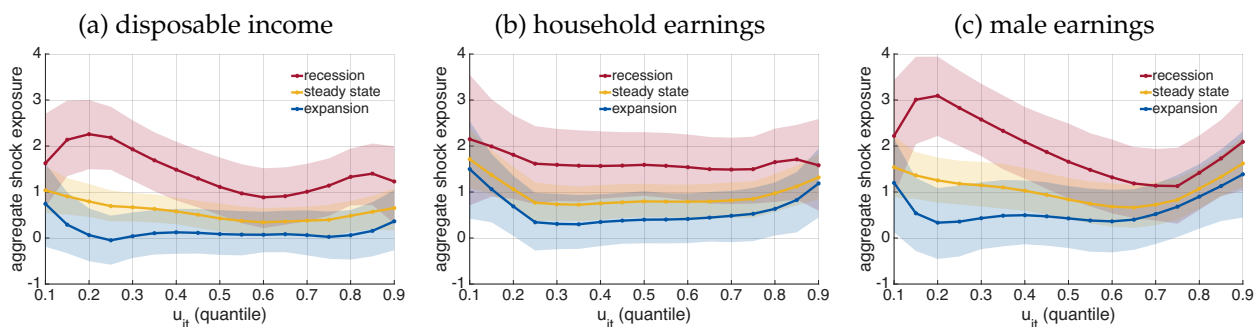


FIGURE 4. Nonlinear exposure to aggregate shocks.

Note: We report the exposure coefficient $\beta(u, \eta, Z_t, Z_{t-1}, x)$ by quantile of the shock $u = u_{it}$ averaged over persistent income $\eta = \eta_{i,t-1}$. Age $x = x_{it}$ is averaged out, $Z_{t-1} = \tilde{Z}_{ss}$ and Z_t is a recession \tilde{Z}_r , the steady state \tilde{Z}_{ss} or an expansion \tilde{Z}_e (see Section 5.1). Shaded areas represent 90% confidence bands.

In Figure 4, we show $\beta(u, \eta, Z_t, Z_{t-1}, x)$ as a function of the micro-level shock $u = u_{it}$ that occurs at the time of the macro shock. We average past income $\eta = \eta_{i,t-1}$ and age $x = x_{it}$, and fix $Z_{t-1} = \tilde{Z}_{ss}$, allowing Z_t to take values compatible with a recession \tilde{Z}_r , steady state \tilde{Z}_{ss} or expansion \tilde{Z}_e (discussed in Section 5.1).

Three patterns stand out. First, the exposure to aggregate shocks depends on Z_t and is countercyclical. Averaged over u and η , in our disposable income calculations, β is 1.3 in a recession, 0.6 in steady state and 0.2 in an expansion. Given the normalization of Z_t in Section 5.1, these numbers can be interpreted as the elasticity of persistent income to an aggregate shock that implies a one percentage point change in GDP per capita relative to its trend. Accordingly, a negative macro shock leads to an aggregate reduction in the persistent component of disposable income that is more or less than one-for-one the fall in GDP depending on whether the economy is already in a recession when the shock hits.

This is a major form of aggregate state-dependence and one that is ruled out by models in which aggregate shocks enter additively. What is more, we argue in Section 7 that this nonlinearity plays a key role in the cost of business cycle risk.

Second, the aggregate exposure coefficient varies across income measures, with male earnings the most and disposable income the least sensitive to Z_t . For example, the recession average β is 1.8 for male earnings, 1.6 for household earnings and 1.3 for disposable income. The same applies to the steady state and expansion β , and at different quantiles of $u = u_{it}$ and $\eta = \eta_{i,t-1}$, as seen in additional results in Supplemental Appendix E.

The last pattern to analyze is the interaction between macro and micro shocks. Units affected by bad micro shocks $u = u_{it}$ at the time when the macro shock occurs are relatively more exposed during recessions (and generally less during expansions) than units subject to neutral or good shocks. In other words, the consequences of recessions are not evenly distributed but are borne mostly by those who experience bad micro luck as the downturn unfolds. This is another feature ruled out by linear models with aggregate shocks.

5.5 A tale of two skewnesses

We conclude by discussing the conditional skewness measure introduced in Equation (6) of Section 2. A full picture of aggregate effects on the income process is completed by measures of dispersion and kurtosis. They appear to be less cyclical than the skewness (in line with [Guvenen et al., 2014](#)), and we report them in Supplemental Appendix E.

The upper panels of Figure 5 show $\text{sk}(\eta, Z_t, Z_{t-1}, x)$ for different quantiles of $\eta = \eta_{i,t-1}$ with $Z_{t-1} = \tilde{Z}_{ss}$ when Z_t ranges from recession \tilde{Z}_r , through steady state \tilde{Z}_{ss} to expansion \tilde{Z}_e . Age $x = x_{it}$ is averaged out. The lower panels display the difference in skewness between recession and expansion including 90% pointwise confidence bands.

Figure 5 provides empirical support for a tale of two skewnesses suggested by our descriptive analysis. The ABB skewness pattern, by which $\text{sk}(\eta, Z_t, Z_{t-1}, x)$ decreases for higher η , is accompanied here by a cyclical skewness pattern: Recessions tend to shift $\text{sk}(\eta, Z_t, Z_{t-1}, x)$ toward the negative plane at all levels of η , often by a large margin. Mirroring nonlinear persistence and exposure to aggregate shocks, cyclical shifts in skewness are strongest for male earnings and weakest for disposable income, that is, after spousal earnings and the tax-transfer system are taken into account.

Since $\text{sk}(\eta, Z_t, Z_{t-1}, x)$ is a measure of the relative strength of upside and downside income shocks, our results characterize recessions as periods when downside shocks become more prevalent in a generalized but heterogeneous way across the income distribution.

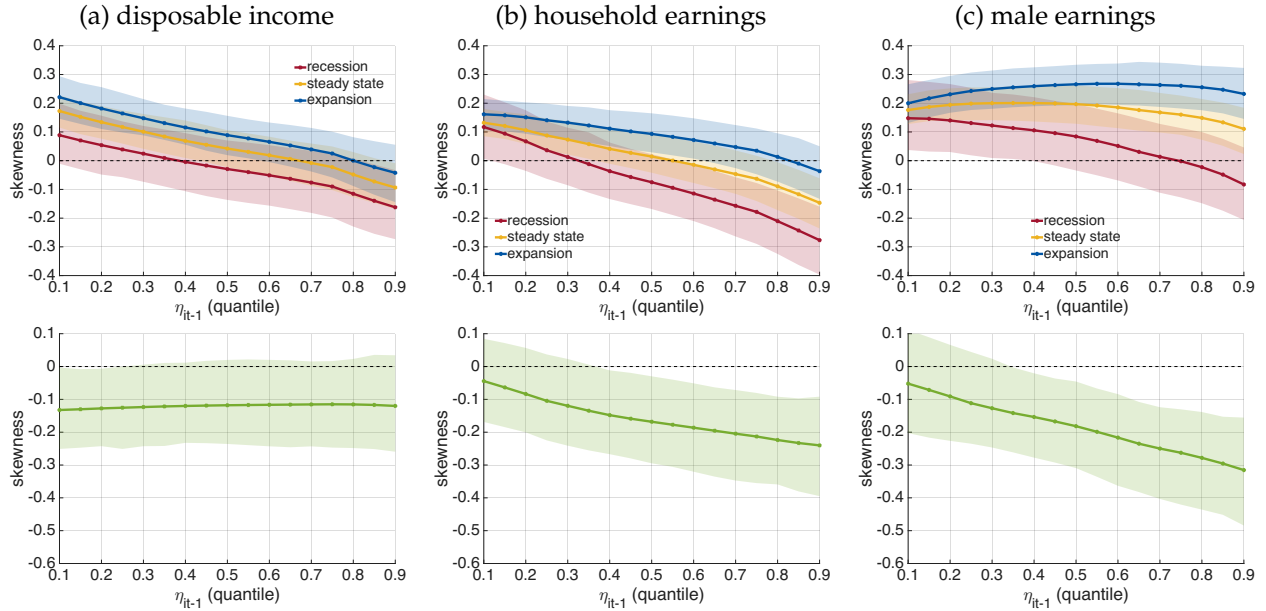


FIGURE 5. Conditional skewness.

Note: The upper panels report skewness $sk(\eta, Z_t, Z_{t-1}, x)$ by past persistent income $\eta = \eta_{i,t-1}$ where age $x = x_{it}$ is averaged out, $Z_{t-1} = \tilde{Z}_{ss}$ and Z_t is a recession \tilde{Z}_r , the steady state \tilde{Z}_{ss} or an expansion \tilde{Z}_e (see Section 5.1). The lower panels show the gap in skewness between recession and expansion, $sk(\eta, \tilde{Z}_r, \tilde{Z}_{ss}, x) - sk(\eta, \tilde{Z}_e, \tilde{Z}_{ss}, x)$. Shaded areas represent 90% confidence bands.

6 Impulse response analysis

In this section, we develop new methodology for measuring impulse responses to macro and micro shocks. We first present our approach and then discuss empirical estimates.

6.1 Measuring the propagation of macro and micro shocks

To develop the idea, we focus on the persistent component η and omit covariates x . From Equations (1) and (3) we get by recursive substitution the representation

$$\eta_{i,t+h} = q_{\eta,h}(\mathbf{u}_{it}^h, \mathbf{V}_{t+1}^{h-1}, \eta_{i,t-1}, Z_t, Z_{t-1}), \quad h = 0, 1, \dots, \quad (13)$$

where $\mathbf{u}_{it}^\ell = (u_{it}, \dots, u_{i,t+\ell})$ and $\mathbf{V}_t^\ell = (V_t, \dots, V_{t+\ell})$. With the distribution of micro and macro shocks, Equation (13) determines the predictive distribution of $\eta_{i,t+h}$ given initial states $(\eta_{i,t-1}, Z_t, Z_{t-1})$. In what follows we assume Z_t is scalar, although it is straightforward to generalize the derivation to the multivariate case.

The macroeconometric tradition often defines macro and micro impulse responses as

$$E[\eta_{i,t+h} \mid V_t = 1] - E[\eta_{i,t+h} \mid V_t = 0] \text{ and } E[\eta_{i,t+h} \mid u_{it} = 1] - E[\eta_{i,t+h} \mid u_{it} = 0],$$

respectively. In linear models, they coincide with the derivatives of $q_{\eta,h}$ with respect to shocks V_t and u_{it} . In our setup, however, this approach suffers from various shortcomings. First, our model features significant nonlinearities in the persistence and interactions between macro and micro states and shocks; impulse responses should account for those. Second, changes in one unit of V_t or u_{it} need not be directly comparable. Third, in our panel data setup, a change of given size in u_{it} may have different impacts for different individuals.

To address these issues, we define impulse responses as the impact on the predictive distribution of $\eta_{i,t+h}$ of perturbations to *past states* (as opposed to shocks), extending [Gallant, Rossi, and Tauchen \(1993\)](#) beyond the time series setup. For this purpose, we introduce a *rule* (denoted g below) that maps perturbations onto a consistent system of measurement. As a result, impulse responses are indexed by past states—allowing us to document nonlinearities—and depend on the normalization rule—allowing us to achieve comparability across shocks and units.

IRFs via perturbations. Let us define a benchmark value for one of the state variables: Z^b for Z_t for the macro impulse response; η^b for $\eta_{i,t-1}$ for the micro impulse response. Let us then define the new state value Z^p (resp., η^p) by means of the perturbation $\Delta = Z^p - Z^b$ (resp., $\Delta = \eta^p - \eta^b$). Considering experiments that perturb a single state at a time, we define impulse responses as suitably scaled differences in the expected trajectory of $\eta_{i,t+h}$ for marginal perturbations Δ , holding every other past and current state constant.

To select the perturbation Δ , we introduce a rule that maps it to a system of comparable units of change δ . The rule is given by a function g such that

$$g(Z^b) = g(Z^b + \Delta(\delta)) - \delta \text{ or } g(\eta^b) = g(\eta^b + \Delta(\delta)) - \delta.$$

The rule g may depend on the benchmark value and the reference value for the remaining states but we omit the dependence from the notation. We will focus on the *unit rule*:

$$g(z) = z, \text{ leading to } \Delta(\delta) = \delta.$$

The unit rule is natural if the perturbed state is measured in money terms, or if it is in logs in which case the impulse consists of the same approximate $100 \times \delta\%$ change applied to all individuals. Other choices are possible, depending on the empirical context. For example, under a *rank rule*, $g(z)$ is the CDF of the perturbed state given the other state and past states $\{\eta_{i,t-1-\ell}, Z_{t-\ell}\}_{\ell>1}$.²³ The rank rule is appropriate when the state is a concept with no natural unit of measurement, such as welfare. For the micro IRF, g can be thought of as an income transfer program implemented by a social planner or policymaker.

Given a rule g , the macro and micro impulse responses are, respectively,

$$\text{IRF}_{\eta_Z}(h, \delta) = \frac{E\left[\eta_{i,t+h} \mid \eta_{i,t-1}, Z_t = Z^b + \Delta(\delta), Z_{t-1}\right] - E\left[\eta_{i,t+h} \mid \eta_{i,t-1}, Z_t = Z^b, Z_{t-1}\right]}{\delta},$$

$$\text{IRF}_{\eta_\eta}(h+1, \delta) = \frac{E\left[\eta_{i,t+h} \mid \eta_{i,t-1} = \eta^b + \Delta(\delta), Z_t, Z_{t-1}\right] - E\left[\eta_{i,t+h} \mid \eta_{i,t-1} = \eta^b, Z_t, Z_{t-1}\right]}{\delta},$$

where we leave the dependence on $\eta_{i,t-1}$, Z_t and Z_{t-1} implicit.

Impulse responses are functions of parameters θ and λ , and can thus be estimated by the plug-in estimators whose properties we covered in Section 4. In addition, it is possible to link impulse responses to the ρ and β measures of Section 5, and to derivatives with respect to locally-defined shocks. We explore those links in Supplemental Appendix F.

6.2 Empirical estimates of impulse responses

Macro impulse responses. We begin with estimates of IRF_{η_Z} for a negative perturbation to Z_t around the steady state benchmark $Z^b = \tilde{Z}_{ss}$. This emulates an aggregate shock that tips the economy into a recession. We calibrate $\delta = -2\sigma_V$ with $\sigma_V^2 = \text{Var}(Z_t \mid Z_{t-1})$ to match a shock comparable to the Great Recession and, to facilitate interpretation, we multiply impulse responses by -1 . Figure 6 shows the results.

It is instructive to consider first the trajectory of Z_t in panel (d), which is annualized by averaging the quarterly responses and scaled by the standard deviation of log GDP. The normalization translates responses into log deviations from the GDP per capita trend. By this account, our underlying experiment implies a GDP roughly 2.3 and 1.3 percentage points (pp) below trend in years zero and one after the shock, respectively, returning to

²³In the macro case, if F_V is the CDF of V_t and Q_Z^{-1} is the inverse of Q_Z with respect to its last argument, $\Delta = Q_Z(Z_{t-1}, F_V^{-1}[F_V(Q_Z^{-1}[Z_{t-1}, Z^b]) + \delta]) - Z^b$. Similarly, in the micro case, if Q_η^{-1} is the inverse of Q_η with respect to its last argument, $\Delta = Q_\eta(\eta_{i,t-2}, Z_{t-1}, Z_{t-2}, Q_\eta^{-1}[\eta_{i,t-2}, Z_{t-1}, Z_{t-2}, \eta^b] + \delta) - \eta^b$.

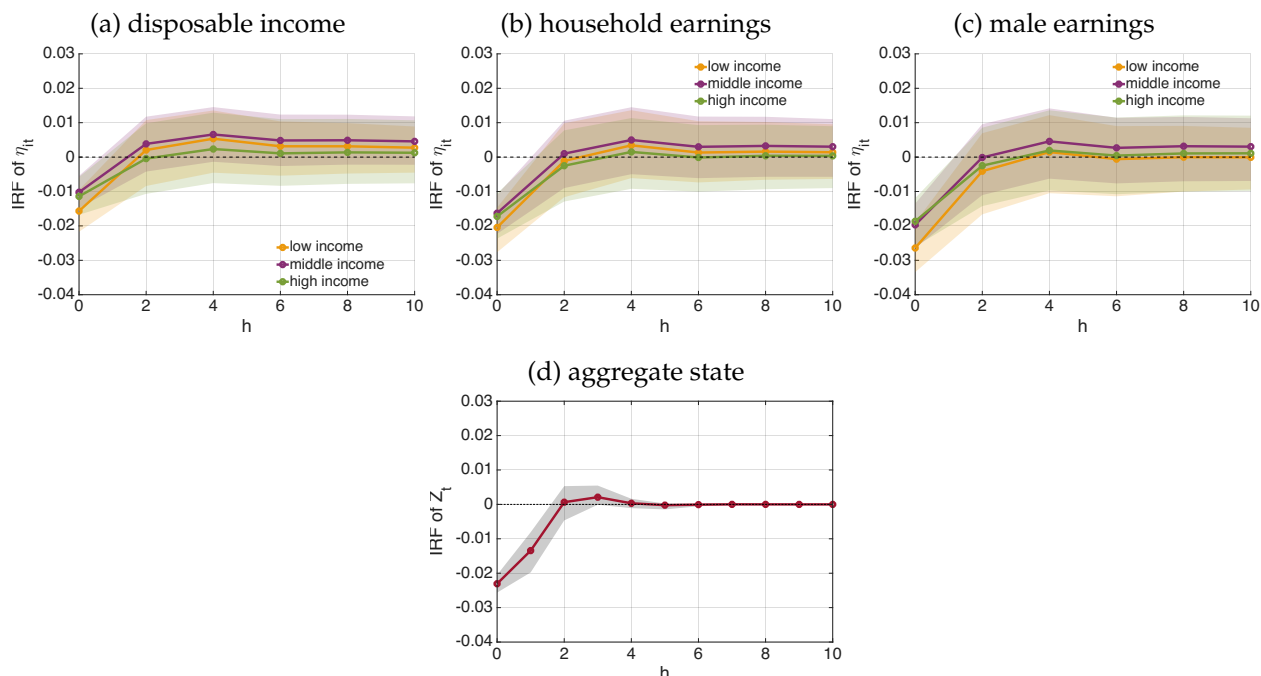


FIGURE 6. Macro impulse responses

Note: Panels (a), (b) and (c) show IRFs of η_{it} to a negative macro shock for different income measures with $Z_t^b = Z_{t-1} = \tilde{Z}_{ss}$ and $\eta_{i,t-1}$ set to the 10th (low), 50th (middle) and 90th (high) percentiles of the persistent income distribution. Panel (d) shows the IRF of Z_t annualized and scaled to detrended log GDP per capita. IRFs are multiplied by -1 . Shaded areas are 90% pointwise confidence bands.

trend afterwards with a slight overshoot in year three.

Next, panels (a), (b) and (c) report macro impulse responses of η_{it} for our three income measures and three initial levels of income: the 10th, 50th and 90th percentiles of the $\eta_{i,t-1}$ distribution. We highlight the following takeaways. First, the responses are quantitatively consistent with the dynamics of GDP described above. For example, averaging over the distribution of $\eta_{i,t-1}$, male earnings fall by 2.1 pp on impact following the macro shock, while household earnings and disposable income fall by 1.7 and 1.1 pp.²⁴ Furthermore, all income measures are near their pre-shock trends after two years; i.e., responses are short-lived.²⁵

Second, household earnings appear as less cyclically sensitive than male earnings but

²⁴In line with the literature, this suggests a mildly procyclical labor share (the ratio of household earnings to total income), although our estimates exclude a small fraction of zero-earnings units.

²⁵Supplemental Appendix F.4 presents local projection estimates of income responses to V_t that uncover similar patterns. They also point to a significant response at $h = 1$ that our biennial setup cannot measure.

more than disposable income. This mimics the discussion of Section 5 and is suggestive of the role of spousal income and the tax-transfer system as potential sources of insurance against aggregate shocks. Third, the macro IRF is U-shaped in $\eta_{i,t-1}$, with lower (but still significant) responses in the middle of the income distribution.

A key advantage of our framework is the possibility to measure the interplay between macro and micro uncertainty. This is illustrated in Figure 7 where we compute a modified version of $\text{IRF}_{\eta Z}(h, \delta)$ that further conditions on the micro shock u_{it} contemporaneous to the macro perturbation, averaged over the distribution of $\eta_{i,t-1}$. This allows us to quantify the micro quantile treatment effects associated with aggregate shocks.



FIGURE 7. Interaction between macro impulse responses and micro shocks

Note: Panels (a), (b) and (c) show macro IRFs conditional on the micro shock u_{it} averaged over the distribution of $\eta_{i,t-1}$ for male earnings, household earnings and disposable income with $Z_t^b = Z_{t-1} = \tilde{Z}_{ss}$. A magenta (cyan) area indicates the response is statistically negative (positive) at the 5% significance level.

Figure 7 shows sizable heterogeneity in the impact of macro shocks along the micro-rank distribution. For disposable income, an individual subject to a bad shock $u_{it} = 0.1$ suffers an expected income loss of 1.6 pp, well above the average of 1.1 pp. The number is 0.9 and 1.1 pp for $u_{it} = 0.5$ and $u_{it} = 0.9$. We find similar (but more pronounced) U-shaped patterns in our household and male earnings estimates; see panels (a) and (b).

Micro impulse responses. We conclude this section with estimates of the micro impulse responses $\text{IRF}_{\eta\eta}$ for a negative perturbation δ that implies a 10% reduction in $\eta_{i,t-1}$. We hold Z_t and Z_{t-1} at their steady state value \tilde{Z}_{ss} and multiply responses by -0.1 for ease of interpretation. See Figure 8.

The main takeaway, consistent across income measures, is that micro responses decay slowly and at different rates that depend on the initial level of income, with less (more) persistence for low- η (high- η) units. This reflects an intrinsic connection between nonlinear persistence ρ and $\text{IRF}_{\eta\eta}$ that we discuss in Supplemental Appendix F.

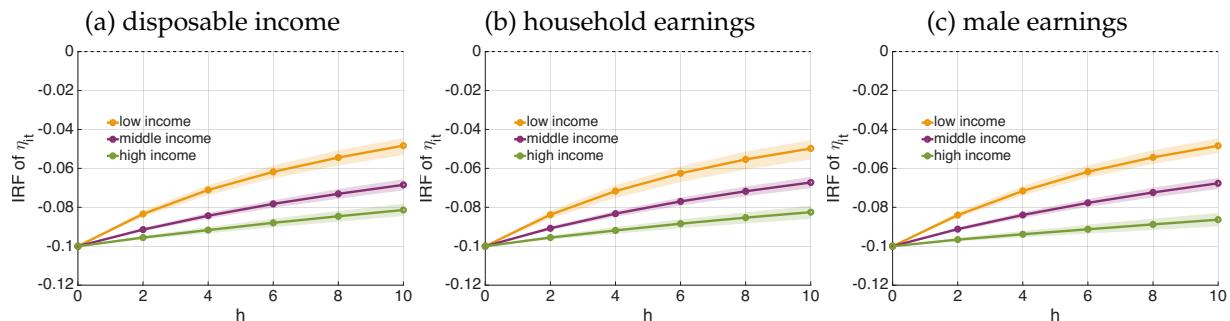


FIGURE 8. Micro impulse responses

Note: Panels (a), (b) and (c) display IRFs of η_{it} to a negative micro shock for different income measures with $Z_t = Z_{t-1} = \tilde{Z}_{ss}$ and $\eta_{i,t-1}^b$ set to the 10th (low), 50th (middle) and 90th (high) percentiles of the distribution of $\eta_{i,t-1}$. Shaded areas are 90% pointwise confidence bands.

Positive shocks. Supplemental Appendix F.5 reports IRFs to positive perturbations.

7 Risk quantification

In this last section, we develop our methodology to quantify aggregate and idiosyncratic contributions to income risk. We conclude by discussing empirical estimates.

7.1 Measuring the contribution to risk of macro and micro shocks

Our approach to risk quantification relies on the indirect utility of persistent income. Let $U(e^{\eta_{it}})$ be the period utility of a household whose persistent log income component at time t is η_{it} . We focus on indirect utility since we do not have consistent data on consumption, and abstract away from the transitory component ε_{it} as it may contain measurement error. Following the macroeconomics tradition after [Lucas \(1987, 2003\)](#), we will measure the risk contribution of shocks by compensating variation: the value CV such that

$$E^* \left[\sum_{h=1}^H \delta^t U((1 - CV) e^{\eta_{i,t+h}}) \middle| \eta_{it}, Z_t \right] = E \left[\sum_{h=1}^H \delta^t U(e^{\eta_{i,t+h}}) \middle| \eta_{it}, Z_t \right]. \quad (14)$$

In (14), E denotes expectations over the actual persistent income process, while E^* denotes expectations under a counterfactual income process where the macro or the micro shocks have been removed. Here CV may depend on (η_{it}, Z_t) but we leave it implicit.

The quantity CV measures the fraction of income the agent would be willing to forego

in every period in order to eliminate a source (macro or micro) of income risk. Much of the literature emphasizes the role of curvature in preferences for the cost of risk. A common finding is that log-utility with an exponential income process implies little aggregate risk, and that high risk-aversion is needed to obtain even moderate costs of business cycles. Here we highlight a different channel that we find matters greatly to infer the magnitude of aggregate income risk: the nonlinear relationship between the income process and the aggregate factor. Indeed, our model allows for a general nonlinear relationship between η_{it} and (Z_t, Z_{t-1}) ; see (1). These nonlinearities, and more specifically a countercyclical β (as documented in Section 5.4), can generate higher amounts of risk than usually found.

To gain intuition, we compute a small-noise expansion of CV around a baseline that sets shocks to their median value. We present below our calculations setting $H = 1$ for the measure of macro risk CV_{macro} where the experiment eliminates the macro shocks only.²⁶ Let $\eta_{i,t+1}(Z_{t+1}) = Q_\eta(\eta_{it}, Z_{t+1}, Z_t, x_{i,t+1}, 0.5)$ and $\beta_{i,t+1}(Z_{t+1}) = \beta(0.5, \eta_{it}, Z_{t+1}, Z_t, x_{i,t+1})$ be the quantile function (1) and the exposure to aggregate shocks (5) at a median micro shock $u_{i,t+1} = 0.5$, leaving the dependence on $(\eta_{it}, Z_t, x_{i,t+1})$ (but not on Z_{t+1}) implicit. Let $\eta_{i,t+1}$ and $\beta_{i,t+1}$ denote their values in the absence of a macro shock. Letting $\gamma(\eta) = -e^\eta U''(e^\eta)/U'(e^\eta)$ be the relative risk aversion, we obtain

$$CV_{\text{macro}} \approx \underbrace{\frac{\sigma_V^2}{2} (\gamma(\eta_{i,t+1}) - 1) \beta_{i,t+1}^2}_{\text{risk aversion channel}} + \underbrace{\frac{\sigma_V^2}{2} \left(-\frac{\partial \beta_{i,t+1}}{\partial Z_{t+1}} \right)}_{\text{cyclical exposure channel}}. \quad (15)$$

Equation (15) shows that CV_{macro} captures the combination of two effects: the curvature of the utility function and nonlinearities in the exposure to aggregate shocks. When risk aversion is moderate, the cost of business cycle risk depends crucially on how nonlinear the income process is with respect to Z_{t+1} . For log utility (i.e., $\gamma(\eta) = 1$) and a linear income process, CV_{macro} is approximately zero, but it can be large when aggregate shock exposures are countercyclical, that is, when $\partial \beta_{i,t+1}/\partial Z_{t+1} < 0$. This reflects the self-amplifying nature of recessions and aligns with the empirical findings we discussed in Section 5.4.

²⁶Details of the expansion and the formula for arbitrary H can be found in Supplemental Appendix G. An analogous approximation applies to the measure of micro risk CV_{micro} .

7.2 Empirical estimates of macro and micro costs of risk

We present below our compensating variation estimates against macro and micro shocks for disposable income.²⁷ To investigate the role of nonlinearities in the aggregate shocks, in addition to computing risk measures from the full model estimated in Section 5, we also consider a version in which $\beta(u, \eta, Z_t, Z_{t-1}, x)$ is constant in (Z_t, Z_{t-1}) , even though it can depend on (u, η, x) .²⁸ In the restricted model, β is acyclical in contrast to the countercyclical β we find in our full model.

Figure 9 illustrates the results. Left panels show risk calculations from our full model while right panels show the restricted model, with macro risk above and micro risk below. Period utility is $U(c) = c^{1-\gamma}/(1-\gamma)$ with risk aversion coefficient $\gamma = 3$. We set the biennial discount factor to $\delta = (0.96)^2$ and we calibrate the horizon for lifetime utility to the number of biennial periods until a notional retirement age of 65, i.e., $H = (65 - x_{it})/2$.

The main highlight from Figure 9 is the striking difference in the cost of business cycle risk between linear and nonlinear income exposures to aggregate shocks. In line with our analytical derivation, with an acyclical β (right), CV_{macro} is virtually zero, supporting the conclusion in Lucas (2003) that eliminating business cycle fluctuations is second-order from a welfare point of view. In stark contrast, a countercyclical β (left) produces CV_{macro} orders of magnitude higher, with households willing to give up between 2.2% and 5% of their income each period in order to avoid cyclical fluctuations. Since our full model nests the restricted one and CV_{macro} is statistically non-zero in the former, the evidence favors the view that aggregate shocks have non-negligible welfare costs.²⁹

Turning to the cost of idiosyncratic risk in Figure 9, perhaps unsurprisingly, CV_{micro} is higher than CV_{macro} even in the countercyclical- β case, but there is ample variation over age and across the income distribution. To give a sense, the cost of macro risk is less than a tenth the cost of micro risk for older and richer households; it is more than a third for young low-income households. This aligns with the tale of two skewnesses documented

²⁷In a model without endogenous labor supply, this is the relevant income measure to study consumption and welfare. In that context, the indirect utility of persistent income $U(e^{\eta_{it}})$ can be rationalized by a variety of consumption functions: e.g., hand-to-mouth households if ε_{it} is mostly measurement error or unconstrained permanent-income households if $E_t[e^{y_{it+h}} | \eta_{it}, \varepsilon_{it}, Z_t] \approx e^{\eta_{it}}$.

²⁸We achieve this by restricting φ to a first-order polynomial in (\tilde{Z}, Z) in the specification described at the end of Section 4. We fit this model to the data using the same approach as for our full model.

²⁹This conclusion changes very little for larger values of the risk aversion coefficient γ . Instead, the cost of risk under countercyclical β is higher for larger persistence and variance of aggregate shocks, confirming that the interaction between $\partial\beta_{it}/\partial Z_t$ and marginal utility is, empirically, the relevant dimension of the income process for risk quantification.

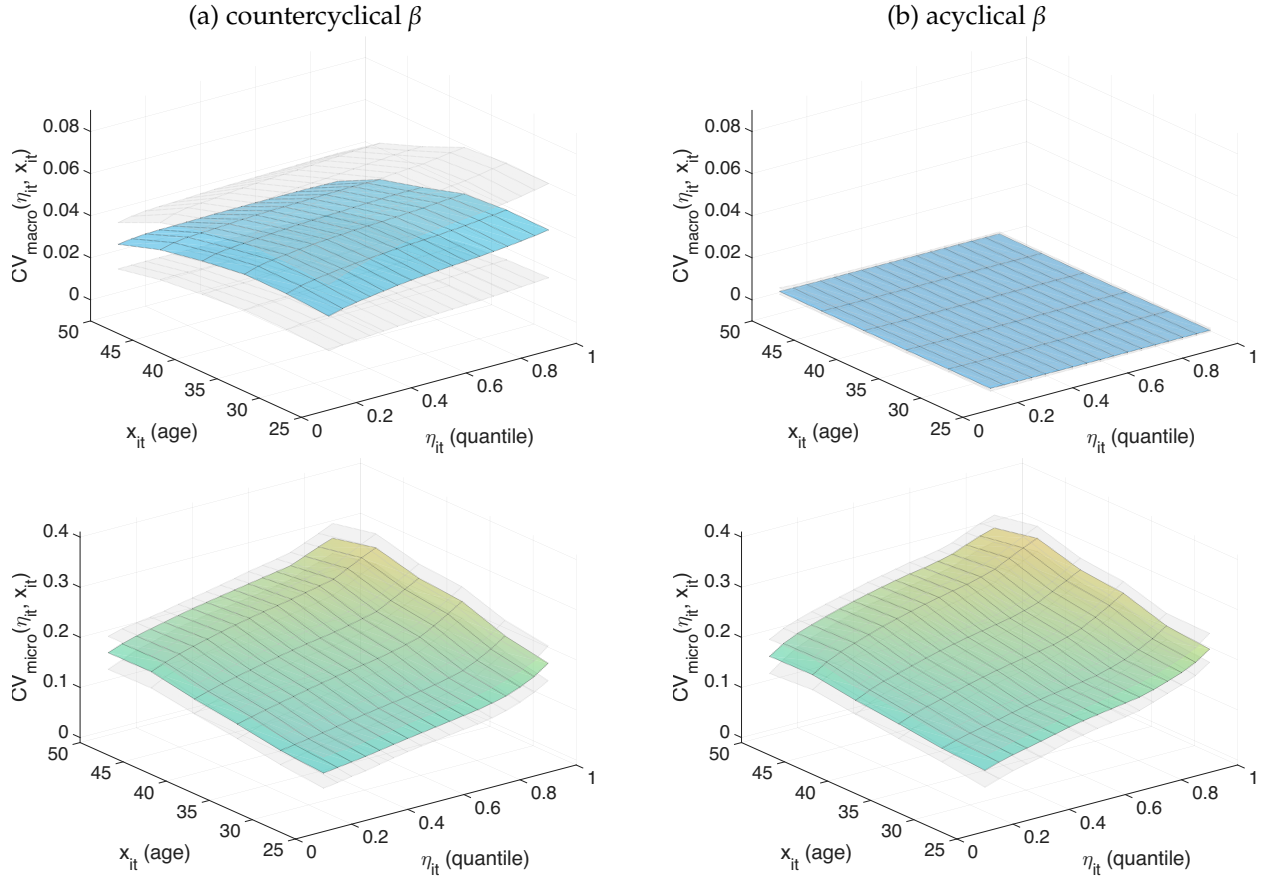


FIGURE 9. Risk quantification (disposable income).

Note: We show compensating variation for aggregate (upper panels) and idiosyncratic risk (lower panels) for various ages $x = x_{it}$ and initial incomes η_{it} . Our full model (where β is countercyclical) is on the left and a restricted acyclical- β model on the right. Lifetime utility is $\sum_{h=1}^{(65-x_{it})/2} \delta^h \frac{e^{(1-\gamma)\eta_{i,t+h}}}{(1-\gamma)}$ with $\delta = (0.96)^2$ and $\gamma = 3$. Gray shaded areas are 90% confidence bands.

in Section 5.5: For low- η units, micro risk is primarily upside risk while aggregate shocks (given their self-amplifying effects) carry downside risks. As η increases (or as households age), conditional skewness becomes more negative and the cost of micro risk increases.

Interestingly, the restricted model and our full model have similar implications for CV_{micro} . The nonlinearities that underlie the aggregate and idiosyncratic components of income risk are separate features, targeted by different parts of our framework. A natural question is whether agents have different amounts of information about the macro- and micro-relevant features, and how much this matters for risk quantification. Another issue is the impact of the assumption that Z_t follows a linear process for welfare calculations. We leave these important questions for future research.

8 Conclusion

In this paper we propose a nonlinear semi-structural framework for heterogeneous-agent models with aggregate shocks. The nonlinear reduced-form relationships our approach can uncover are useful to assess the fit of such models and their implications. Allowing for general nonlinear dynamics and rich macro-micro feedback, we study identification, estimation and inference tools that leverage both macro and micro data.

In our empirical analysis of U.S. panel data on income, we find that the nonlinear persistence and conditional skewness patterns documented in previous work are affected by business cycle fluctuations. The persistence surface tilts in recessions, decreasing for high-income households hit by a bad shock and increasing for low-income households, while skewness declines throughout the income distribution. We also find evidence of nonlinear exposures to aggregate shocks, with higher sensitivity of income to macro shocks during recessions and nontrivial interactions between macro and micro shocks. Our results suggest that nonlinearities with respect to aggregate shocks matter for risk quantification. One avenue to explore is how to account for model uncertainty in our risk measurement approach.

Three natural extensions come to mind which we leave to future work. The first one is to examine the pass-through of aggregate and idiosyncratic income shocks to consumption. This would follow [Arellano et al. \(2017\)](#) and [Arellano et al. \(2023\)](#), where consumption data from the PSID was combined with the nonlinear income process to examine the impact of income shocks on consumption. The results in those studies found a differential response according to initial household assets and household heterogeneity. The framework developed in this paper would enable estimation of the transmission of aggregate shocks through income to consumption as well as the differential impact of idiosyncratic income shocks by aggregate state. Thus allowing an assessment of the extent by which insurance to persistent income shocks varies across the cycle for different types of households, and providing a more complete analysis of partial insurance and the welfare costs of income risk across the business cycle.

The second extension relates to the primary components of household income: male and female earnings. These two earnings measures embody the labor supply responses of the spouses. In a linear partial insurance framework, [Blundell, Pistaferri, and Saporta-Eksten \(2016\)](#) investigate the responses to wage shocks of spousal hours of work and consumption in the PSID, finding significant responses to permanent and transitory wage

shocks. However, they do not consider non-linear wage processes and ignore the impact of aggregate factors. [Arellano et al. \(2017\)](#) found that the permanent component of male wages in the PSID follows a nonlinear persistent process. However, they did not examine the joint process of male and female wages, nor the impact of aggregate factors. The potential for uncovering important wage dynamics and effects of aggregate factors for couples makes this an exciting area for future research.

A third extension, related to the second one, concerns the zeros in the income data. [Figure 2](#) showed a small but growing, and cyclical, proportion of zeros for male earnings in our couples sample, a more stable and smaller proportion of zeros for household earnings and no zeros for disposable income. This is one reason why we have focused attention on disposable income. Nonetheless, we have also drawn conclusions in comparison with male and household earnings. Modeling the extensive margin requires some assumptions on the process of selection. A natural framework is to assume random missingness conditional on history of the income process. But it is likely that aggregate shocks also play a role here. [Braxton, Herkenhoff, Rothbaum, and Schmidt \(2021\)](#) develop a Kalman filter and EM algorithm approach to incorporate observations with zero (or missing) earnings. By specifying a law of motion for persistent earnings they show how assuming a model for earnings upon re-entry can deliver identification under selection. In another recent paper, [Gobillon, Magnac, and Roux \(2022\)](#) study earnings dynamics in French administrative data assuming zeros are missing at random conditional on a set of factors drawn from earnings history. We believe the nonlinear framework with aggregate factors developed in our paper provides a promising setting for incorporating the extensive margin into the study of income dynamics over the cycle. We leave this extension to future work.

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