Why HANK Matters for Stabilization Policy

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Preliminary

Abstract

When do optimal inflation and consumption differ between Heterogeneous-Agent (HA) and Representative-Agent (RA) models? What are the underlying mechanisms? To answer these questions, we derive jointly the optimal fiscal and monetary Ramsey policy in HA and RA models, incorporating both price and wage stickiness. The allocation in HA economies diverges significantly from RA economies when the severity of credit constraints varies over time, which is captured by a new statistics, the Marginal Value of the Credit Constraint. We identify the relevant fiscal tools to maximize welfare over the business cycle. Time-varying labor tax appears to be a useful stabilizing tool.

Keywords: Heterogeneous agents, wage-price spiral, inflation, monetary policy, fiscal policy.

JEL codes: D31, E52, D52, E21.

1 Introduction

Standard heterogeneous-agent (HA) models describe economies in which agents face incomplete insurance markets for idiosyncratic risk and credit constraints. When combined with nominal frictions, these models—known as HANK models—are now widely used to identify and quantify new transmission channels following shocks or policy changes. Yet their implications for optimal stabilization policy remain unclear. Do these models imply genuinely new normative predictions compared to simpler representative-agent (RA) models? Does the heterogeneity merely matter for positive questions?

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To answer these questions, we solve for the optimal joint monetary and fiscal policy in HA models featuring both wage and price stickiness, under different assumptions about the fiscal system. We derive the optimal policy response to the main aggregate shocks considered in the literature: a TFP shock, a public spending shock, a discount factor shock, and an uncertainty shock. Our analysis addresses two central questions. First, under what conditions does fiscal policy make it optimal for inflation to deviate from price stability in HA economies? Second, does the optimal aggregate allocation differ between RA and HA economies and, if so, does this difference rely on HA-specific mechanisms?

Our first contribution is to analyze deviations from price stability in HA and RA models. The environment we consider features both sticky prices and sticky wages, under a standard separable utility function. Our investigation of fiscal-monetary interactions starts with characterizing a benchmark "complete fiscal system" comprising three types of labor taxation (employer contributions, employee contributions, and total labor income tax), a capital tax, and public debt. When all these instruments are optimally time-varying, the optimal rates of price and wage inflation are shown to be exactly zero, regardless of the aggregate shock that is considered. This result extends the equivalence findings of Correia et al. (2008) or LeGrand et al. (2022) to an environment with both sticky prices and sticky wages. Building on this flexible-price benchmark, our analysis of fiscal-monetary interactions then involves solving for optimal policy in RA and HA economies, when the planner is prevented from using fiscal instruments that are set constant. This approach allows us to identify which missing fiscal tools generate the largest welfare losses and the most significant deviations from price and wage stability.

Our second contribution is to show that HA economies generate new first-order mechanisms that are relevant to optimal stabilization policy. These mechanisms are driven by time-varying precautionary savings, and are hence specific to HA models. In a tractable setting, we prove that aggregate allocations differ between HA and RA economies after an aggregate shock when the severity of the credit constraint is time-varying—specifically, when the Lagrange multiplier on household borrowing constraints varies over time. To build economic intuition, we introduce a new sufficient statistic that we call the Marginal Value of the Credit Constraint (MVCC). We show that the time variations of this quantity are instrumental in determining when HA and RA allocations diverge. The MVCC measures by how much to increase the interest rate for relaxing the borrowing limit of credit-constrained households. In both simple and quantitative models, we demonstrate that the behavior of the MVCC depends on the nature of the aggregate shock and on the set of available fiscal instruments. The MVCC is time-varying after discount factor and uncertainty shocks but nearly constant after TFP and public spending shocks, at least when the set of fiscalinstruments is not too restricted. Consequently, HA and RA economies differ significantly in their responses to discount factor and uncertainty shocks but not so much to the TFP and public spending shocks, helping reconcile contrasting results in the literature. When fewer fiscal instruments are available, the MVCC becomes more volatile, regardless of the aggregate shock. The gap between HA and RA economies also grows with the lack of available instruments. We also compare the MVCC to the discount factor wedge (DFW) used in Nakajima (2005), Werning (2015), Acharya and Dogra (2021) and Berger et al. (2023) to compare allocations between HA and RA economies. While the two statistics are distinct, they are quantitatively close in our simulations.

The two contributions show that when labor taxes cannot vary over time, the optimal HA allocation differ from the optimal RA one and sizable deviations from price stability arise optimally in HA models. This holds for instance for TFP shocks, that imply a labor wedge and make the real wage differ from the marginal productivity of labor. With sticky nominal wages, price inflation becomes a second-best instrument to mitigate this wedge. Time-varying labor taxes thus serve as a key stabilizer, reducing both deviations from price stability and aggregate fluctuations (that become closer to those in the HA model). We call these taxes non-Keynesian stabilizers because they dampen the fall in consumption and inflation after a negative shock, whereas traditional Keynesian stabilizers reduce the fall in consumption by increasing inflation through a lower output gap. Furthermore, when the planner is prevented from optimally levying capital tax, short-lived variations in optimal inflation responses in HA models act as an imperfect substitute for time-varying capital taxes. Conversely, the absence of optimal capital tax has no consequence in RA models. The HA allocation therefore deviates from the RA one. However, substantial inflation adjustments through this channel require relatively flexible prices.

As a final side result, we also derive predictions on the optimal path of public debt. In HA models, the variation at impact is always smaller in absolute value than the one in RA models. Given the role of public debt as a buffer saving in HA models, the planner tends to limits its variations compared to the complete market case. The public debt is highly persistent both in RA and HA models, but slightly less so in the latter than in the latter..

Related literature. This paper contributes to three strands of the literature: optimal fiscal and monetary policy in heterogeneous-agent (HA) and representative-agent (RA) economies, fiscal-monetary interactions in HANK models, and models with both price and wage stickiness.

First, deriving optimal Ramsey policy in HA models with aggregate shocks is both theoretically and computationally challenging. Some papers rely on numerical methods to solve for the optimal path of instruments (Dyrda and Pedroni, 2022), while others use continuous-time techniques to derive first-order conditions for the planner (Nuño and Moll, 2018 and Nuño and Thomas, 2022 among others). Acharya et al. (2022) solve for optimal monetary policy using the tractability of a CARA-normal environment without capital. Bhandari et al. (2021) provide a quantitative solution for optimal policies in a New Keynesian model with aggregate shocks. Yang (2022) analyzes optimal monetary policy by optimizing the coefficients of a Taylor rule. McKay and Wolf (2022) develop a general quadratic-linear framework to characterize optimal policy rules. In this paper, we use the tools of LeGrand and Ragot (2022a) and their subsequent improvements, which rely on constructing a finite state-space representation of HANK models. This approach makes it possible to solve for optimal policy with multiple instruments, various nominal frictions, and different aggregate shocks. Using this framework, monetary policy after a TFP shock is studied in LeGrand et al. (2022), while fiscal policy after a public spending shock is examined in LeGrand and Ragot (2025). Within this body of work, our contribution is to clarify the new insights that HA environments provide for optimal policy design.

Second, the literature on fiscal and monetary interactions has been reinvigorated by the HANK framework. In these models, monetary policy has redistributive implications, fiscal shocks exert novel effects on inflation due to realistic marginal propensities to consume (MPCs), and public debt influences the real interest rate. Recent surveys by Auclert et al. (2025) and Kaplan (2025) provide a synthesis of this literature. In this context, the analysis of Ramsey policy serves both as a tool to derive policy implications and as a framework to identify the mechanisms that matter most.

Finally, this paper relates to the literature on environments with both price and wage stickiness, sometimes referred to as the "inflation spiral" literature. Early contributions in RA settings include Blanchard (1986), Galí (2015, chapter 6), or Blanchard and Gali (2007). Erceg et al. (2000) study optimal monetary policy in this framework, while Chugh (2006) analyzes both optimal monetary policy and optimal labor taxation. More recently, Lorenzoni and Werning (2023) provide a detailed analysis of optimal policy and real wage dynamics in such an environment. Introducing both nominal frictions generates real wage rigidity, which creates new and important roles for fiscal and monetary policy.

2 Understanding the difference between RA and HA economies in a simple environment

In this section, we analyze a tractable model, where the difference between the optimal policies and allocations of HA and RA economies can be characterized analytically. The key simplifying assumption is to consider deterministic productivity fluctuations, following the approach of Woodford (1990) and LeGrand and Ragot (2025), among others. To further streamline the exposition, we focus on the flexible price economy.

Production. The production function transforms each unit of labor L_t into Z_t units of output, such that aggregate output is given by $Y_t = Z_t L_t$. Since prices are flexible, the real wage is equal to TFP: $\tilde{w}_t = Z_t$.

The agents. The economy is populated by two types of agents, denoted by A and B. A unit mass of agent A has a productivity 1 in every odd period and a productivity 0 in every even period. Conversely, a unit mass of agent B has a productivity 1 in every even period and a productivity 0 in every odd period. Thus, in each period, there is a unit mass of agents with productivity 1, and income fluctuations are deterministic. Agents with positive productivity are referred to as "employed" (subscript e), while those with zero productivity are referred to as "unemployed" (subscript u).

Agents' preferences are represented by a Greenwood-Hercowitz-Huffman (GHH) utility function: $U(c,l) = \log(c - \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}})$, where c and l are individual consumption and labor supply respectively, and $\varphi > 0$ is the Frisch elasticity of labor supply. We assume that agents discount utility from period $t+1$ in period t by a possibly time-varying discount factor β_t, with $0 < \beta_t < 1$. Following Galí (2015) among others, we interpret changes in β_t as preference shocks(). The discount factor for utility in period t as of period 0 is:

$$\Theta_t = \Pi_{k=0}^{t} \beta_t, \tag{1}$$

which simplifies to β^t if β_t is constant over time.

The only friction in the economy is a credit constraint: Agents cannot borrow. We denote $c_{e,t}, a_{e,t}, c_{u,t}, a_{u,t} \geq 0$ as the consumption and saving levels of employed and unemployed agents in period t, respectively. The budget constraints for employed and unemployed agents are:

$$c_{e,t} + a_{e,t} = R_t a_{u,t-1} + w_t l_{e,t}, \tag{2}$$

$$c_{u,t} + a_{u,t} = R_t a_{e,t-1}, \tag{3}$$

where R_t and w_t are the gross real post-tax interest rate and wage rate, respectively. For simplicity, we assume that initial wealth is zero: $a_{e,-1} = a_{u,-1} = 0$. The Euler equations for employed and unemployed agents are:

$$\left(c_{e,t} - \frac{l_{e,t}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right)^{-1} \geq \beta_t R_{t+1} c_{u,t+1}^{-1}, \tag{4}$$

$$c_{u,t+1}^{-1} \geq \beta_t R_{t+1} \left(c_{e,t+1} - \frac{l_{e,t+1}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right)^{-1}, \text{ with equality if } a_{u,t} > 0. \tag{5}$$

Because they have a null productivity, unemployed agents do not work: $l_{u,t} = 0$. Due to the GHH utility function, the labor supply of employed employed agents is pinned down by the real wage. Aggregate labor supply L_t is therefore:

$$L_t = l_{e,t} = w_t^{\varphi}. \tag{6}$$

Government. The government issues a quantity of public debt B_t. Financial market clearing requires:

$$a_{e,t} + a_{u,t} = B_t. \tag{7}$$

The government raises linear capital tax τ_t^K and labor tax τ_t^W to finance interest payments on public debt. If $\tilde{r}_t$ denotes the net pre-tax interest rate, the gross post-tax interest rate is $R_t = 1 + (1 - \tau_t^K)\tilde{r}_t$. Similarly, the post-tax wage rate is $w_t = (1 - \tau_t^W)\tilde{w}_t$. The government budget constraint writes as $(1 + \tilde{r}_t)B_{t-1} \leq B_t + \tau_t^K \tilde{r}_t(a_{e,t-1} + a_{u,t-1}) + \tau_t^W \tilde{w}_t L_t$, which with $\tau_t^W \tilde{w}_t = \tilde{w}_t - w_t = Z_t - w_t$ simplifies to:

$$R_t B_{t-1} = (Z_t - w_t)L_t + B_t. \tag{8}$$

The Ramsey allocation. For a given sequence of MIT shocks, known at date 0, $\{\beta_t, Z_t\}_{t\geq 0}$, the Ramsey program selects the path of instruments $\{\tau_t^K, \tau_t^W, B_t\}_{t\geq 0}$ that implements the competitive equilibrium achieving the highest aggregate welfare (given the initial conditions). The aggregate welfare criterion used is the standard Utilitarian objective, in which both agents are equally weighted. Since the discount factors correspond to those of the agents, they may therefore be time-varying. The Ramsey program can be expressed in post-tax terms as follows:

$$\max_{(c_{e,t},c_{u,t},a_{e,t},a_{u,t},l_{e,t}B_t,A_t,R_t,w_t)_t} \sum_{t=0}^{\infty} \Theta_t \left(\log\left(c_{e,t} - \frac{l_{e,t}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right) + \log(c_{u,t}) \right) \tag{9}$$

$$\text{subject to: } c_{e,t}, c_{u,t}, a_{e,t}, a_{u,t}, l_{e,t}, l_{u,t} \geq 0, \tag{10}$$

$$a_{e,-1} = a_{u,-1} = 0, \tag{11}$$

and subject to: the constraints (2)–(8) guaranteeing the optimality of individual choices (budget constraints, Euler equations and labor FOC with GHH utility function, respectively); the resource constraint (8); and the financial market clearing condition (7).

2.1 Two benchmark economies: The representative-agent and no-policy economies

Before characterizing the Ramsey allocation, we present two benchmark economies that will serve as a baseline for analyzing the effects of the different aggregate shocks we will consider.

The representative-agent economy. The representative-agent (RA) economy is an economy populated by a unique agent endowed with the same GHH utility function as in the general case. In this setting, the Ramsey problem involves maximizing the agent's intertemporal welfare subject to the Euler equation and the resource constraint. The optimal policy is straightforward to characterize, as the first-best allocation can be achieved. Specifically, the planner sets $\tau_t^K = \tau_t^W = B_t = 0$. Under this policy, the labor supply is given by $L_t^{RA} = Z_t^{\varphi}$, and optimal consumption is $C_t^{RA} = Z_t^{1+\varphi}$.

The no-policy economy. In the no-policy (NP) economy, the planner does not intervene in the economy with credit constraints. The laissez-faire policy implies $B_t = \tau_t^W = \tau_t^K = 0$. The wage equals labor productivity of labor, and the employed agent supplies labor such that $l_{e,t} = L_t = Z_t^{\varphi}$. Since there is value storage, the employed agent consumes their entire income: $c_{e,t} = Z_t^{1+\varphi}$, while the unemployed agent consumes nothing: $c_{u,t} = 0$. There is no consumption smoothing and the aggregate consumption, denoted as C_t^{NP} in this economy, is equal to the consumption of employed agent and verifies $C_t^{NP} = Z_t^{1+\varphi} = C_t^{RA}$.

As a consequence, these two economies yield identical aggregate consumption paths following both TFP shocks $(Z_t)_t$ and discount factor shocks $(\beta_t)_t$.

2.2 The incomplete-market economy with optimal policy

The Ramsey program is expressed in equations (9)–(11). As a preliminary step, we prove that the planner does not increase public debt to the point where credit constraints cease to bind for unemployed agents. Such a policy would imply a very high debt level and necessitate an excessively high and distortionary labor tax. We therefore assume that credit constraint remains binding for unemployed agents. By substituting the labor supply expression (6), the program can considerably be simplified. All technical details can be found in Appendix A, here we present only the main result.

We denote the aggregate consumption in this economy by $C_t^{HA} = c_{u,t} + a_{u,t}$. Our first result, stated in the following proposition, summarizes how the optimal allocations in HA and RA economies differ in their response to shocks to the discount factor and TFP.[1]

Proposition 1 *For any path of TFP $(Z_t)_t$ and discount factor $(\beta_t)_t$, the optimal allocations in the HA and RA economies are related as follows:*

$$\frac{C_t^{HA}}{C_t^{RA}} = \left(\frac{1}{2} + \varphi\right)^{\varphi} \left(\varphi + \frac{1}{1+\beta_t}\right)^{-\varphi}.$$

Proposition 1, whose proof can be found in Appendix A highlights two key insights regarding the role of aggregate shocks on why HA and RA models differ.

TFP shocks $(Z_t)_t$ only. When the economies are subject to TFP shocks alone, then discount factors remain constant and $\beta_t = \beta$ for all t. Proposition 1 implies that the ratio $\frac{C_t^{HA}}{C_t^{RA}}$ also stays constant. This further yields $\hat{C}_t^{HA} = \hat{C}_t^{RA}$, where $\hat{x}_t$ denotes the proportional deviation of variable x_t from its steady state value. Although RA and HA economies have different steady states, their aggregate response to TFP shocks is identical.

[1] In the literature, the former are typically referred to as demand shocks, while the latter are termed supply shocks. However, since we also consider other shocks in the quantitative sections, we adhere to their precise denominations to avoid any ambiguity.

Discount factor shocks $(\beta_t)_t$ ***only.*** When the economies are subject to shocks to the discount factor only, the ratio $\frac{C_t^{HA}}{C_t^{RA}}$ becomes time-varying. As a result, the RA and HA economies exhibit heterogeneous aggregate responses, with $\hat{C}_t^{HA} \neq \hat{C}_t^{RA}$.

Consequently, only discount factor shocks induce different aggregate responses between the HA and RA economies. However, for both shocks, the paths of the policy instruments (labor tax and public debt) differ between the twoeconomies. In the RA economy, setting $B_t = \tau_t^W = 0$ achieves the first-best allocation in each period, and the paths of all instruments adjust in response to both types of shocks. The difference in the optimal allocation response between the HA and RA economies is our primary focus, as it is a key indicator capturing the distinct policy implications of the two economies for both types of shocks.

2.3 Interpreting the results: Two statistics

We introduce two statistics to understand the differences between optimal allocations in RA and HA economies.

The Marginal Value of the Credit Constraints (MVCC). We define as ν_t the Lagrange multiplier associated to the credit constraint of unemployed agents. It is given by: $\nu_t := U_c(c_{u,t}, 0) - \beta R_{t+1} U_c(c_{e,t+1}, l_{e,t+1})$. Intuitively, words, ν_t measures the gap between the current and future discounted marginal utilities of agents u and therefore reflects how slack the Euler equation (5) of unemployed agents is. It equals 0, when credit constraints do not bind and is positive otherwise. We then define the Marginal Value of Credit Constraints (MVCC) as the Lagrange multiplier ν_t normalized by the current marginal utility of agents u:

$$MVCC_t := \frac{\nu_t}{U_c(c_{u,t}, 0)}.$$

This unitless quantity reflects how binding the credit constraint is. If credit constraints do not bind, the MVCC equals zero as saving is already optimal by construction.

Using the definition of ν_t, the Euler equation of unemployed agents can be written as:

$$U_c(c_{u,t}, 0) = \beta_t \frac{R_{t+1}}{1 - MVCC_t} U_c(c_{e,t+1}, l_{e,t+1}).$$

This implies that $\frac{R_{t+1}}{1-MVCC_t}$ is the gross interest rate at which zero savings would be optimal for unemployed agents. In other words, $MVCC_t$ measures by how much the interest rate should be increased for the credit constraints of the unemployed agent to be relaxed.

When the $MVCC$ is constant, the interest rate R_{t+1} becomes a sufficient statistic for the growth rate of the marginal utility of unemployed agents—and hence for the growth rate of their consumption. The same logic applies to employed agents, as their Euler equation holds with equality. Thus, when the MVCC is constant, the consumption dynamics of both employed and unemployed agents depend solely on the real interest rate, and their consumption growth rates are identical to those in the RA economy. Therefore, only the dynamic properties of the MVCC – and not its steady-satte value – determine the extent to which the dynamic responses of RA and HA differ following an aggregate shock.

The MVCC can be explicitly computed in this economy. As derived in Appendix A.2, its expression is:

$$MVCC_t = 1 - \frac{(1+\varphi(1+\beta_{t+1}))\,(1+\varphi(1+\beta_t))}{(1+2\varphi)^2}.$$

This expression depends on the discount factor β_t but not on TFP Z_t. In particular, the MVCC remains constant when β is constant, regardless of TFP shocks. This explains why the dynamics of aggregate consumption in RA and HA economies are identical following a TFP shock, but differ after a discount factor shock.

The Discount Factor Wedge (DFW). Following Nakajima (2005), Werning (2015), Acharya and Dogra (2021) and Berger et al. (2023), it is well established that the aggregate allocation of HA models can be interpreted as the equilibrium outcome of a RA model, augmented by the appropriate wedges. In particular, the *discount factor wedge* (DFW) is defined such that the allocation and interest rate in the HA economies emerge as an equilibrium outcome of an RA economy, in which the discount factor is scaled by this wedge—with all other elements held constant between the HA and RA economies. The DFW thus enables the replication of the HA economy's allocations and prices as an equilibrium outcome of a modified RA economy. It can also be seen as a measure of the distortions in saving incentives arising from market incompleteness. Formally, the DWF, denoted DFW_t, is defined to ensure that the Euler equation of a representative agent—endowed with the allocation of the HA economy and a scaled discount factor— holds with equality. Formally:

$$\left(C_t^{HA} - \frac{l_{HA,t}^{1+1/\varphi}}{1+1/\varphi}\right)^{-1} = (1+DFW_t)\beta_t R_{t+1}^{HA}\left(C_{t+1}^{HA} - \frac{l_{HA,t+1}^{1+1/\varphi}}{1+1/\varphi}\right)^{-1}.$$

After some algebra, derived in Appendix A.3, we obtain the following expression:

$$DFW_t = \frac{(1+1/\varphi)\,(1+\varphi\,(1+\beta_{t+1})) - (1/2+\varphi)\,(1+\beta_{t+1})}{(1+1/\varphi)\,(1+\varphi\,(1+\beta_t)) - (1/2+\varphi)\,(1+\beta_t)} \times \frac{1+2\varphi}{1+\varphi\,(1+\beta_{t+1})} - 1.$$

The DFW DFW_t pertains to the aggregate consumption of all agents, both constrained and unconstrained. However, since the Euler equation of unconstrained agents holds with equality, movements in the DFW are primarily driven by the choices of unconstrained agents. Actually, when the steady-state $MVCC$ is close to 0, it can be shown that the deviations of DFW_t and $MVCC_t$ are proportional, i.e., $\widehat{DFW}_t \propto \widehat{MVCC}_t$. As a consequence, both statistics convey the same information. The advantage of the MVCC lies in its micro-foundation, as it directly reflects the underlying credit constraint.

In the more general HA model studied below, the MVCC will be different from 0 for all credit-constrained agents. In this case, we will verify that the average value of the MVCC behave similarly to the DFW and that both statistics are observationally equivalent (see Section 5).

Sufficient statistics and optimal policies. The simple model delivers three results, which will help understanding the general model. First, it demonstrates that a specific statistis can be employed to assess the degree to which the dynamics of HA and RA allocations differ. This statistic, that we call MVCC, captures the intensity of the credit constraint, which is the central imperfection responsible for the discrepancies between HA and RA outcomes.. Second, the model reveals that the optimal aggregate allocation in the HA economy *can* replicate the dynamic properties of its RA counterpart. This equivalence arises when the optimal path of policy instruments effectively neutralizes the impact of shocks on the credit constraint, thereby ensuring that the MVCC remains constant over time. Third, the MVCC, while informative, does not serve as a sufficient statistic for evaluating the optimality of observed policies. For instance, maintaining a constant MVCC in response to a discount factor shock would be suboptimal, whereas such a policy is indeed optimal in the context of TFP shocks. This distinction highlights that optimal policy design must account for the nature of the shock and its implications for credit constraints.

3 The general model

We now relax many of the simplifying assumptions of the previous Section, and we introduce both sticky prices and sticky wages, along with a rich fiscal structure.

We consider a discrete-time economy populated by a continuum of size one of ex-ante identical agents. These agents are assumed to be distributed along a set J, with the non-atomic measure ℓ: $\ell(J) = 1$.[2]

3.1 Risk

Aggregate risks. Agents face an aggregate shock $(S_t)_t$. The shock is persistent but known at date 0 and should hence be considered as a MIT shock. The aggregate shock can affect the economy through different channels: the TFP, Z_t; agents' discount factors, β_t; government public spending, G_t; or individual productivity levels, y_{it}. In the main text, we primarily focus on two channels: the TFP and the discount factors, as in the theoretical section. These two cases are sufficient to contrast Ramsey policies in HA and RA economies. Results for the other channels are summarized in Section XX, with a more more detailed providedin the appendix.

[2]We follow Green (1994) and assume that the law of large numbers holds.

Idiosyncratic risk. Agents also face idiosyncratic productivity risk. The productivity process, denoted y, follows a first-order Markov chain with transition matrix $\boldsymbol{\pi} = (\pi_{yy'})_{y,y'}$ and takes value in a finite set $\mathcal{Y}_t$, which may depend on the aggregate shock S_t. With wage w and labor supply l, an agent with productivity y earns pre-tax labor income wyl. In each period, the fraction of agents with productivity y is constant and denoted by n_y. We normalize average productivity to 1, i.e., $\sum_y n_{y \in \mathcal{Y}_t} y = 1$. The history of idiosyncratic productivity shocks up to date t for agent i is denoted by $y_i^t = \{y_{i,0}, \ldots, y_{i,t}\}$, where $y_{i,\tau}$ is the productivity at date τ. The measure of idiosyncratic histories up to date t, denoted θ_t, can be computed using the initial distribution and the transition matrix.

3.2 Preferences

Households are expected-utility maximizers with time-separable preferences and possibly time-varying discount factors. As in Section 2, the discount factor from $t+1$ to t is denoted $\beta_t \in (0,1)$, and the compounded discount factor from t to 0 by$\Theta_t = \Pi_{s=0}^{t-1} \beta_s$. In each period, households derive utility $U(c,l)$ from consuming the economy's unique consumption good c and experience disutility from supplying labor l. We further assume that in each period, the instantaneous utility is separable in consumption and labor: $U(c,l) = u(c) - v(l)$, where $u, v : \mathbb{R}_+ \to \mathbb{R}$ are twice continuously differentiable and increasing. Furthermore, u is concave, with $u'(0) = \infty$, and v is convex.

3.3 Labor taxes

For generality, and for theoretical reasons developed in Section 3.8, we introduce a rich set of four linear taxes. We here present the two labor taxes, and introduce income and capital tax below.

First, unions bargain over the nominal wage rate, denoted $\hat{W}_t$. Workers pay a linear labor tax τ_t^W on this income, so their post-tax nominal wage is $(1-\tau_t^W)\hat{W}_t$. We will interpret τ^W as a worker social contribution. Second, firms pay an additional labor tax, τ_t^E, creating a wedge between the labor cost per efficient unit of labor, $\tilde{W}_t$, paid by firms and the bargained wage $\hat{W}_t$. This tax can be interpreted of as an employer social contribution that does not appear on workers' payroll. Formally, the relationship is: $\hat{W}_t = (1-\tau_t^E)\tilde{W}_t$. The tax τ_t^E affects labor demand, which unions internalize in their bargaining strategy. Similarly, the tax τ_t^W affects labor income and is also internalized. The key between is that τ_t^E has a direct effect on employment for a given bargained wage $\hat{W}_t$, but not on the wage W_t, whereas τ_t^W has a direct effect on the wage W_t for a given wage $\hat{W}_t$, but no direct effect on employment.[3]

[3] By *direct effect*, we refer tothe partial equilibrium effect of each variable. In general equilibrium (with endogenous income), these taxes affect all variables through price variations. The assumption on the incidence of the two taxes, τ^W and τ^E, is based on the empirical literature (e.g., Saez et al., 2012 and Lehmann et al., 2013).

3.4 Production

The specification of the production sector follows the New-Keynesian literature on price stickiness, adapted to the tax structure described above. The consumption good Y_t is produced by a unique profit-maximizing representative firm that combines intermediate goods $(y^f_{j,t})_j$ from different sectors indexed by $j \in [0,1]$ using a standard Dixit-Stiglitz aggregator with an elasticity of substitution ε_P:

$$Y_t = \left[\int_0^1 {y^f_{j,t}}^{\frac{\varepsilon_P - 1}{\varepsilon_P}} dj\right]^{\frac{\varepsilon_P}{\varepsilon_P - 1}}.$$

For any intermediate good $j \in [0,1]$, the production $y^f_{j,t}$ is realized by a monopolistic firm and sold at price $p_{j,t}$. Aggregate labor productivity Z_t is affected in period 0 by a shock ϵ^Z_0 and follows a AR(1) process: $Z_t = e^{z_t}$,with

$$z_0 = 1 + \epsilon^Z_0 \text{ and } z_t = \rho^Z z_{t-1} \text{ for } t \geq 1, \rho^Z < 1.$$

Intermediate firms face quadratic price adjustment costs à la Rotemberg, proportional to the magnitude of relative price changes: $\frac{\psi_p}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - 1\right)^2$. Denoting the price inflation rate as $\pi^P_t = \frac{P_t}{P_{t-1}} - 1$ where P_t is the implied price index, we obtain the standard ePhillips curve (see Appendix XXX for the details):

$$\pi^P_t(1 + \pi^P_t) = \frac{\varepsilon_P - 1}{\psi_P}(m_t - 1) + \beta_t \mathbb{E}_t\left[\pi^P_{t+1}(1 + \pi^P_{t+1})\frac{Y_{t+1}}{Y_t}\right], \ Y_t = Z_t L_t. \tag{12}$$

3.5 Labor market: Labor supply and Union wage decision

Following the New Keynesian sticky-wage literature, labor hours are supplied monopolistically by unions (Erceg et al., 2000; Chugh, 2006; Hagedorn et al., 2019; Auclert et al., 2022 among others). There is a continuum of unions of size 1 indexed by k. Each union k supplies L_{kt} hours of labor at date t with a nominal wage $\hat{W}_{kt}$. Each union k sets its wage $\hat{W}_{kt}$ to maximize the intertemporal welfare of its members internalizing the labor demand by firms. We assume quadratic utility costs for the adjustment of the nominal wage: $\frac{\psi_W}{2}(\hat{W}_{kt}/\hat{W}_{kt-1} - 1)^2 dk$. The objective of union k is:

$$\max_{(\hat{W}_{ks})_s} \mathbb{E}_t \sum_{s=t}^{\infty} \Theta_s \int_i \left(U(c_{i,s}, l_{i,s}) - \frac{\psi_W}{2}\left(\frac{\hat{W}_{ks}}{\hat{W}_{ks-1}} - 1\right)^2\right)\ell(di),$$

This maximization yields the New-Keynesian wage-Phillips curve:

$$\pi^W_t(\pi^W_t + 1) = \frac{\varepsilon_W}{\psi_W}\underbrace{\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}(1 - \tau^W_t)\hat{w}_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right)}_{\text{labor gap}} L_t + \beta_t \mathbb{E}_t\left[\pi^W_{t+1}(\pi^W_{t+1} + 1)\right], \tag{13}$$

where $\pi_t^W = \frac{\hat{W}_t - \hat{W}_{t-1}}{\hat{W}_{t-1}}$ is the wage inflation rate, and $\hat{w}_t = \hat{W}_t/P_t$ is the real pre-tax wage.

3.6 Assets

The only asset is nominal public debt, with supply B_t at date t, paying a pre-determined before-tax nominal interest rate i_{t-1}. Public debt, issued by the government, is assumed to be default free. The financial market clearing implies that the net total savings of households, denoted A_t, equals public debt:

$$A_t = B_t. \tag{14}$$

The real before-tax (net) interest rate for public debt, denoted by $\tilde{r}_t$, is defined by:

$$\tilde{r}_t = \frac{1 + i_{t-1}}{1 + \pi_t^P} - 1. \tag{15}$$

3.7 Agents' program

Each agent enters the economy with an initial endowment of public debt $a_{i,-1}$ and productivity level $y_{i,0}$. The joint initial distribution over public debt and productivity levels is Λ_0. In subsequent periods, agents learn their productivity $y_{i,t}$, supply labor, and earn savings payoffs. Since labor supply L_t is chosen by unions, labor income is $(1-\tau_t^W)\hat{w}_t y_{i,t} L_t$. The before-tax real financial payoff amounts to $\tilde{r}_t a_{i,t-1}$.

We assume that agents pay two additional taxes. First, a capital tax τ_t^K is levied on interest payment, implying a net asset payoff of $(1-\tau_t^K)\tilde{r}_t a_{i,t-1}$. Second, an income tax τ_t^L is levied on total labor income, $\tau_t^L(1-\tau_t^W)\hat{w}_t y_{i,t} L_t$. The latter income tax τ_t^L is not internalized by the unions as each union's marginal contribution to total income is negligible. As a consequence, the post-tax total income is equal to $(1-\tau_t^L)(1-\tau_t^W)\hat{w}_t y_{i,t} L_t$.[4]

Agents earn this net total income and use it together with their past savings, to consume $c_{i,t}$ and save $a_{i,t}$. Their budget constraint can be expressed as follows:

$$c_{i,t} + a_{i,t} = a_{i,t-1} + (1-\tau_t^K)\tilde{r}_t a_{i,t-1} + (1-\tau_t^L)\big((1-\tau_t^W)\hat{w}_t y_{i,t} L_t\big). \tag{16}$$

To simplify the notation, we define the post-tax real interest and wage rates as:

$$r_t = (1-\tau_t^K)\tilde{r}_t, \tag{17}$$

$$w_t = (1-\tau_t^L)(1-\tau_t^W)\hat{w}_t = (1-\tau_t^L)(1-\tau_t^W)(1-\tau_t^E)\tilde{w}_t. \tag{18}$$

Given that $\hat{W}_t/P_t = w_t/(1-\tau_t^L)(1-\tau_t^W)$, we derive the law of motion for the post-tax real wage

[4]An alternative specification would treat τ^L as a tax on all income, including capital income. However, results would remain unchanged, as the set of feasible optimal allocations would be identical. In our setting, τ^K should be interpreted as the sum of corporate income tax and household capital income tax.

as a function of inflation and taxes

$$(1+\pi_t^W)\frac{w_{t-1}}{(1-\tau_{t-1}^W)(1-\tau_{t-1}^L)} = \frac{w_t}{(1-\tau_t^W)(1-\tau_t^L)}(1+\pi_t^P). \tag{19}$$

The agent's program can be finally be written as:

$$\max_{\{c_{i,t},a_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Theta_t \left(U(c_{i,t}, L_t)\right), \tag{20}$$

$$c_{i,t} + a_{i,t} = (1+r_t)a_{i,t-1} + w_t y_{i,t} L_t, a_{i,t}, \tag{21}$$

and subject to the credit constraint $a_{i,t} \geq -\underline{a}$, and the consumption positivity constraint $c_{i,t} > 0$. The notation $\mathbb{E}_0$ represents the expectation operator over both idiosyncratic and aggregate risks. The solution to the agent's program is a sequence of functions, defined over $([-\bar{a}; +\infty) \times \mathcal{Y}) \times \mathcal{Y}^t \times \mathbb{R}^t$ and denoted by $(c_t, a_t)_{t\geq 0}$, such that:[5]

$$c_{i,t} = c_t((a_{i,-1}, y_{i,0}), y_i^t, z^t),\ a_{i,t} = a_t((a_{i,-1}, y_{i,0}), y_i^t, z^t). \tag{22}$$

For simplicity, we retain the i-index notation. Denoting by $\nu_{i,t}$ the discounted Lagrange multipliers of the credit constraint, the Euler equation corresponding to the agent's program (20) is:

$$u'(c_{i,t}) = \beta_t \mathbb{E}_t\Big[(1+r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t}. \tag{23}$$

with the complementary slackness condition:

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0,\ \nu_{i,t} \geq 0. \tag{24}$$

The Marginal Value of Cash Constraint (MVCC). As in Section 2, the MVCC for agent i is defined as $MVCC_{i,t} = \nu_{i,t}/u'(c_{i,t})$. The Euler equation (23) of agent i becomes:

$$u'(c_{i,t}) = \beta_t \mathbb{E}_t\Big[\frac{1+r_{t+1}}{1-MVCC_{i,t}}u'(c_{i,t+1})\Big].$$

In the HA economy, there is a distribution of MVCC values. When agents are unconstrained, $MVCC_{i,t} = 0$; and $0 < MVCC_{i,t} < 1$. In the quantitative section, we will discuss the average value of MVCC across agents.

3.8 Government and market clearing conditions

The government finances an exogenous public good expenditure G_t by raising four taxes and issuing one-period riskless public debt. The government raises four linear taxes: (i) a tax τ_t^E

[5]See e.g. Miao (2006), Cheridito and Sagredo (2016), and Açikgöz (2018) for a proof of the existence of such functions.

based on labor cost $\tilde{w}_t$ and paid by employers, (ii) a tax τ_t^W based on bargained wage $\hat{w}_t$ and paid by workers, (iii) a tax τ_t^L based on income and paid by workers, and (iv) a capital tax τ_t^K. Importantly, the three labor instruments (τ_t^W, τ_t^E and τ_t^L) are independent and non-redundant. On the one hand, τ_t^E creates a wedge between the labor cost and the bargained wage, while τ_t^W and τ_t^L create wedges between the bargained wedge and the net wage. On the other hand, τ_t^W is internalized by unions, while τ_t^L is not. These three labor taxes operate on different margins and enable us to derive our equivalence result below. They should be understood as theoretical tools necessary to generate price and wage stability. Each tax will be removed in turn to consider more realistic fiscal settings and to assess how each fiscal instrument contributes to inflation volatility.

In addition to capital and labor taxes and public debt, the government also taxes firms' profits, Ω_t, which limits the distortions implied by profit distribution.[6]The government budget constraint can be expressed as:

$$
\begin{aligned}
G_t + \frac{1+i_{t-1}}{1+\pi_t^P} B_{t-1} \leq \Omega_t + B_t + \tau_t^L (1-\tau_t^W)\hat{w}_t L_t \\
+ \tau_t^K \tilde{r}_t \int_i a_{i,t-1}\ell(di) + \tau_t^W \hat{w}_t L_t + \tau_t^E \tilde{w}_t L_t.
\end{aligned}
$$

Using the financial market clearing condition (14), the post-tax interest rate $\tilde{r}_t$ (15) and the post-tax rate definitions (17), we simplify the government budget constraint to:

$$
G_t + r_t B_{t-1} + w_t L_t \leq \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big) Y_t + B_t - B_{t-1}, \tag{25}
$$

We also express the financial market clearing condition and the economy's resource constraints as:

$$
\int_i a_{i,t}\ell(di) = B_t, \tag{26}
$$

$$
\int_i c_{i,t}\ell(di) + G_t = \left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right) Z_t L_t. \tag{27}
$$

Equilibrium definition. We now formulate our definition of competitive equilibrium.

Definition 1 (Sequential equilibrium) *For any exogenous paths of aggregate shocks (S_t), characterizing TFP $(Z_t)_t$, public spending $(G_t)_t$, discount factors $(\beta_t)_t$, and productivity levels, $(\mathcal{Y}_t)_t$, a sequential competitive equilibrium is a collection of individual allocations $(c_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$, aggregate quantities $(L_t, A_t, Y_t, \Omega_t, m_t)_{t\geq 0}$, price processes $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$, monetary policy $(i_t)_{t\geq 0}$, fiscal policies $(\tau_t^W, \tau_t^E, \tau_t^L, \tau_t^K, B_t)_{t\geq 0}$, and inflation dynamics $(\pi_t^W, \pi_t^P)_{t\geq 0}$ such that, for an initial wealth and productivity distribution $(a_{i,-1}, y_{i,0})_{i\in\mathcal{I}}$, and for an initial value of public debt satisfying $B_{-1} = \int_i a_{i,-1}\ell(di)$, we have:*

[6]Alternative modeling strategies could involve distributing profits to agents or introducing a fund that receives interest payments and profits (see LeGrand et al., 2022 for a discussion and references). We adopt the current assumption to simplify the algebra, as these alternatives yield quantitatively similar results.

1. *given prices, the allocations* $(c_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$ *solve the agent's optimization program (20)–(21);*

2. *financial and goods markets clear at all dates: for all* $t \geq 0$*, equations (26) and (27) hold;*

3. *the government budget is balanced at all dates: equation (25) holds for all* $t \geq 0$*;*

4. *firms' profits* Ω_t *and the mark-up* m_t *are consistent with firms profit maximization.*

5. *the price inflation path* $(\pi_t^P)_{t\geq 0}$ *is consistent with the price Phillips curve (12), while the wage inflation path* $(\pi_t^W)_{t\geq 0}$ *is consistent with the wage Phillips curve (13);*

6. *the real and nominal rates* $(\tilde{r}_t, i_t)_{t\geq 0}$ *verify (15);*

7. *post tax rates* $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$ *are defined in equations (17)–(18).*

Social Welfare Function. We assume that the planner maximizes a generalized Social Welfare Function (SWF), where the weights ω on each period's utility can depend on the agent's current productivity. The planner's objective is:

$$W_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \Theta_t \int_i \omega(y_t^i) U(c_t^i, l_t^i) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right]. \tag{28}$$

This expression encompasses the utilitarian case, where $\omega(y) = 1$ for all y. This generalization of the standard SWF is now used either in both quantitative work (e.g., LeGrand and Ragot, 2025 and McKay and Wolf, 2022) and theoretical investigations as a deviation from the utilitarian case (see Dávila and Schaab, 2022). A theoretical foundation is provided in LeGrand et al. (2025). We use it here to facilitate simulations and comparisons of economies in Section 5.

We assume that the economy starts from a steady-state situation where the fiscal system is optimally determined. In period 0, the economy is hit by an aggregate shock affecting either G_t, Z_t, β_t or the productivity levels y_t. The entire path of these shocks is known in period 0, and the planner optimally sets its available instruments under commitment.

We have introduced five fiscal instruments $(\tau_t^W, \tau_t^E, \tau_t^L, \tau_t^K, B_t)_{t\geq 0}$. This rich fiscal system is a theoretical device to understand differences between HA and RA economies with both price and wage stickiness. As shown below, this fiscal system is the minimal one required to ensure no deviation from price and wage stability in all cases. In what follows, we consider different fiscal systems, where only some fiscal instruments, rather than all, are available to the planner to smooth the effect of the aggregate shock. Specifically, we solve for optimal monetary policy considering subsets $\mathcal{I} \subset \{\tau^W, \tau^E, \tau^L, \tau^K\}$ of available fiscal instruments. Public debt is always optimally set, which is theoretically and empirically relevant in this environment. The set $\mathcal{I}$ is fixed and does not change across periods. For all other instruments $I \in \{\tau^W, \tau^E, \tau^L, \tau^K, B\} \setminus \mathcal{I}$, we assume that the instrument is constant and set to its steady-state value: $I_t = I_{ss}$ at all dates. Ramsey equilibrium definitions.

We start with the definitions when all instruments are available.

Definition 2 (Ramsey equilibrium) *For a given path of aggregate shocks $(S_t)_{t\geq 0}$, a Ramsey equilibrium with all instruments is the path of monetary policy $(i_t)_{t\geq 0}$, fiscal instruments $(\tau_t^W, \tau_t^E, \tau_t^L, \tau_t^K, B_t)_{t\geq 0}$, which selects a sequential equilibrium following Definition 1 and maximizing the SWF (28).*

Definition 3 (Ramsey steady state) *A steady-state Ramsey equilibrium is a Ramsey equilibrium where aggregate real variables $(L_t, A_t, Y_t, \Omega_t, m_t)_{t\geq 0}$, prices $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$, monetary policy $(i_t)_{t\geq 0}$, fiscal policies $(\tau_t^W, \tau_t^E, \tau_t^L, \tau_t^K, B_t)_{t\geq 0}$, and inflation dynamics $(\pi_t^W, \pi_t^P)_{t\geq 0}$ are constant. The value of the instruments are denoted as $(\tau_{ss}^W, \tau_{ss}^E, \tau_{ss}^L, \tau_{ss}^K, B_{ss})$.*

We then turn to the case of a limited set of instruments.

Definition 4 (Ramsey equilibrium with limited instruments) *For a given path of aggregate shocks $(S_t)_{t\geq 0}$ and a given set of available instruments $\mathcal{I} \subset \{\tau^W, \tau^E, \tau^L, \tau^K\}$, a Ramsey equilibrium with a limited number of instruments is the path of monetary policy $(i_t)_{t\geq 0}$, public debt $(B_t)_{t\geq 0}$, and fiscal instruments $(\mathcal{I}_t)_{t\geq 0}$, which selects a competitive equilibrium maximizing the SWF (28) given that the unavailable instruments are set to their steady-state values: $I_t = I_{ss}$ for all $I \in \{\tau^W, \tau^E, \tau^L, \tau^K\} \setminus \mathcal{I}$.*

We first solve the Ramsey model without aggregate shock to compute the steady-state values of instruments, $(\tau_{ss}^W, \tau_{ss}^E, \tau_{ss}^L, \tau_{ss}^K)$, and then we solve for the optimal dynamics of the available instruments. For the simulation of the dynamics for a given set of available instruments $\mathcal{I}$, observe that unavailable instruments are set to their steady-state value. Therefore, regardless of the choice of $\mathcal{I}$, the Ramsey equilibrium will feature the same steady-state allocation, as the aggregate shock is transitory.

3.9 The representative agent economy

We compare HA results to the optimal allocation in the RA economy, which is standard. A representative agent maximizes utility in an economy without financial constraints. The planner uses distorting tools and public debt to finance public spending. The algebra is provided in Appendix C.

4 Optimal policies with heterogeneous agents

4.1 Characterizing the Ramsey allocation

We derive optimal policies in HA economies for all aggregate shocks, given a set of available fiscal instruments $\mathcal{I}$. The Ramsey planner's program is:

$$\max_{\left(\mathcal{I}_t, w_t, r_t, L_t, B_t, \pi_t^P, \pi_t^W, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Theta_t \left[\int_i \omega(y_t^i) \left(u(c_t^i) - v(L) \right) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right], \quad (29)$$

subject to the government budget constraint (25), the individual budget constraints (21), the individual Euler equation (23), the individual slackness condition (24), the individual positivity constraints $c_t^i, l_t^i \geq 0$ (for given initial wealth a_{-1}^i), the price Phillips curve (12), the wage Phillips curve (13), the real wage dynamics (19), and that unavailable instruments verify $I_t = I_{ss}$ for all $I \notin \mathcal{I}$. We provide the full program in Appendix D, where we also derive the first-order conditions of the planner. As in LeGrand and Ragot (2025), we use aspects of Marcet and Marimon (2019) to factorize the Lagrangian. On a technical note, the factorization of the price and wage Phillips curve is straightforward, as both can interpreted as Euler equations, for firms and unions, respectively.

This economy features several frictions, which are worth summarizing. The monetary side of the economy exhibits two sets of market imperfections. The first set relates to the goods market. Intermediary firms posses monopoly power, which implies a price markup m_t that can differ from one. There is also a Rotemberg cost for price adjustment, preventing firms from freely setting their price. Notably, these two market imperfections are complementary: one vanishes when the other is absent, as evident from the price Phillips curve (12). The second set of imperfections pertains to the labor market. The presence of unions means that agents' labor supply is not set optimally, while the Rotemberg cost for wages prevents unions from freely adjusting wages. In the absence of Rotemberg cost, labor supply remains sub-optimal, as it still determined at the union level and characterized by: $v'(L_t) = w_t \int_i y_{i,t} u'(c_{i,t}) \ell(di)$. If agents could choose their own labor supply, $l_{i,t}$, it would be individual-specific and satisfy: $v'(l_{i,t}) = w_t y_{i,t} u'(c_{i,t})$.

To gain further insights into the optimal allocation in the HA economy, it is useful to introduce the notion of *social valuation of liquidity (SVL) for agent i,* which represents the value to the planner of transferring one additional unit of the consumption good to agent i in period t. In LeGrand and Ragot (2025); LeGrand et al. (2025), we show that this statistics simplifies the derivation of the FOCs of the planner, and that it is related to the *Generalized Social Marginal Welfare Weights* (GSMWW) introduced by Saez and Stantcheva (2016).

More precisely, we denote by $\Theta_t \lambda_{i,t}$ the Lagrange multipliers of the Euler equations (23 of agent i at date t. The Lagrange multiplier of the government budget constraint is $\Theta_t \mu_t$ (25) and the Lagrange multipliers on the price and wage Phillips curves (12) and (13) are denoted $\Theta_t \gamma_{P,t}$

and $\Theta_t\gamma_{W,t}$, respectively. From the Lagrangian denoted $\mathcal{L}$, we define the SVL $\psi_{i,t}$ as:

$$\psi_{i,t} := \frac{\partial \mathcal{L}}{\partial c_{i,t}},$$

which can also be expressed as:

$$\psi_{i,t} := \underbrace{\omega_t^i u'(c_{i,t})}_{\text{direct effet}} - \underbrace{(\lambda_{i,t} - (1+r_t)\lambda_{i,t-1})\, u''(c_{i,t})}_{\text{effect on savings}} - \underbrace{\frac{\varepsilon_W - 1}{\psi_W}\gamma_{W,t}\frac{w_t y_{i,t} L_t}{1-\tau_t^L}u''(c_{i,t})}_{\text{effect on the bargained wage}} \tag{30}$$

As shown in equation (30), this valuation consists of three terms. The first term, $\omega_t^i u'(c_{i,t})$, represents the private valuation of liquidity for agent i scaled by the planner's current weight for agent i. The second term in (30) accounts for the impact of an additional unit of consumption on saving incentives from periods $t-1$ to t and from periods t to $t+1$. An extra unit of consumption makes the agent more willing to smooth out consumption between periods t and $t+1$, making the Euler equation more binding. This more "binding" constraint reduces the utility by the algebraic quantity $u''(c_{i,t})\lambda_{i,t}$. Conversely, this extra consumption unit also makes the agent less willing to smooth consumption between periods $t-1$ and t and therefore "relaxes" the constraint from period $t-1$, as reflected in $\lambda_{i,t-1}$. The third term captures the effect of the transfer on the unions' marginal incentives to bargain over wages and is hence proportional to the Lagrange multiplier γ_W on the wage Phillips curve.

This expression for $\psi_{i,t}$ is common across all HA economies we consider, regardless of the set of available tools. The planner's FOCs will depend on the specific set of available instruments. We provide all derivations in Appendix D.

The corresponding RA economy. We solve the same problem with a representative agent instead of the HA structure. For brevity, we present the problem and the FOCs in Appendix C, as the solution techniques are more standard. However, to the best of our knowledge, this problem has not been solved with such a rich fiscal structure.

4.2 The equivalence result

We can now state our main equivalence result.

Proposition 2 (An equivalence result) *In the HA economy, when all instruments* $(\tau^L, \tau^E, \tau^W, \tau^K)$ *are optimally chosen, the planner exactly implements* $\pi_t^P = 0$ *and* $\pi_t^W = 0$*, for every path of aggregate shocks* $(S_t)_{t\geq 0}$*.*

Proposition 2 generalizes the equivalence result of Correia et al. (2008) and Correia et al. (2013) for RA economies and LeGrand et al. (2022) for HA economy, to the case where both sticky prices and sticky wages are present. Notably, compared to LeGrand et al. (2022), we need two additional instruments (τ_t^L, τ_t^E), while introducing one additional nominal constraint. Specifically, one instrument is needed to prevent wage inflation (which destroys resources), and another is required to replicate the flexible-price labor supply and neutralize the market power of unions. In the presence of a sufficiently large fiscal system, monetary policy plays no role beyond ensuring stability. This result holds for all possible paths of aggregate shocks, regardless of whether they affect TFP, public spending, discount factors, or productivity levels.

Importantly, the result hindges on the presence of two labor taxes. The first labor tax τ^E (internalized by the planner) enables the planner to "isolate" the pre-tax rate $\tilde{w}_t$, which is determined by the allocation (with zero price inflation), from the union wage $\hat{w}_t$, which is determined by the inflation path $(\pi_t^W)_t$. Removing τ^E as an independent instrument imposes a constraint linking the factor price $\tilde{w}_t$ and the wage inflation path. In other words, the planner would have to balance the effects of price inflation (determining $\tilde{w}_t$) and of wage inflation (determining $\hat{w}_t$). The second labor tax τ_t^L allows the planner to simultaneously set the labor supply optimally and close the wage gap in the wage Phillips curve. Without τ^L, the planner would face a tradeoff between two inefficiencies: (i) the sub-optimal labor supply due to union market power and (ii) the cost of wage inflation. If either of these two instruments were removed, Proposition 2 would no longer hold, and the economy would exhibit positive inflation in wages or prices.

Overall, the first part of Proposition 2 rationalizes our tax system, which represents the minimal tax system for which price stability is optimal.[7]

To assess the deviation in the allocations and in price and wage stability, we now provide a quantitative investigation of economies in which we vary the set of available fiscal instruments.

5 Quantitative analysis of optimal policies

We now focus on two pairs of economies to derive the main lessons regarding the differences between HA and RA allocations, as well as the deviations from price and wage stability. In each case, the ies differ based on the set of taxes to the planner (with all other taxes held constant at their steady-state value). Given four taxes $(\tau^L, \tau^E, \tau^W, \tau^K)$, we can analyze 15 subsets of time-varying tools (since $2^4 - 1 = 15$, as at least one tax must be adjustable to prevent public debt divergence). We examine the effects of four shocks (Z_t, β_t, G_t, y_t) in both RA and HA economies. Rather than presenting results for all these 120 combinations, we concentrate on the economies that best clarify the relevant mechanisms.

More precisely, for each pair of economies (HA and RA), we contrast the implications of a TFP shock with those of a discount factor shock. This approach is a consistent extension of

[7] More precisely, while other tax systems could also achieve price and wage stability—such as introducing a time-varying consumption tax, as in Correia et al. (2008)—the number of independent instruments would not be reduced. We consider our tax system to be realistic.

our theoretical investigation in Section 2. These two shocks yield very different implications for optimal policy. We consider first the case where all fiscal instruments are available: $\mathcal{I}^{(1)} = (\tau^K, \tau^W, \tau^E, \tau^L)$. We then analyze two scenarios where we remove some labor taxes. In the second economy, τ^W is held constant $\mathcal{I}^{(2)} = (\tau^K, \tau^E, \tau^L)$. In the third economy, τ^E is the only time-varying labor tax. $\mathcal{I}^{(3)} = (\tau^K, \tau^W)$. The rationale for this selection will become clear in the interpretation of the results.

Finally, we discuss the effects of other missing instruments (notably τ^K) and the two remaining aggregate shocks in Section 5.6.

To conduct these simulations, we first calibrate the model in Section 5.1. We explain how to compute optimal policies in HA economies in Section 5.2.

5.1 The calibration and steady-state distribution

The time period is a quarter.

Aggregate shock. The aggregate shock (S_t) follows an AR(1) process, such that $S_t = \rho S_{t-1}$, where we set $\rho = 0.95$ to ensure the same persistence across all channels of the aggregate shock.

Technology and TFP shock. The production function is: $Y = ZL$. The TFP process is a standard AR(1) process and satisifies $Z_t = \exp(z_0 S_t)$, where $z_0 < 0$ represents the initial negative TFP shock.

Preferences. The steady-state discount factor is $\beta = 0.99$, and the period utility function is: $\frac{c^{1-\sigma}-1}{1-\sigma} - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}$. The Frisch elasticity of labor supply is set to $\varphi = 0.5$, which is the value recommended by Chetty et al. (2011) for the intensive margin in HA models. The scaling parameter is $\chi = 0.01$, which implies an aggregate labor supply of roughly one-third.

The process for β_t verifies $\beta_t = \beta \times \exp(b_0 S_t)$, where $b_0 > 0$ is a period-0 positive shock to the discount factor.

Idiosyncratic risk. We use a standard productivity process: $\log y_t = \rho_y \log y_{t-1} + \varepsilon_t^y$, with $\varepsilon_t^y \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_y^2)$. We calibrate the persistence of the productivity process as $\rho_y = 0.994$ and the standard deviation as $\sigma_y = 0.06$. These values are consistent with empirical estimates (Krueger et al., 2018), and generate a steady-state Gini coefficient of wealth of 0.78, which aligns with the data.[8] Finally, we use the Rouwenhorst (1995) procedure to discretize the productivity process into 10 idiosyncratic states $\{y_1, \ldots, y_{10}\}$ with a constant transition matrix.

To consider an increase in the variance of the productivity process, we define $y_{i,t} := y_i + (-1)^{1_{i\leq 5}} n_i^{-1} u_t \delta_0$, where $\delta_0 > 0$ is the initial variance shock, n_i is the share of agents with productivity y_i and $(-1)^{1_{i\leq 5}} = -1$ if $i \leq 5$ and 1 otherwise (recall that there are 10 productivity

[8]The Gini coefficient of wealth is 0.78 using the SCF data in 2007, before the 2008 Great Recession.

levels). This specification leaves the average productivity level unchanged but increases the variance. Low productivity levels decrease further, while high-productivity levels increase.

Steady state taxes and public debt, and public spending shock. We first solve the model with constant exogenous taxes and explain the choice of the SWF below. We assume that employer social contributions and capital taxes are 0, $\tau^E = \tau^K = 0$.[9] The income tax τ^L is set to $\frac{1}{\varepsilon_W}$ to offset distortions on the labor market due to the monopoly power of unions. We assume $\tau^W = 16\%$. This value, combined with the value of public debt level described below, implies that public spending amounts to 15 of GDP, which is close to the US value in 2007. The amount of public debt (which is the only asset in this economy) is set to an annual value of 128% of GDP. Since public debt is the sole asset, we calibrate this level to achieve an average Marginal Propensity to Consume (MPC) of 0.3.[10]

The process for G_t is a AR(1) process, with $G_t = G\exp(g_0 S_t)$ where $g_0 > 0$ is the period-0 positive shock to public spending.

Monetary parameters. Following the literature, particularly Schmitt-Grohé and Uribe (2005), we assume that the elasticity of substitution is $\varepsilon_P = 6$ across goods and $\varepsilon_W = 21$ across labor types. The price adjustment cost is set to $\psi_P = 100$, such that the slope of the price Phillips curve is $\frac{\varepsilon_P - 1}{\psi_P} = 5\%$ (see Bilbiie and Ragot, 2021, for a discussion and references). The wage adjustment cost is set to $\psi_W = 2100$, such that the slope of the wage Phillips curve is 1%, , reflecting the assumption that wages are stickier than prices.[11] Since there is no steady-state inflation in prices or wages: $\pi^P = \pi^W = 0$, these coefficients only affect the dynamics.

Table 1 summarizes the model parameters.

Calibration of the RA economy. The calibration of the RA economy retains the same preference parameters as in the HA economy. Allocations in the RA (resp., HA) economy are denoted with a superscript RA (HA). In the RA economy, the first-best allocation is achieved at the steady-state. The steady-state labor supply, L^{RA} (with $\pi^W = 0$), is determined by the FOC: $v'(L^{RA}) = u'(c^{RA})$. We set public spending in the RA economy, G^{RA}, such that the public-spending-to-GDP ratio is equalized across the two economies: $G^{RA}/Y^{RA} = G^{HA}/Y^{HA}$.

[9] Setting a zero capital tax is necessary to facilitate the comparison bewteen HA and RA models. In the latter, the optimal steady-state capital tax is 0 (when it exists), which is not necessarily the case in HA framework. See LeGrand and Ragot (2025) for a discussion.

[10] We thus adopt a liquid one-asset liquid wealth calibration to match a realistic MPC (Kaplan and Violante, 2022).

[11] Sensitivity analysis confirms that our qualitative results are robust to these values, although the volatility of price and wage inflation obviously increases with the slopes of Phillips curves.

Parameter	Description	Value	Target
	Preference and technology		
β	Discount factor	0.99	Quarterly calibration
σ	Curvature utility	2	
$\bar{a}$	Credit limit	0	
χ	Scaling param. labor supply	0.01	$L = 1/3$
φ	Frisch elasticity labor supply	0.5	Chetty et al. (2011)
	Shock process		
ρ_y	Autocorrelation idio. income	0.993	Krueger et al. (2018)
σ_y	Standard dev. idio. income	6%	$Gini = 0.78$
ρ_z	Autocorrelation TFP shock	0.95	
	Tax system		
τ^W	Worker social contribution	16%	$G/Y = 15\%$
τ^L	Income tax	4.74%	$1/\varepsilon_w$
τ^E,τ^L,τ^K	Other tax	0%	
B/Y	Public debt over yearly GDP	128%	$MPC = 0.3$
G/Y	Public spending over yearly GDP	15%	Targeted
	Monetary parameters		
ε_p	Elasticity of sub. between goods	6	Schmitt-Grohé and Uribe (2005)
ψ_p	Price adjustment cost	100	Price PC 5%
ε_w	Elasticity of sub. labor inputs	21	Schmitt-Grohé and Uribe (2005)
ψ_w	Wage adjustment cost	2100	Wage PC 1%

Table 1: Parameter values for the baseline calibration. See the text for descriptions and calibration targets.

5.2 Simulating optimal policies in the HA economies

To investigate the optimal dynamics of the model, we perform the following experiment – which is standard in the New Keynesian RA literature, but which must be adapted to the HA case. We first solve for the optimal policy for a given set of instruments and consider the steady-state allocation – which represents the long-run allocation in the absence of aggregate shock. We then assume that the economy starts from this Ramsey steady state and implement a period-0 persistent MIT shock, either a TFP or a discount factor shock. This procedure allows us to quantify the extent to which the economy deviates from the steady state before converging back to it. The magnitude of the economy's response to the shock depends both on the nature of the shock and on the set of instruments available to the planner.

The steady state crucially depends on the SWF used in the Ramsey program, as well as in general on the tools available to the planner. To overcome this difficulty and ensure that all simulations start from the same steady state, we employ the inverse optimal taxation approach, as in Heathcote and Tsujiyama (2021) and LeGrand and Ragot (2025). Specifically, we fix thesteady-state fiscal instruments, defined by $\tau^E = \tau^K = 0$, $\tau^L = 1/\varepsilon_w$ and $\tau^W > 0$, and estimate the weights of the SWF for each set of fiscal tools to ensure that this steady state is optimal. Each instrument available to the planner generates a FOC, imposing a restriction on the SWF.[12]Nonetheless, these constraints alone are insufficient to uniquely determine the SWF. We therefore select the SWF closest to the utilitarian SWF (where all weights are equal) that satisfies these restrictions. We also verify that the choice of the SWF does not quantitatively affect the first-order dynamics of the allocation.

The Ramsey problem in HA models cannot be solved with standard simulation techniques. The Ramsey equilibrium involves a joint distribution across wealth and Lagrange multipliers, which is a high-dimensional object. While computing the steady-state values of Lagrange multipliers is already challenging, the Ramsey solution further requires solving the dynamics of this joint distribution. To address this complexity, we employ the truncation method developed by LeGrand and Ragot (2022a) to determine the joint distribution of individual wealth and Lagrange multipliers.[13] The accuracy of optimal policies, both in the steady state and dynamics, has been analyzed in LeGrand and Ragot (2023). In addition, LeGrand and Ragot (2022b) propose an improvement to efficiently reduce the state space. Further details on the method and its application in the present setup are provided in Appendix.

To determine the steady-state values of the Lagrange multipliers and SWF for a given fiscal policy, we use the following algorithm:

1. Set a truncation structure (with a maximum truncation length N) and set the desired values for the fiscal instruments.

2. Solve the steady-state allocation of the full-fledged Bewley model, using standard techniques and the specified instrument values.

3. Compute the truncated representation of the economy by aggregating over truncated histories.

4. Solve the steady-state Ramsey problem in the truncated economy through the following steps:

    (a) Derive FOCs of the planner for each instrument in the truncated representation.

[12]This strategy ensures the existence of a consistent steady-state. An alternative approach would involve specifyinga given SWF function and solving for the optimal Ramsey steady state. However, this method may yield unrealistic steady-state allocations (Auclert et al., 2024 or LeGrand and Ragot, 2025 for a discussion). As in standard New Keynesian models, optimal steady-state price and wage inflation rates are zero, regardless of the SWF. Consequently, steady-state price stability does not impose any additional restriction on the SWF.

[13]Optimizing simple rules in the spirit of Krusell and Smith (1998) is also difficult to implement due to the large number of many independent instruments.

(b) Compute the SWF weights that are the closest to 1, for which all the planner's FOCs hold.

(c) Compute the associated Lagrange multipliers.

(d) The truncated representation, combined with the fiscal instruments, the estimated SWF, and the Lagrange multipliers constitutes a steady-state optimal Ramsey allocation for the truncated representation.

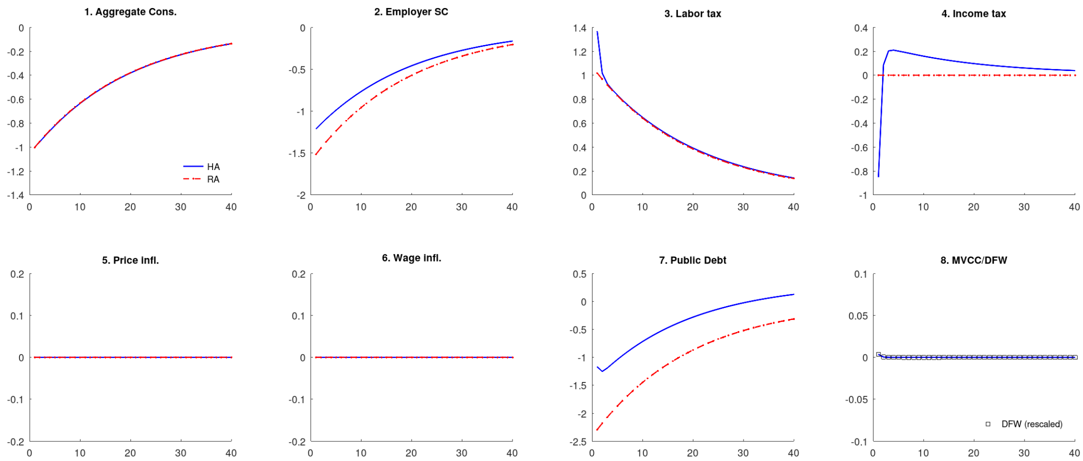
5. Compute the optimal dynamics of the instruments and allocation in the truncated economy using the planner's FOCs, as is standard in finite state space models.

We use the refined truncation approach, setting the refinement truncation length to $N = 8$. We check that the results do not depend on the choice of the truncation length. Consistent with LeGrand and Ragot (2022a), the truncation method provides accurate results, thanks to the introduction of the ξs parameters.

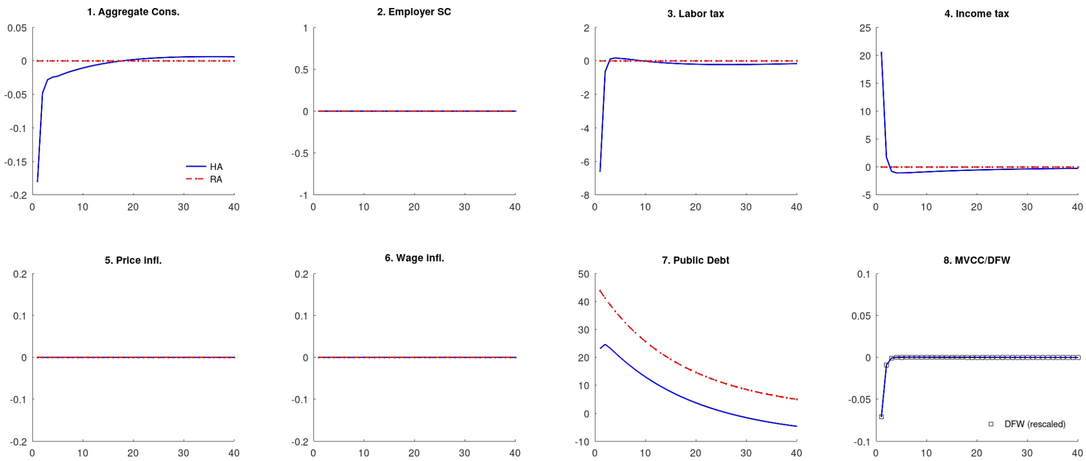
5.3 The economies with all instruments

We consider an economy where the planner has access to all fiscal instruments, with no restriction. We plot the Impulse Response Functions (IRFs) of allocations and planner's instruments following either a transitory negative TFP shock or a transitory positive discount factor shock. The solid blue line corresponds to the HA economy and the red dashed line to the RA economy. We report the IRFs for eight key variables over 40 periods: aggregate consumption (panel 1), expressed as a percentage deviation from the steady state; the three labor taxes, all in level deviations: employer social contribution τ_t^E (panel 2), the worker social contribution tax τ_t^W (panel 3), and the labor tax τ_t^L (panel 4); the two inflation rates, for prices (π^P, panel 5) and wages (π^W, panel 6), both also in level deviations; the public debt (B_t, panel 7); the MVCC and the DFW (panel 8). In the latter panel, we only report the MVCC for the HA economy using a solid blue plain line, as it is inapplicable for the RA economy, where credit constraints never bind. We also report the DFW using black squares. We have rescaled the series for the DFW by the same constant (approximately equal to 2), which depends solely on steady-state parameters and is identical across economies. As will be evident, the (rescaled) series of DFW and MVCC are nearly—though not exactly—identical across all simulations.

First, both price and wage inflation rates remain at zero along the dynamics, as expected. This price and wage stability is independent of the nature of the shock and holds in Figures 1a and 1b. Second, the response of aggregate allocation differs markedly depending on the shock. In the case of the TFP shock, the aggregate consumption response is virtually identical between the HA and RA economies. In contrast, following a discount factor shock, aggregate consumption does not react in the RA economy but falls on impact before rapidly converging back to zero in the HA economy. The RA economy implements the first-best allocation, which is independent



(a) TFP shock.



(b) Discount factor shock.

Figure 1: Dynamics of the economy when all instruments are available, after a TFP and discount factor shock. The Heterogeneous-Agent economy (HA) is in blue and the Representative Agent (RA) is in red. Variables are in percentage proportional change, except tax rates and inflation rates which are in percentage level change.

of agents' discount factor. Only two quantities react to the discount factor shock in the RA economy: the interest rate and the public debt. The interest rate does not affect the allocation and is merely a residual of the Euler equation. Public debt, computed from the government's budget constraint, increases to reflect the changes in the interest rate.

In the HA economy, the aggregate allocation is sensitive to the discount factor shock. Agents become more patient, leading them to save more. Because of credit constraints, the economy is non-Ricardian and these larger individual savings translate into lower aggregate consumption. This explains the drop on impact of aggregate consumption in Panel 1. Similarly, the rise in individual savings implies that public debt must increase to accommodate the extra saving demand coming from more patient agents. As explained in the theoretical model of Section 2, the MVCC (panel 8) is a strong indicator of whether HA and RA economies do differ. The MVCC measures the extent to which the credit constraints are binding in the economy and, thus, the degree to which the economy is non-Ricardian. For the TFP shock in Figure 1a, the MVCC barely changes, indicating that HA and RA allocations do not differ significantly. For the discount factor shock in Figure 1b, the MVCC responds strongly on impact, highlighting the quantitative importance of the non-Ricardian dimension in this case and the differing responses of aggregate consumption in HA and RA economies. As noted, the responses of the MVCC and DFW are observationally equivalent.

Finally, the reaction of fiscal tools differs across economies and shocks. After the TFP shock, the planner substantially reduces social the employer contribution (τ^E). This adjustment aligns the cost and the marginal productivity of labor more closely. Subsidizing labor demand helps mitigate the negative impact of the contraction. Conversely, the planner increases the worker social contribution (τ^W) by a comparable amount to prevent an excessive reduction in the wage targeted by unions (which would be costly) and to avoid heavily distorting household supply of labor. These two sizable movements occur in both i the HA and RA cases. Finally, regarding the labor tax (τ^L), it remains unaffected in the RA economy, as it is a redundant instrument that provides no benefit .[14] However, in the HA economy, τ^L is slightly reduced on impact to ease households' budget constraints, which explains why the increase in τ^W on impact is slightly larger in the HA economy than in RA one.

5.4 The economies when τ^W is not available

Figure 2 plots the response of the HA and RA economies to a TFP and discount factor shocks, when both the labor tax τ^L and the employer social contribution, τ^E, are available to the planner. For both shocks, the worker social contribution τ^W remains unchanged by construction.

[14] Individual and government budget constraints can be combined inyo the resource constraint, where the wage and the interest rates play no role. The wage rate w thus influences the Phillips curves (12)–(13) and the consistency equation (19) between inflation rates. This role is analogous to that of $w/(1-\tau^L)$ making w and τ^L substitute.

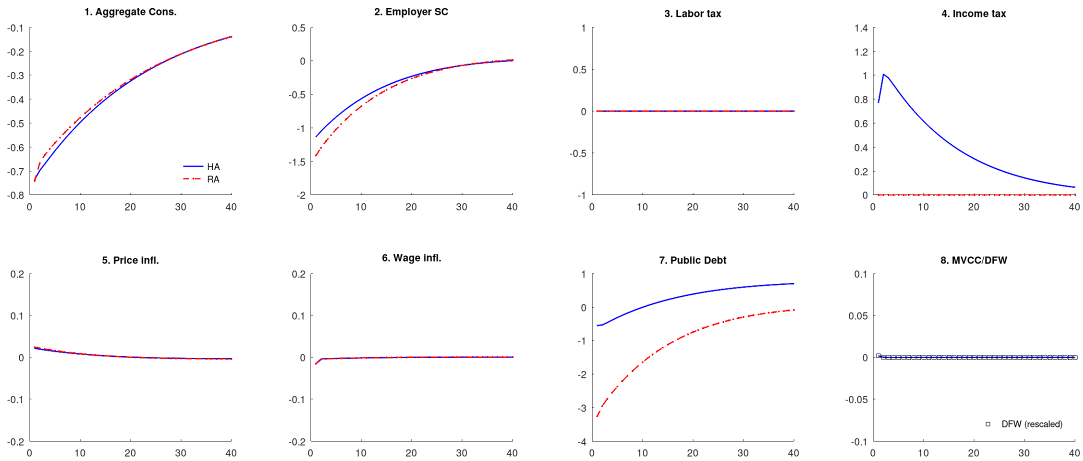
The outcomes differ across the two shocks. First, for the TFP shock as shown in Figure 2a, the real effect of the shock is similar in the HA and RA economies, even despite the unavailability of τ^W. This is corroborated by the MVCC and DFW, which exhibit negligible movement. Compared to the case where all instruments are available, there is a (small) inflation response, observed both in prices and wages, but it remains similar across the HA and RA economies. Unlike in the full-instrument case, the income tax sharply increases in the HA economy at impact, while it had slightly decreased when all instruments were available. The reason is that this increase acts as a substitute for the increase in the worker social contribution tax observed in the full instrument case, as the worker social contribution is here constrained to remain constant. The overall similarity between the full-instrument case and this one suggests that the labor tax τ^L can serve as a reasonably effective substitute for the unavailability of the worker social contribution, τ^W.

Second, in the case of the discount factor shock plotted in Figure 2b, the picture is slightly different. As in the full-instrument case, the real response differs between the HA and RA economies. This shock continues to have no impact in the RA economy, as reflected in the MVCC and DFW. However, the drop in consumption is much larger when τ^W is not available, highlighting the absence of this instrument. In the full-instrument case the worker social contribution had sharply decreased, while the employer social contribution was kept constant and the income tax increased. Here, the unavailability of a time-varying worker social contribution forces the employer social contribution to act as a substitute, since the income tax must still increase at impact and thus cannot fulfill this role. This explains why the employer social contribution now decreases at impact. However, this instrument is a poor substitute for the missing worker social contribution, because it affects the bargained wage differently. In addition to the larger real effects observed, the planner must also deviate from price and wage stability to compensate for the unavailability of the worker social contribution.

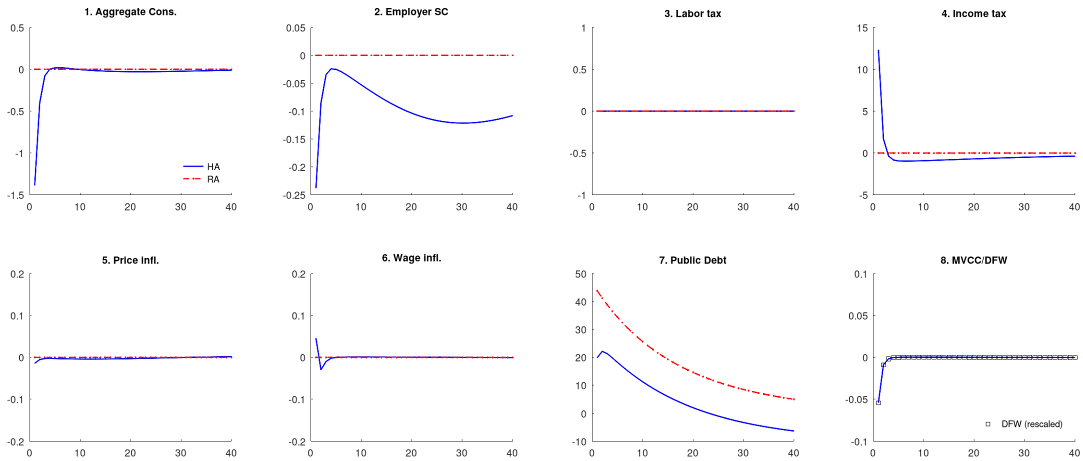
For the sake of conciseness, we do not report the cases where only τ^L or τ^E are unavailable. The absence of the labor tax τ^L yields results broadly similar to those documented here whenτ^W is unavailable. This again shows that these two instruments are reasonably good substitutes. When τ^E is missing, the outcomes closely resemble the full-instrument case, indicating that this instrument plays a less critical role than the other two. This finding is further confirmed in the following section.

5.5 The economies when τ^W and τ^L are not available

Figure 3 illustrates the response of the HA and RA economies to a TFP and a discount factor shock, when the employer social contribution, τ^E, is the sole labor fiscal instrument available to the planner. By construction, worker social contribution (τ^W) and labor tax (τ^L) remain unchanged for both shocks.



(a) TFP shock.



(b) Discount factor shock.

Figure 2: Dynamics of the economy when the worker social contribution τ^W is the only unavailable fiscal instrument, following a TFP and a discount factor shock. The HA economy is represented in blue and the RA one in red. All variables are expressed in percentage proportional changes, except for tax and inflation rates, which are presented in percentage level change.

The results differ markedly from the full-instrument case presented in Figure 1, and even from the scenario without the worker social contribution (τ^W) in Figure 2. For both shocks, the real response differs between HA and RA economies when only τ^E is available. This is again reflected in the MVCC and DFW (Panel 8). Additionally, the planner deviates from price and wage stability—for both shocks in the HA economy and for the TFP shock in the RA economy.

In response to the TFP shock, both price and wage inflations are used to implement a decline in the real wage following a negative TFP shock, reflecting that τ^E is not an effective instrument for the planner. This adjustment occurs in both the RA and HA economies, but the deviation from price stability is more pronounced in the HA economy. This experience suggests that when no efficient tax is available to reduce the labor wedge, i.e the gap between the real wage and the marginal productivity of labor, then the planner relies on monetary policy, using price and wage inflations as substitutes for the missing instrument.

For the discount factor shock, both the allocation and the dynamics of price and wage inflation differ between the HA and RA economies. The planner implements an increase in the real wage, leveraging both price and wage inflation to achieve this adjustment. As for the TFP shock, the departure frm price and wage stability is more pronounced in the HA economy, thereby illustrating that that the absence of τ^W or τ^L is costlier.

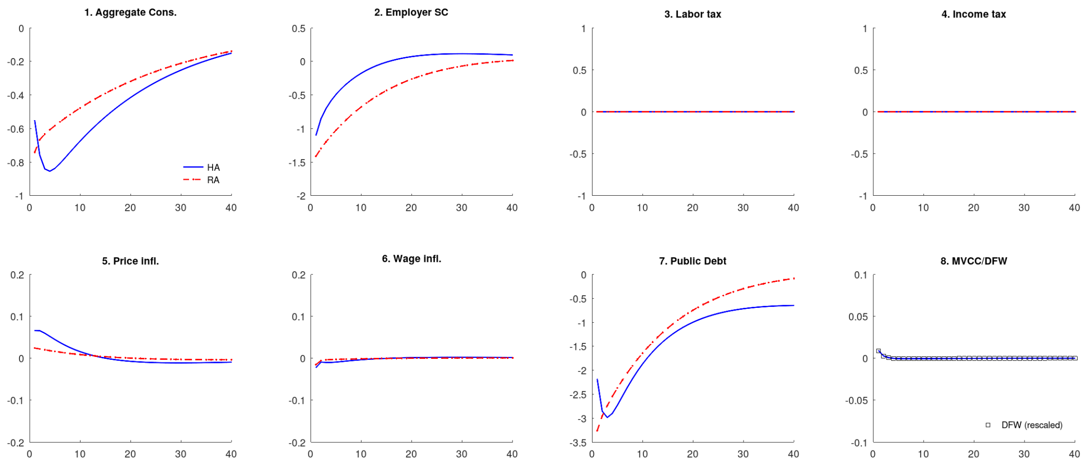
In summary, we find that allocations and price dynamics differ most significantly in response to the discount factor shock (which directly affects the MVCC), and also when no efficient labor tax is available to reduce the labor wedge.

5.6 The effect of missing instruments and other shocks

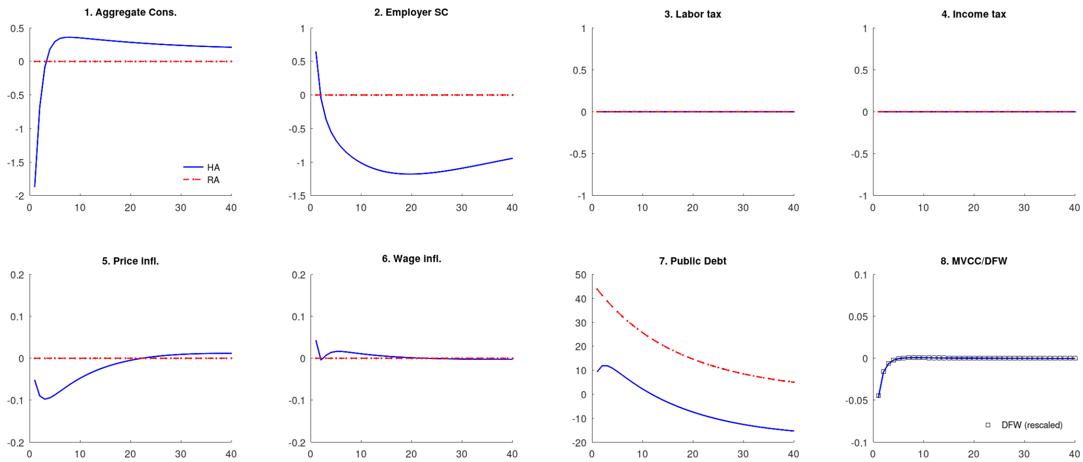
We now present results from additional simulations. To save some space, we summarize key findings and refer to the IRFs in the Appendix.

First, we consider an economy in which the capital tax is held constant ($\tau_t^K = \tau_{ss}^K$), while the remaining labor fiscal instruments ($\tau_t^E, \tau_t^W, \tau_t^L$) are optimally time-varying (see Figure XXX in Appendix XXX). For the given calibration, inflation moves slightly on impact. When we further reduce the coefficient of price stickiness from 100 to a low value of $\psi_p = 10$, inflation increases significantly on impact for one period. This spike in inflation acts as a substitute for the missing capital tax, as it lowers the real interest rate for one period through the Fisher effect, given its unexpected nature. Thus, inflation can substitute for the unavailable capital tax on impact, provided that prices are sufficiently flexible. This result aligns with the findings of LeGrand et al. (2022).

Second, we implement two other aggregate shocks: a public spending shock and a pure idiosyncratic uncertainty shock. The latter is implemented through a time-varying, mean-preserving change in idiosyncratic productivity $y_{i,t}$. The allocations after a public spending shock closely resemble those resulting from negative TFP shocks. This can be seen in Figure XXX of



(a) TFP shock.



(b) Discount factor shock.

Figure 3: Dynamics of the economy when the employed social contribution τ^E is the only available labor fiscal instrument, following a TFP and a discount factor shock. The HA economy is represented in blue and the RA one in red. All variables are expressed in relative deviations from their steady-state values, except for tax and inflation rates which are presented in level deviations from their steady-state values.

Appendix XXX. Conversely, the effects of the uncertainty shock mirror those of a discount factor shock, as both operate through the mechanism of a time-varying MVCC. In the RA economy, this shock has no effect by construction, while in the HA economy, this shock induces time-varying precautionary saving, which directly affects the MVCC. Results are reported in Figure XXX of Appendix XXX.

Finally, we also derive two main results regarding instruments. First, the path of public debt varies significantly between the HA and RA economies, even when allocations are similar. The variation of public debt at impact (in absolute terms) is always lower in the HA economy than in the RA one. Similarly, the optimal response of public debt is always less volatile in the RA economy than in the HA one. This comes from the fact that public debt plays very different roles in the two setups. In the HA economy, the planner adjusts public debt issuance to agents' demand for liquidity to facilitate self-insurance, while it has no such role in the RA economy. Second, the income tax or the worker social contribution are substitute instruments and both reasonably effective in maintaining price and wage stability, as well as keeping the HA allocation close to the RA one—regardless of the shock. Conversely, when only the employer social contribution is available, the departure from price and wage stability is sizable (except for public spending shocks) and HA and RA allocations markedly differ for all shocks. This tends to show that the employer social contribution is the least effective fiscal tool.

6 Conclusion

We have derived the optimal fiscal and monetary policy in an economy characterized by both sticky prices and sticky wages, considering a range of fiscal instruments. We compare the outcomes of a RA economy and a HA economy in response to four distinct shocks: a TFP shock, discount factor shock, a public spending shock, and a shock to idiosyncratic risk. As a benchmark, we first establish a complete fiscal system, defined as a set of fiscal instruments in which both price and wage inflation are zero following any of the four shocks. In other words, under this complete fiscal system, any potential benefits of inflation are more efficiently achieved through time-varying taxes, thereby avoiding the additional costs associated to inflation. This complete fiscal system serves as a theoretical benchmark, which we use to analyze both allocations and inflation dynamics when certain fiscal instruments are fixed.

When all instruments are available, the allocation in the HA and RA economies are very similar for both the TFP and the public spending shock, but they differ substantially in response to the discount factor shock and the idiosyncratic risk shock. This result stems from the differential effects of each shock on the average tightness of agents' credit constraints, a statistic we label the Marginal Value of Credit Constraint (MVCC). We demonstrate both theoretically and quantitatively that this statistic explains the divergence in allocations between the HA and RA economies. A time-varying MVCC induces changes in precautionary saving, which influence consumption and saving decisions in the HA economy—a mechanism absent in the RA economy. This ingfind helps reconcile disparate results in the literature, which appear to be shock specific.

We then examine the effects of missing fiscal instruments on optimal inflation. Specifically, we identify the fiscal tools necessary to achieve price stability. This analysis clarifies the wedges that inflation affects to improve welfare, and offers insights for monetary policy in HA settings. We find that persistent inflation is employed to reduce the labor wedge,—the gap between the real wage rate and the marginal productivity of labor— when certain labor taxes are not-time-varying. Additionally, short-lived inflation can act as a substitute for missing capital taxes, but only when prices are sufficiently flexible.

Time-varying labor subsidies prove to be a valuable policy tool for reducing the labor wedge following a TFP shock. Notably, such policies have been recently implemented in Europe to stabilize employment. For instance, Germany's *kurzarbeit* program and France's *activité partielle* scheme functioned as wage subsidies aimed at reducing layoffs during the Covid-19 crisis. We refer to these instruments as non-Keynesian stabilizers, since their primary objective is not to stimulate aggregate demand or boost economic activity through fiscal multipliers. Instead, they directly target the labor wedge, which is the gap between the real wage and labor productivity..

From this analysis, we conclude that HA economies offer new insights into optimal stabilization policy, which are highly dependent on the nature of the shock and the set of available fiscal instruments.

References

Açıkgöz, O. T. (2018): "On the Existence and Uniqueness of Stationary Equilibrium in Bewley Economies with Production," *Journal of Economic Theory*, 173, 18–55.

Acharya, S., E. Challe, and K. Dogra (2022): "Optimal Monetary Policy According to HANK," Working Paper, Federal Reserve Bank of New York.

Acharya, S. and K. Dogra (2021): "Understanding HANK: Insights from a PRANK," *Econometrica*, 88, 1113–1158.

Auclert, A., M. Cai, M. Rognlie, and L. Straub (2024): "Optimal Long-Run Fiscal Policy with Heterogeneous Agents," *Working Paper.*

Auclert, A., M. Rognlie, and L. Straub (2022): "The Intertemporal Keynesian Cross," Working paper, Stanford University.

——— (2025): "Fiscal and Monetary Policy with Heterogeneous Agents," *Annual Review of Economics Volume*, 17.

Berger, D., L. Bocola, and A. Dovis (2023): "Imperfect Risk Sharing and the Business Cycle," *Quarterly Journal of Economics*, 138, 1765–1815.

Bhandari, A., D. Evans, M. Golosov, and T. J. Sargent (2021): "Inequality, Business Cycles, and Monetary-Fiscal Policy," *Econometrica*, 89, 2559–2599.

Bilbiie, F. and X. Ragot (2021): "Optimal Monetary Policy and Liquidity with Heterogeneous Households," *Review of Economic Dynamics*, 41, 71–95.

Blanchard, O. (1986): "The wage-price spiral," *Quarterly Journal of Economics*, 101, 543–566.

Blanchard, O. and J. Gali (2007): "Real wage Rigidities and the New Keynesian Model," *Journal of Money, Credit and Banking*, 39, 35–65.

Cheridito, P. and J. Sagredo (2016): "Existence of Sequential Competitive Equilibrium in Krusell-Smith Type Economies," Working Paper, ETH Zurich.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011): "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins," *American Economic Review*, 101, 471–475.

Chugh, S. (2006): "Optimal fiscal and monetary policy with sticky wages and sticky prices," *Review of Economic Dynamics*, 9, 683–714.

Correia, I., E. Farhi, J.-P. Nicolini, and P. Teles (2013): "Unconventional Fiscal Policy at the Zero Bound," *American Economic Review*, 4, 1172–1211.

Correia, I., J.-P. Nicolini, and P. Teles (2008): "Optimal Fiscal and Monetary Policy: Equivalence Results," *Journal of Political Economy*, 1, 141–170.

Dávila, E. and A. Schaab (2022): "Welfare Assessments with Heterogeneous Individuals," .

Dyrda, S. and M. Pedroni (2022): "Optimal Fiscal Policy in a Model with Uninsurable Idiosyncratic Shocks," *Review of Economic Studies*, forthcoming.

Erceg, C., D. Henderson, and A. Levin (2000): "Optimal monetary policy with staggered wage and price contractsl," *Journal of Monetary Economics*, 46, 281–313.

Galí, J. (2015): *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Application*, Princeton University Press, 2nd ed.

Green, E. (1994): "Individual-Level Randomness in a Nonatomic Population," Working Paper, University of Minnesota.

Hagedorn, M., M. Iouri, and K. Mitman (2019): "The Fiscal Multiplier," *Working Paper.*

Heathcote, J. and H. Tsujiyama (2021): "Optimal Income Taxation: Mirrlees Meets Ramsey," *Journal of Political Economy*, 129, 3141–3184.

Kaplan, G. (2025): "Implications of Fiscal-Monetary Interaction from HANK Models," Working Paper 34117, National Bureau of Economic Research.

Kaplan, G. and G. L. Violante (2022): "The Marginal Propensity to Consume in Heterogeneous Agent Models," *Annal Review of Economics*, 14, 747–775.

Krueger, D., K. Mitman, and F. Perri (2018): "On the Distribution of the Welfare Losses of Large Recessions," in *Advances in Economics and Econometrics: Volume 2, Eleventh World Congress of the Econometric Society*, ed. by B. Honoré, A. Pakes, M. Piazzesi, and L. Samuleson, Cambridge University Press, 143–184.

Krusell, P. and A. A. J. Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867–896.

LeGrand, F., A. Martin-Baillon, and X. Ragot (2022): "Should Monetary Policy Care About Redistribution? Optimal Fiscal and Monetary Policy with Heterogeneous Agents," Working Paper, SciencesPo.

LeGrand, F. and X. Ragot (2022a): "Managing Inequality over the Business Cycle: Optimal Policies with Heterogeneous Agents and Aggregate Shocks," *International Economic Review*, 63, 511–540.

——— (2022b): "Refining the Truncation Method to Solve Heterogeneous-Agent Models," *Annals of Economics and Statistics*, 146, 65–92.

——— (2023): "Optimal Policies with Heterogeneous Agents: Truncation and Transitions," *Journal of Economic Dynamics and Control*, 156.

——— (2025): "Optimal Fiscal Policy With Heterogeneous Agents and Capital: Should We Increase or Decrease Public Debt and Capital Taxes?" *Journal of Political Economy*.

LeGrand, F., X. Ragot, and D. Rodrigues (2025): "From Homo Economicus to Homo Moralis: A Bewley Theory of the Social Welfare Function," *Working Paper*.

Lehmann, E., F. Marical, and L. Rioux (2013): "Labor income responds differently to income-tax and payroll-tax reforms," *Journal of Public Economics*, 99, 66–84.

Lorenzoni, G. and I. Werning (2023): "Wage Price Spirals," Working paper, MIT.

Marcet, A. and R. Marimon (2019): "Recursive Contracts," *Econometrica*, 87, 1589–1631.

McKay, A. and C. Wolf (2022): "Optimal Policy Rules in HANK," Working Paper, FRB Minneapolis.

Miao, J. (2006): "Competitive Equilibria of Economies with a Continuum of Consumers and Aggregate Shocks," *Journal of Economic Theory*, 128, 274–298.

Nakajima, T. (2005): "A business cycle model with variable capacity utilization and demand disturbances," *European Economic Review*, 49, 1331–1360.

Nuño, G. and B. Moll (2018): "Social Optima in Economies with Heterogeneous Agents," *Review of Economic Dynamics*, 48, 150–180.

Nuño, G. and C. Thomas (2022): "Optimal Redistributive Inflation," *Annals of Economics and Statistics*, 146, 3–64.

Rouwenhorst, G. K. (1995): "Asset Pricing Implications of Equilibrium Business Cycle Models," in *Structural Models of Wage and Employment Dynamics*, ed. by T. Cooley, Princeton: Princeton University Press, 201–213.

Saez, E., M. Matsaganis, and P. Tsakloglou (2012): "Earnings Determination and Taxes: Evidence From a Cohort-Based Payroll Tax Reform in Greece," *The Quarterly Journal of Economics*, 127, 493–533.

Saez, E. and S. Stantcheva (2016): "Generalized Social Marginal Welfare Weights for Optimal Tax Theory," *American Economic Review*, 106, 24–45.

Schmitt-Grohé, S. and M. Uribe (2005): "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model," *NBER Macroeconomics Annual*, 20, 383–425.

Werning, I. (2015): "Incomplete Markets and Aggregate Demand," NBER Working Paper 21448, National Bureau of Economic Research.

Woodford, M. (1990): "Public Debt as Private Liquidity," *American Economic Review*, 80, 382–388.

Yang, Y. (2022): "Redistributive Inflation and Optimal Monetary Policy," Working paper, Princeton University.

Appendix

A The simple model

The program of the planner in the economy with credit constraint (presented in Section 2) is:

$$\max_{(c_{e,t},c_{u,t},a_{e,t},a_{u,t},l_{e,t}B_t,A_t,R_t,w_t)} \sum_{t=0}^{\infty} \Theta_t \Big(U(c_{e,t}, l_{e,t}) + U(c_{u,t}, 0) \Big) \tag{31}$$

$$\text{s.t. } c_{e,t} + a_{e,t} = R_t a_{u,t-1} + w_t l_{e,t}, \tag{32}$$

$$c_{u,t} + a_{u,t} = R_t a_{e,t-1}, \tag{33}$$

$$U_c(c_{e,t}, l_{e,t}) = \beta_t R_{t+1} U_c(c_{u,t+1}, 0), \tag{34}$$

$$U_c(c_{u,t}, 0) \geq \beta_t R_{t+1} U_c(c_{e,t+1}, l_{e,t+1}), \text{ with equality if } a_{u,t} > 0, \tag{35}$$

$$-U_l(c_{e,t}, l_{e,t}) = w_t U_c(c_{e,t}, l_{e,t}), \tag{36}$$

$$R_t B_{t-1} + w_t l_{e,t} = Z_t l_{e,t} + B_t. \tag{37}$$

$$A_t = a_{e,t} + a_{u,t}, \tag{38}$$

$$A_t = B_t, \tag{39}$$

$$a_{e,t}, a_{u,t} \geq 0, \tag{40}$$

$$c_{e,t}, c_{u,t} > 0 \text{ and } l_{e,t}, l_{u,t} \geq 0. \tag{41}$$

We first assume that $a_u = 0$, and show below (in Section A.1) that it is the case.

If $a_{u,t} = 0$, using the GHH property, the first-order condition for labor supply is $l_t = (\chi w_t)^{\varphi}$. The Euler equation of employed agents is (using $c_{u,t+1} = R_{t+1} a_{e,t}$):

$$\left(w_t \left(\chi w_t\right)^{\varphi} - a_{e,t} - \chi^{-1} \frac{\left(\chi w_t\right)^{1+\varphi}}{1 + 1/\varphi} \right)^{-1} = \beta_t R_{t+1} \left(R_{t+1} a_{e,t}\right)^{-1},$$

One finds the saving function:

$$a_{e,t} = \frac{\beta_t}{1+\beta_t} \frac{1}{1+\varphi} w_t^{1+\varphi}$$

From that we get the relationships:

$$c_{e,t} = \left(1 - \frac{\beta_t}{1+\beta_t} \frac{1}{1+\varphi}\right) w_t^{1+\varphi} \tag{42}$$

$$c_{e,t} - \frac{l^{1+1/\varphi}}{1+1/\varphi} = \left(\frac{1}{1+\varphi} - \frac{\beta_t}{1+\beta_t} \frac{1}{1+\varphi}\right) w_t^{1+\varphi}$$

$$c_{u,t} = R_t \frac{\beta_{t-1}}{1+\beta_{t-1}} \frac{1}{1+\varphi} w_{t-1}^{1+\varphi} \tag{43}$$

We will use the change of variables:

$$\eta_t := \frac{1}{1+\varphi}\frac{\beta_t}{1+\beta_t} \tag{44}$$
$$x_t := w_t^{1+\varphi}$$

and $x_t := w_t^{1+\varphi}$, we have $\frac{1}{1+\varphi}\frac{1}{1+\beta_t} = \frac{1}{1+\varphi} - \eta_t$.

Using $B_t = a_{e,t}$, the budget of the state can be written as:

$$G_t + R_t\eta_{t-1}x_{t-1} + = Z_t x_t^{\frac{\varphi}{1+\varphi}} - x_t + \eta_t x_t.$$

Using these relationships in the program of the planner, one finds

$$\begin{aligned}\max_{(R_t,w_t)} \sum_{t=0}^{\infty} \Xi_t \Big(\log x_t + \log R_t + \log x_{t-1}\Big) \\ - \sum_{t=0}^{\infty} \Xi_t \mu_t \left(\frac{G_t}{\chi^{\varphi}} + R_t\eta_{t-1}x_{t-1} - Z_t x_t^{\frac{\varphi}{1+\varphi}} + x_t - \eta_t x_t\right).\end{aligned}$$

The two first-order conditions and the budget constraint of the state form a system of 3 equations in the three unknowns R_t, μ_t, x_t

$$\begin{aligned}(1+\beta_{t+1})\frac{1}{x_t} &= \mu_t\left(-Z_t\frac{\varphi}{1+\varphi}x_t^{-\frac{1}{1+\varphi}} + 1 - \eta_t\right) + \beta_{t+1}\mu_{t+1}R_{t+1}\eta_t \\ 1 &= R_t\mu_t\eta_{t-1}x_{t-1} \\ R_t\eta_{t-1}x_{t-1} &= Z_t x_t^{\frac{\varphi}{1+\varphi}} - x_t + \eta_t x_t\end{aligned}$$

Substituting for R_t and μ_t, one finds $\left(\frac{1+\varphi}{1/2+\varphi}(1-\eta_t)\right)^{-(1+\varphi)} Z_t^{1+\varphi} = x_t$. Then:

$$R_t = \frac{x_t}{\eta_{t-1}x_{t-1}}\frac{1}{1+2\varphi}(1-\eta_t) \tag{45}$$

Then, using the expression of $c_{e,t}$ and $c_{u,t}$ one finds $\frac{c_{e,t}}{c_{u,t}} = 1+2\varphi$ and $c_{e,t} = (1-\eta_t)\,x_t$.

Then:

$$C_t^{HA} = c_{e,t} + c_{u,t} = \left(\frac{1+\varphi}{1/2+\varphi}\right)^{-\varphi}(1-\eta_t)^{-\varphi} Z_t^{1+\varphi}. \tag{46}$$

Steady-state interest rate

At the steady state, $Z_t = 1$ and $\eta = \frac{1}{1+\varphi}\frac{\beta}{1+\beta}$, thus:

$$R = \frac{1+\beta}{\beta}\frac{1+\varphi}{1+2\varphi} - \frac{1}{\beta}\frac{\beta}{1+2\varphi} = \frac{1+(1+\beta)\varphi}{1+2\varphi} \times \frac{1}{\beta}$$

As $\beta < 1$, $\frac{1+(1+\beta)\varphi}{1+2\varphi} < 1$ and $R < \frac{1}{\beta}$, and credit constraints are binding at the steady-state.

Comparison with RA economy\log\left(c_{e,t}-\frac{l_{e,t}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\var

Using $C_t^{RA} = Z_t^{1+\varphi}$, and the expression of C_t^{HA} in (46), we deduce:

$$\frac{C_t^{HA}}{C_t^{RA}} = \left(\frac{1+\varphi}{1/2+\varphi}\right)^{-\varphi} \left(1 - \frac{1}{1+\varphi}\frac{\beta_t}{1+\beta_t}\right)^{-\varphi}$$

A.1 An existence proof

We now prove that $a_u = 0$, and that credit constraints are binding at the steady state. The proof is is related to the one presented in LeGrand and Ragot (2025), but it simpler, as we consider an economy without capital. We first prove that, when credit constraints are not binding, then the optimal level of public debt implements $a_u = 0$. Second, we show that when $a_u = 0$, the planner wants the credit constraint to bind and $R < \frac{1}{\beta}$.

First, in a Ramsey equilibrium where credit constraint doesn't bind, the economy is a simple two-agents economy with distorting taxes. The planner minimizes distortions (i.e labor taxes) to a level consistent with credit constraints not binding. This implies that B reached the minimum level for which $a_{u,t} \geq 0$,and $a_u = 0$in the Ramsey steady-state.

Second, the proof of the previous Section has assumed that $a_{u,t} = 0$ to conclude that in this case, the planner wants to credit constraints to bind. In other words, starting from $a_{u,t} = 0$, the previous proof shows that the optimal program implements $R < \frac{1}{\beta}$. The gain of a marginal increase in production efficiency by reducing labor taxes is higher that the marginal cost of reducing allocation efficiency, by decreasing public debt and the real interest rate.

A.2 Expression of the MVCC

One can compute the value of Lagrange multiplier on the credit constraint of unemployed agent.

$$\nu_t = c_{u,t}^{-1} - \beta R_{t+1} \left(c_{e,t} - \frac{\chi l_{e,t}^{1/\varphi+1}}{1+1/\varphi}\right)^{-1}$$

Using (42)-(43), one finds, after some algebra

$$MVCC_t = \frac{\nu_t}{u'(c_{u,t})} = 1 - \frac{\left(1+\varphi\left(1+\beta_{t+1}\right)\right)\left(1+\varphi\left(1+\beta_t\right)\right)}{\left(1+2\varphi\right)^2}$$

Note that $MVCC_t > 0$ is equivalent to

$$\left(1+\varphi\left(1+\beta_{t+1}\right)\right)\left(1+\varphi\left(1+\beta_t\right)\right) < \left(1+2\varphi\right)^2$$

which is always true as $0 < \beta_{t+1} < 1$.

A.3 Expression of the DFW

First, the real interest rate in the HA economy is, from (45) and the expression of x_t and η_t:

$$R_t^{HA} = \frac{\left(1 - \frac{1}{1+\varphi}\frac{\beta_t}{1+\beta_t}\right)^{-(1+\varphi)}}{\frac{1}{1+\varphi}\frac{\beta_{t-1}}{1+\beta_{t-1}}\left(1 - \frac{1}{1+\varphi}\frac{\beta_{t-1}}{1+\beta_{t-1}}\right)^{-(1+\varphi)}}\left(\frac{Z_t}{Z_{t-1}}\right)^{1+\varphi}\frac{1}{1+2\varphi}\left(1 - \frac{1}{1+\varphi}\frac{\beta_t}{1+\beta_t}\right) \tag{47}$$

As as $l_{HA,t} = x_t^{\frac{\varphi}{1+\varphi}}$ the DFW β_t^{wedge} is defined by:

$$\frac{1}{R_{t+1}^{HA}} = \beta_t \beta_t^{wedge}\left(\frac{C_{t+1}^{HA} - \frac{l_{HA,t+1}^{1+1/\varphi}}{1+1/\varphi}}{C_t^{HA} - \frac{l_{HA,t+1}^{1+1/\varphi}}{1+1/\varphi}}\right)^{-1}$$

Substituting for $C_t^{HA}, C_t^{HA}, R_{t+1}^{HA}$ one finds after some algebra:

$$\beta_t^{wedge} = \frac{(1+1/\varphi)\left(1+\varphi\left(1+\beta_{t+1}\right)\right) - (1/2+\varphi)\left(1+\beta_{t+1}\right)}{(1+1/\varphi)\left(1+\varphi\left(1+\beta_t\right)\right) - (1/2+\varphi)\left(1+\beta_t\right)} \times \frac{1+2\varphi}{1+\varphi\left(1+\beta_{t+1}\right)}$$

A.4 Relationship between MVCC and DFW

Using the expression of MVCC, one finds:

$$\beta_t^{wedge} = \frac{1}{1-MVCC_t}\left(1 - \frac{1+2\varphi}{2(1+\varphi)+\varphi(1+\beta_t)}MVCC_t\right)$$

Assume $MVCC_t \simeq 0$, then:

$$\beta_t^{wedge} = 1 + \frac{1+\varphi(1+\beta_t)}{2(1+\varphi)+\varphi(1+\beta_t)}MVCC_t$$

Thus at the first order $\tilde{\beta}_t^{wedge} = \left(1 - \frac{1}{\beta^{wedge}}\right)M\tilde{V}CC_t$, where variables with tilda represent proportional deviations.

B Derivation of the wage-Phillips curve

There is a continuum of unions of size 1 indexed by k and each union k supplies L_{kt} hours of labor at date t with nominal wage $\hat{W}_{kt}$. Union-specific labor supplies are then aggregated into aggregate labor supply by a competitive technology featuring a constant elasticity of substitution ε_W:

$$L_t = \left(\int_k L_{kt}^{\frac{\varepsilon_W - 1}{\varepsilon_W}} dk\right)^{\frac{\varepsilon_W}{\varepsilon_W - 1}}. \tag{48}$$

The competitive aggregator demands the union labor supplies $(L_{kt})_k$ that minimize the total labor cost $\int_k \hat{W}_{kt} L_{k,t} dk$ subject to the aggregation constraint (48), where $\hat{W}_{kt}$ is the bargained nominal wage of the members of union k. The demand for labor of union k depends on the total labor cost paid by the firm $\tilde{W}_{kt}$: $L_{kt} = \left(\frac{\tilde{W}_{kt}}{\tilde{W}_t}\right)^{-\varepsilon_W}$, where $\tilde{W}_t = \left(\int_k \tilde{W}_{kt}^{1-\varepsilon_W} dk\right)^{\frac{1}{1-\varepsilon_W}}$ is the total nominal wage index. As the labor demand depends on relative wages, and $\frac{\tilde{W}_{kt}}{\tilde{W}_t} = \frac{\hat{W}_{kt}}{\hat{W}_t}\frac{1-\tau_t^E}{1-\tau_t^E} = \frac{\hat{W}_{kt}}{\hat{W}_t}$, total labor demand can be written as:

$$L_{kt} = \left(\frac{\hat{W}_{kt}}{\hat{W}_t}\right)^{-\varepsilon_W} L_t, \tag{49}$$

where $\hat{W}_t = \left(\int_k \hat{W}_{kt}^{1-\varepsilon_W} dk\right)^{\frac{1}{1-\varepsilon_W}}$ is the bargained nominal wage index. Each union k sets its wage $\hat{W}_{kt}$ so as to maximize the intertemporal welfare of its members subject to fulfilling the demand of equation (49). We assume the presence of quadratic utility costs related to the adjustment of the nominal wage and equal to $\frac{\psi_W}{2}(\hat{W}_{kt}/\hat{W}_{kt-1} - 1)^2 dk$. The objective of union k is thus:

$$\max_{(\hat{W}_{ks})_s} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \int_i \left(u(c_{i,s}) - v(l_{i,s}) - \frac{\psi_W}{2}\left(\frac{\hat{W}_{ks}}{\hat{W}_{ks-1}} - 1\right)^2\right)\ell(di),$$

$$\max_{(\hat{W}_{ks})_s} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \int_i \left(u(c_{i,s}) - v(l_{i,s}) - \frac{\psi_W}{2}\left(\frac{\hat{W}_{ks}}{\hat{W}_{ks-1}} - 1\right)^2\right)\ell(di),$$

subject to (49) and where $c_{i,t}$ and $l_{i,t}$ are the consumption and labor supply of agent i. The first-order condition with respect to W_{kt} thus writes as:

$$\pi_t^W(\pi_t^W + 1) = \frac{\hat{W}_{kt}}{\psi_W} \int_i \left(u'(c_{i,t})\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} - v'(l_{i,t})\frac{\partial l_{i,t}}{\partial \hat{W}_{kt}}\right)\ell(di) + \beta \mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W + 1)\right], \tag{50}$$

where the wage inflation rate is denoted by:

$$\pi_t^W = \frac{\hat{W}_{k,t}}{\hat{W}_{k,t-1}} - 1.$$

The labor supply l_{it} of agent i is the sum of her hours l_{ikt} supplied to union k, summed over all unions: $l_{it} = \int_k l_{ikt} dk$. Each union is assumed to request its members to supply an uniform number of hours, such that: $l_{ikt} = L_{kt}$. We thus deduce from (49):

$$\hat{W}_{kt}\frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} = \hat{W}_{kt}\frac{\partial\left(\int_k \left(\frac{\hat{W}_{kt}}{\hat{W}_t}\right)^{-\varepsilon_W} L_t dk\right)}{\partial \hat{W}_{kt}} = -\varepsilon_W L_{kt}. \tag{51}$$

To compute the derivative of consumption $\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}}$, it should observed that it is equal to the derivative of its net total income. The net total income of agent i writes as $(1 - \tau_t^W)\hat{W}_{kt} y_{i,t} l_{i,t}/P_t$, where τ_t^W is the labor tax. Formally:

$$\begin{aligned}
\frac{1}{c_{i,t}}\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} &= \frac{1}{\hat{W}_{kt}} + \frac{1}{l_{i,t}}\frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} \\
&= \frac{1}{\hat{W}_{kt}} - \frac{\varepsilon_W}{\hat{W}_{kt}}\frac{L_{kt}}{l_{i,t}} \\
\hat{W}_{kt}\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} &= (1-\varepsilon_W)(1-\tau_t^W)\hat{W}_{kt}y_{i,t}l_{i,t}/P_t \qquad (52)
\end{aligned}$$

We focus on the symmetric equilibrium where all unions choose to set the same wage $\hat{W}_{kt} = \hat{W}_t$, hence all households work the same number of hours, equal to $l_{it} = L_t$. Combining (50) with the partial derivatives (51) and (52), we deduce the following Phillips curve for wage inflation:

$$\pi_t^W(\pi_t^W+1) = \frac{\varepsilon_W}{\psi_W}\underbrace{\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}(1-\tau_t^W)\hat{w}_t\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)}_{\text{labor gap}}L_t + \beta\mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W+1)\right], \qquad (53)$$

where $\hat{w}_t = \hat{W}_t/P_t$ is the real pre-tax wage.

C Characterization of the Ramsey allocation in the RA case

We solve for the Ramsey allocation in the RA case, for both demand and supply shocks.

C.1 First-best allocation

In the first-best allocation, the resource constraint imposes that total consumption is financed out of production: $G_t + C_t = Z_tL_t$. The labor supply is thus determined by the solution to the following program: $\max_{L_t} u(Z_tL_t - G_t) - v(L_t)$. The first-order condition defines the first-best labor supply L_t^{FB} as the solution of:

$$Z_tu'(Z_tL_t^{FB} - G_t) = v'(L_t^{FB}), \qquad (54)$$

which can be shown to admit a unique solution under standard assumption (u increasing concave with $u'(0) = \infty$ and $u'(\infty) = 0$ and v increasing convex).

Consider the following particular case. We set $G_t = 0$, $u'(c) = c^{-\gamma}$, and $v'(L) = \chi^{-1}L^{1/\phi}$ such that $\gamma > 0$ is the inverse of the IES and $\phi > 0$ is the Frisch elasticity of labor supply. We obtain: $L_t^{FB} = \chi^{\frac{1}{\frac{1}{\phi}+\gamma}}Z_t^{\frac{1-\gamma}{\frac{1}{\phi}+\gamma}}$.

C.2 Representative-agent model with a full set of instruments

We show that when the planner has access to the full set of instrument, the first-best allocation can be implemented for both demand and supply shocks. This requires $\pi^W = \pi^P = 0$, to avoid price or wage adjustment costs. The equations defining the equilibrium allocation are

$$w_t = \left(1-\tau_t^L\right)(1-\tau_t^W)\left(1-\tau_t^E\right) Z_t,$$

$$v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}\frac{w_t}{1-\tau_t^L} u'(C_t) = 0 \tag{55}$$

$$\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^W)(1-\tau_t^E)\left(1-\tau_t^L\right)} = 1 \tag{56}$$

$$1 = \frac{w_t}{w_{t-1}}\frac{(1-\tau_{t-1}^W)}{(1-\tau_t^W)} \tag{57}$$

$$C_t = Z_t L_t - G_t$$

$$G_t + (1+r_t) B_{t-1} + (w_t - Z_t) L_t = B_t \tag{58}$$

$$u'(C_t) = \beta (1+r_{t+1}) u'(C_{t+1}) \tag{59}$$

Note that the Euler equation (59) determines the real interest r_t^{FB} $t \geq 1$ from the first-best path of consumption C_t^{FB}. Importantly, this equations don't determine the period-0 interest rate r_0.

Equation (58à is the budget constraint of the government.

Equation (57) implies that there is a α such that $1-\tau_t^W := \alpha w_t$. For the allocation to be the first best, equations (55) and (54) implies that

$$1-\tau_t^L \quad := \quad \frac{\varepsilon_W - 1}{\varepsilon_W}\frac{w_t}{Z_t}$$

Then equation (56) implies

$$1-\tau_t^E := \frac{\varepsilon_W}{\varepsilon_W - 1}\frac{1}{\alpha w_t}$$

Then the budget of the government implies

$$w_t = Z_t - \frac{G_t + (1+r_t) B_{t-1} - B_t}{L_t} \tag{60}$$

Implementation results: For any path of G_t, Z_t and path of public debt B_t, for $t \geq 0$, the first best can be implemented.

The proof is direct. Consider a path G_t, Z_t, B_t and the first-best labor supply L_t^{FB}. It gives a path of consumption determining the real interest rate r_t, $t \geq 1$. For any r_0 (which is an additional free variable), the equation (60) determines a path for the real wage rate. Then for any α, $1-\tau_t^W = \alpha w_t$, $1-\tau_t^L = \frac{\varepsilon_W - 1}{\varepsilon_W}\frac{w_t}{Z_t}$, $1-\tau_t^E = \frac{\varepsilon_W}{\varepsilon_W - 1}\frac{1}{\alpha w_t}$ is a market equilibrium.

Note thus that public debt is not determined in this implementation.

Note that in a steady-state equilibrium (where $Z = 1$ and B, G, w are constant) , we have

$$B_{SS} = \frac{\beta}{1-\beta}\left(\left(1-w\right)L_{SS}^{FB} - G_{SS}\right)$$

C.3 RA model : Representative agents without time-varying τ_t^L

We assume that the economy is in steady state, where public debt is B_{SS}. and hit by the shock at period 0. In the previous analysis, we can impose $\tau_t^L = \tau_{SS}^L$.

$$w_t = \frac{\varepsilon_W}{\varepsilon_W - 1}\left(1-\tau_{ss}^L\right)Z_t$$

Then, $1-\tau_t^W = \alpha\frac{\varepsilon_W}{\varepsilon_W-1}\left(1-\tau_{ss}^L\right)Z_t,$, $1-\tau_t^E = \frac{1}{\alpha(1-\tau_{ss}^L)Z_t}$.

The budget of the state implies (for $t \geq 0$, with the notation $B_{-1} = B_{SS}$)

$$\left(1+r_t\right)B_{t-1} - B_t = \Theta_t$$

with

$$\Theta_t := \left(1 - \frac{\varepsilon_W}{\varepsilon_W - 1}\left(1-\tau_{ss}^L\right)\right)Z_t L_t^{FB} - G_t$$

The variable Θ_t is uniquely determined. This uniquely determines the path of public converging back to the steady state. To see that, first observe that the period-0 interest rate 0 r_0 is a free parameter determined by period-0 capital tax $\hat{\tau}_0^K$.

$$B_{-1}\left(1+r_0\right) = \sum_{t=0}\frac{G_t}{R_{0,t}} + \lim_{T\to\infty}\frac{B_T}{R_{t,T}}$$

To have $\lim_{T\to\infty}\frac{B_T}{R_{t,T}} = 0$, we must choose the initial capital tax such that

$$\left(1+r_0\right)B_{-1} = \sum_{k=t}^{\infty}\frac{\Theta_k}{\prod_{j=t+1}^{k}\left(1+r_j^{FB}\right)} + \lim_{T\to\infty}\frac{B_T}{\prod_{j=t+1}^{T}\left(1+r_j^{FB}\right)}$$

(with the notation $\prod_{j=t+1}^{t} = 1$). The term $\lim_{T\to\infty}\frac{B_T}{\prod_{j=t+1}^{T}(1+r_j)} = 0$ if the economy converges back to the steady state. The unique period 0 allowing the public debt to converge back to the steady state is

$$1 + r_0 = \frac{\sum_{k=t}^{\infty}\frac{\Theta_k}{\prod_{j=t+1}^{k}\left(1+r_j^{FB}\right)}}{B_{SS}},$$

which is uniquely determined.

C.4 RA with Demand shock

We now show that whatever the fiscal system (economy 3 and 4), the first-best allocation can be implemented with demand shocks. The proof follows the consideration of the previous Section.

We now assume that $\tau^L = \tau^L_{SS}$ and $\tau^L = \tau^L_{SS}$. We now focus on the case where $1-\tau^L_{ss} = \frac{\varepsilon_W - 1}{\varepsilon_W}$, to determine uniquely the path of the instruments. Any other value would not quantitatively change the allocation, and qualitatively the path of the instruments.

In this case, we have $w = Z = 1$. Then $1 - \tau^W_t = \alpha$, and $1 - \tau^E = \frac{\varepsilon_W}{\varepsilon_W - 1}\frac{1}{\alpha}$ and

$$1 + r_0 = \frac{1-\beta}{\beta}\sum_{k=t}^{\infty}\frac{G_k/G_{SS}}{\prod_{j=t+1}^{k}\left(1 + r^{FB}_j\right)},$$

implements the first-best allocation.

C.5 The RA economy without time-varying τ^L_t and τ^E_t, with optimal τ^W_t and supply shocks

The first-best cannot be implemented, and we must solve for the Ramsey allocation. We provide equations for both demand and supply shocks and then discuss each case in turn.

The program is:

$$\max_{\left(\tau^W_t,\tau^E_t,\tau^K_t,B_t,T_t,\pi^P_t,\pi^W_t,w_t,r_t,\Omega_t,\tilde{R}^N_t,L_t,c_t,a_t\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t) - v(L_t)\right) - \frac{\psi_W}{2}(\pi^W_t)^2\right],$$

$$G_t + (1+r_t)B_{t-1} + w_t L_t \leq \left(1 - \frac{\psi_P}{2}(\pi^P_t)^2\right)Z_t L_t + B_t,$$

$$c_t + a_t = (1+r_t)B_{t-1} + w_t L_t,$$

$$u'(c_t) = \beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{t+1})\Big],$$

$$\pi^W_t(\pi^W_t + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}\frac{w_t}{1-\tau^L_{ss}}u'(c_t)\right)L_t + \beta\mathbb{E}_t\Big[\pi^W_{t+1}(\pi^W_{t+1}+1)\Big],$$

$$\pi^P_t(1+\pi^P_t) = \frac{\varepsilon_P - 1}{\psi_P}\left(\frac{1}{Z_t}\frac{w_t}{(1-\tau^W_t)(1-\tau^L_{ss})} - 1\right) + \beta\mathbb{E}_t\left(\pi^P_{t+1}(1+\pi^P_{t+1})\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\right),$$

$$(1+\pi^W_t)\frac{w_{t-1}}{1-\tau^W_{t-1}} = \frac{w_t}{1-\tau^W_t}(1+\pi^P_t),$$

Define

$$T_t = (1+r_t)B_{t-1} - B_t$$
$$x_t = \frac{w_t}{1-\tau^W_t}$$

Then the program is (using $\frac{\varepsilon_W - 1}{\varepsilon_W} = 1 - \tau^L_{ss}$)

$$
\begin{aligned}
&\max_{\left(x_t T_t, \pi^P_t, \pi^W_t, w_t, L_t, c_t\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(u(c_t) - v(L_t)\right) - \frac{\psi_W}{2}(\pi^W_t)^2\right], \\
&G_t + T_t + \left(1 - \tau^W_t\right) x_t L_t \leq \left(1 - \frac{\psi_P}{2}(\pi^P_t)^2\right) Z_t L_t, \\
&c_t = T_t + \left(1 - \tau^W_t\right) x_t L_t, \\
&\pi^W_t(\pi^W_t + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \left(1 - \tau^W_t\right) x_t u'(c_t)\right) L_t + \beta \mathbb{E}_t\left[\pi^W_{t+1}(\pi^W_{t+1} + 1)\right], \\
&\pi^P_t(1 + \pi^P_t) = \frac{\varepsilon_P - 1}{\psi_P}\left(\frac{\varepsilon_W}{\varepsilon_W - 1}\frac{1}{Z_t} x_t - 1\right) + \beta \mathbb{E}_t\left(\pi^P_{t+1}(1 + \pi^P_{t+1})\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\right), \\
&(1 + \pi^W_t) x_{t-1} = x_t(1 + \pi^P_t),
\end{aligned}
$$

while the corresponding Lagrangian becomes $c_t = T_t + \left(1 - \tau^W_t\right) x_t L_t$

$$
\begin{aligned}
\mathcal{L} = {} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) - v(L_t) - \frac{\psi_W}{2}(\pi^W_t)^2) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1}) \pi^W_t (1 + \pi^W_t) \\
& + \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \left(1 - \tau^W_t\right) x_t u'(c_t)\right) L_t \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1}) \pi^P_t (1 + \pi^P_t) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{\varepsilon_W}{\varepsilon_W - 1} x_t - Z_t\right) L_t \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left((1 - \frac{\psi_P}{2}(\pi^P_t)^2) Z_t L_t - G_t - T_t - \left(1 - \tau^W_t\right) x_t L_t\right) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1 + \pi^W_t) x_{t-1} - x_t (1 + \pi^P_t)\right)
\end{aligned}
$$

We now turn to the computation of the FOCs.

Consider

$$
\psi_t := \frac{d\mathcal{L}}{dc} = u'(c_t) - \underbrace{\frac{\varepsilon_W}{\psi_W} \gamma_{W,t} \left(1 - \tau^W_t\right) x_t L_t u''(c_t)}_{\text{effect on wage inflation}}
$$

FOC wrt π^W_t.

$$
-\psi_W \pi^W_t - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi^W_t + 1) + \Lambda_t x_{t-1} = 0.
$$

FOC wrt π^P_t.

$$
-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi^P_t + 1) - \mu_t \psi_P \pi^P_t - \frac{\Lambda_t}{Z_t L_t} x_t = 0.
$$

FOC wrt x_t.

$$0 = \left(1-\tau_t^W\right) L_t\psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(1-\tau_t^W\right)u'(c_t)L_t + \frac{\varepsilon_P-1}{\psi_P}\gamma_{P,t}\frac{\varepsilon_W}{\varepsilon_W-1}L_t$$
$$-\mu_t\left(1-\tau_t^W\right)L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

FOC wrt L_t.

$$0 = \left(1-\tau_t^W\right)x_t\psi_t - v'(L_t) + \mu_t\left(\left(1-\frac{\psi_P}{2}(\pi_t^P)^2\right)Z_t - \left(1-\tau_t^W\right)x_t\right)$$
$$+\frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t) - \left(1-\tau_t^W\right)x_t u'(c_t)\right)$$
$$-(\gamma_{P,t}-\gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P-1}{\psi_P}\gamma_{P,t}\left(\frac{\varepsilon_W}{\varepsilon_W-1}x_t - Z_t\right).$$

FOC wrt T_t.

$$\mu_t = u'(c_t).$$

FOC wrt $1-\tau_t^W$.

$$0 = L_t x_t\psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}x_t u'(c_t)L_t - \mu_t x_t L_t.$$

Simplifying

FOC wrt T_t.

$$\mu_t = u'(c_t).$$

FOC wrt $1-\tau_t^W$.

$$0 = \psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}u'(c_t) - \mu_t.$$
$$0 = \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}u'(c_t)\left(1 - \frac{-\left(c_t-T_t\right)u''(c_t)}{u'(c_t)}\right)$$

In this case, one can check that one has $\gamma_{W,t}=0$ (The wage Phillips curve is not a constraint)

FOC wrt π_t^W.

$$-\psi_W\pi_t^W + \Lambda_t x_{t-1} = 0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t}-\gamma_{P,t-1})(2\pi_t^P+1) - \mu_t\psi_P\pi_t^P - \frac{\Lambda_t}{Z_t L_t}x_t = 0.$$

FOC wrt x_t.

$$0 = \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\frac{\varepsilon_W}{\varepsilon_W - 1}L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

Using the FOC wrt to τ^W, we have:

FOC wrt L_t.

$$\begin{aligned} 0 = &-v'(L_t) + \mu_t\Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t \\ &- (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(\frac{\varepsilon_W}{\varepsilon_W - 1}x_t - Z_t\right). \end{aligned}$$

Simplifying

$$\psi_W\pi_t^W = \Lambda_t x_{t-1}.$$

and

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) = \mu_t\psi_P\pi_t^P + \frac{\Lambda_t}{Z_t L_t}x_t.$$

FOC wrt x_t.

$$0 = \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\frac{\varepsilon_W}{\varepsilon_W - 1}L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

FOC wrt L_t.

$$\begin{aligned} v'(L_t) = &\mu_t\Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t \\ &- (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(\frac{\varepsilon_W}{\varepsilon_W - 1}x_t - Z_t\right). \end{aligned}$$

Determining the path of public debt from the path of T_t

The dynamics of public debt is

$$B_t = (1+r_t)B_{t-1} + T_t$$

At the moment of the shock, at period 0, the planner can change capital tax.

$$1 + r_0 = \frac{-\sum_{k=t}^{\infty}\frac{T_k}{\prod_{j=t+1}^{k}(1+r_j^{FB})}}{B_{SS}},$$

C.6 RA analysis without (time-varying) τ_t^L, τ_t^W, with time-varying τ_t^E

With the same change of variable as in the previous case,

$$\max_{\left(\tau_t^W,\tau_t^E,\tau_t^L,\tau_t^K,\pi_t^P,\pi_t^W,w_t,r_t,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-v(L_t)\right)\ell(di)-\frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t+T+w_tL_t\leq\Big(1-\frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_tL_t,$$

$$c_t=T_t+w_tL_t,$$

$$\pi_t^W(\pi_t^W+1)=\frac{\varepsilon_W}{\psi_W}\Big(v'(L_t)-w_tu'(c_t)\ell(di)\Big)L_t+\beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W+1)\Big],$$

$$\pi_t^P(1+\pi_t^P)=\frac{\varepsilon_P-1}{\psi_P}\Big(\frac{1}{Z_t}\frac{w_t}{(1-\tau_{ss}^W)(1-\tau_t^E)(1-\tau_{ss}^L)}-1\Big)+\beta\mathbb{E}_t\Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_tL_t}\Big),$$

$$(1+\pi_t^W)w_{t-1}=w_t(1+\pi_t^P).$$

Define

$$z_t=\frac{1}{(1-\tau_{ss}^W)(1-\tau_t^E)(1-\tau_{ss}^L)}=\frac{1-\tau_{ss}^E}{1-\tau_t^E}$$

$$\begin{aligned}
\mathcal{L}=&\,\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big(u(c_t)-v(L_t)-\frac{\psi_W}{2}(\pi_t^W)^2\Big)\\
&-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(\gamma_{W,t}-\gamma_{W,t-1})\pi_t^W(1+\pi_t^W)\\
&+\frac{\varepsilon_W}{\psi_W}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma_{W,t}\left(v'(L_t)-w_tu'(c_t)\right)L_t\\
&-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(\gamma_{P,t}-\gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_tL_t+\frac{\varepsilon_P-1}{\psi_P}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma_{P,t}\left(wz_t-Z_t\right)L_t\\
&+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mu_t\Big((1-\frac{\psi_P}{2}(\pi_t^P)^2)Z_tL_t-G_t-T_t-w_tL_t\Big)\\
&+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Lambda_t\Big((1+\pi_t^W)w_{t-1}-w_t(1+\pi_t^P)\Big)
\end{aligned}$$

FOC wrt π_t^W.

$$-\psi_W\pi_t^W-(\gamma_{W,t}-\gamma_{W,t-1})(2\pi_t^W+1)+\Lambda_tw_{t-1}=0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t}-\gamma_{P,t-1})(2\pi_t^P+1)-\mu_t\psi_P\pi_t^P-\frac{\Lambda_t}{Z_tL_t}w_t=0.$$

FOC wrt w_t.

$$0 = L_t\psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}u'(c_t)L_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}L_t - \mu_t L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

FOC wrt L_t.

$$0 = w_t\psi_t - v'(L_t) + \mu_t\left(\Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t - w_t\right) + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t) - w_t u'(c_t)\right)$$
$$-(\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(wz_t - Z_t\right).$$

FOC wrt T_t.

$$\mu_t = u'(c_t).$$

FOC wrt z_t.

$$0 = \gamma_{P,t}.$$

Simplifying

FOC wrt π_t^W.

$$\psi_W\pi_t^W = -(\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t w_{t-1}.$$

FOC wrt π_t^P.

$$-\mu_t\psi_P\pi_t^P = \frac{\Lambda_t}{Z_tL_t}w_{t-1}.$$

FOC wrt w_t.

$$0 = -\frac{\varepsilon_W}{\psi_W}\gamma_{W,t}u'(c_t)L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

FOC wrt L_t.

$$v'(L_t) = +\mu_t\Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t) - w_t u'(c_t)\right)$$

$$\mu_t = u'(c_t).$$

D Ramsey program for HA models

D.1 Flexible-price equilibrium

We here assume here that the planner must choose a common labor supply for all agents, in a flexible price economy: $\pi_t^P = \pi_t^W = 0$. The program is:

$$\max_{\left(\tau_t^W,\tau_t^E,\tau_t^K,w_t,r_t,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega(y_t^i)\left(u(c_t^i)-v(L_t)\right)\ell(di)\right],$$

$$G_t+(1+r_t)\int_i a_{i,t-1}\ell(di)+w_tL_t+T_t\leq Z_tL_t+\int_i a_{i,t}\ell(di),$$

for all $i\in\mathcal{I}$: $c_{i,t}+a_{i,t}=(1+r_t)a_{i,t-1}+w_ty_{i,t}L_t,$

$$a_{i,t}\geq-\overline{a}, \nu_{i,t}(a_{i,t}+\overline{a})=0,\ \nu_{i,t}\geq 0,$$

$$u'(c_{i,t})=\beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{i,t+1})\Big]+\nu_{i,t}.$$

$$u'(c_{i,t})=\frac{\beta}{1-MVCC_{i,t}}\mathbb{E}_t\Big[(1+r_{t+1})\,u'(c_{i,t+1})\Big]$$

The Lagrangian can be written as:

$$\mathcal{L}=\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i\omega_t^i(u(c_{i,t})-v(L_t))\ell(di)-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i(\lambda_{i,c,t}-(1+r_t)\lambda_{i,c,t-1})\,u'(c_{i,t})\ell(di)$$
$$+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mu_t\left(Z_tL_t+\int_i a_{i,t}\ell(di)-G_t-(1+r_t)\int_i a_{i,t-1}\ell(di)-w_tL_t-T_t\right).$$

We recall that $\psi_{i,t}=\omega_t^iu'(c_{i,t})-(\lambda_{i,c,t}-(1+r_t)\lambda_{i,c,t-1})\,u''(c_{i,t})$. Compared to (30), we drop the FP subscript for the sake of simplicity. We compute the FOCs wrt four independent instruments: r_t, w_t, L_t and $(a_{i,t})_i$. The other instruments can be recovered from the constraints.

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di)+\int_i\lambda_{i,c,t-1}u'(c_{i,t})\ell(di)=0. \tag{61}$$

FOC wrt w_t.

$$\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di)=0.$$

FOC wrt L_t. Using the FOC on w_t:

$$\int_i\omega_{i,t}\ell(di)v'(L_t)=\mu_tZ_t=Z_t\int_i y_{i,t}\psi_{i,t}\ell(di).$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{i,t+1} \right].$$

D.2 The HA economy with all instruments

The program is:

$$\max_{\left(\tau_t^W, \tau_t^E, \tau_t^L, \tau_t^K, \pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(L_t) \right) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right],$$

$$G_t + (1 + r_t) \int_i a_{i,t-1} \ell(di) + w_t L_t \leq \left(1 - \frac{\psi_P}{2} (\pi_t^P)^2 \right) Z_t L_t + \int_i a_{i,t} \ell(di),$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1 + r_t) a_{i,t-1} + w_t y_{i,t} L_t,$

$$a_{i,t} \geq -\bar{a}, \nu_{i,t}(a_{i,t} + \bar{a}) = 0, \ \nu_{i,t} \geq 0,$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \left[(1 + r_{t+1}) u'(c_{i,t+1}) \right] + \nu_{i,t},$$

$$\pi_t^W (\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} \frac{w_t}{1 - \tau_t^L} \int_i y_{i,t} u'(c_{i,t}) \ell(di) \right) L_t + \beta \mathbb{E}_t \left[\pi_{t+1}^W (\pi_{t+1}^W + 1) \right],$$

$$\pi_t^P (1 + \pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P} \left(\frac{1}{Z_t} \frac{w_t}{(1 - \tau_t^W)(1 - \tau_t^E)(1 - \tau_t^L)} - 1 \right) + \beta \mathbb{E}_t \left(\pi_{t+1}^P (1 + \pi_{t+1}^P) \frac{Z_{t+1} L_{t+1}}{Z_t L_t} \right),$$

$$(1 + \pi_t^W) \frac{w_{t-1}}{1 - \tau_{t-1}^W} = \frac{w_t}{1 - \tau_t^W} (1 + \pi_t^P).$$

We can set:

- τ_t^E such that $1 - \tau_t^E = \frac{1}{Z_t} \frac{w_t}{(1-\tau_t^W)(1-\tau_t^L)}$, hence $\frac{1}{Z_t} \frac{w_t}{(1-\tau_t^W)(1-\tau_t^E)(1-\tau_t^L)} - 1$ and $\pi_t^P = 0$.
- τ_t^L is a free parameter that can be deduced from π_t^W and the allocation. Hence, the wage Phillips curve is not a constraint.
- π_t^W only reduces utility and is an independent parameter that can be set through τ^W, hence $\pi_t^W = 0$

The program then reduces to the same one as in the flexible-price economy without union:

Recovering taxes from the allocation We then have

$$1 - \tau_t^L = \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \frac{\int_i y_{i,t} u'(c_{i,t}) \ell(di)}{v'(L_t)}$$

$$1 - \tau_t^W = \alpha w_t$$

$$1 - \tau_t^E = \frac{1}{Z_t} \frac{w_t}{(1 - \tau_t^W)(1 - \tau_t^L)}$$

D.3 The HA economy without τ_t^L

We impose $\tau_t^L = 0$. The program is otherwise the same as in Section D.2. In particular, τ_t^E only appears in the price Phillips curve. As consequence, this equation is not a constraint and τ_t^E is set, such that $\pi_t^P = 0$. Inflation indeed only destroys resources here. We then obtain the following program:

$$\max_{\left(\tau_t^W, B_t, T_t, \pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(L_t)\right) \ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t + (1 + r_t) \int_i a_{i,t-1}\ell(di) + w_t L_t + T_t \leq Z_t L_t + \int_i a_{i,t}\ell(di),$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1 + r_t)a_{i,t-1} + y_{i,t} w_t L_t + T_t$,

$$u'(c_{i,t}) = \beta \mathbb{E}_t\Big[(1 + r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t} = \left(\frac{Z_t}{2}\right)^{\varphi},$$

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) L_t + \beta \mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W + 1)\Big],$$

Because of $\tau_t^L = 0$, we cannot have simultaneously optimal labor supply and $\pi_t^W = 0$: the planner has to balance the relative costs of wage inflation with the suboptimal provision of labor supply. The Lagrangian is:

$$\begin{aligned}
\mathcal{L} = {} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_{i,c,t} - (1 + r_t)\lambda_{i,c,t-1}\right) u'(c_{i,t})\ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W(1 + \pi_t^W) \\
& + \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) L_t \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1 + r_t) \int_i a_{i,t-1}\ell(di) - w_t L_t - T_t\right).
\end{aligned}$$

We recall that in this economy, we have $\psi_{i,t} = \omega_t^i u'(c_{i,t}) - \left(\lambda_{i,c,t} - (1 + r_t)\lambda_{i,c,t-1}\right) u''(c_{i,t}) - \frac{\varepsilon_W - 1}{\psi_W} \gamma_{W,t} w_t y_{i,t} u''(c_{i,t}) L_t$, where we also drop the superscript.

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t})\ell(di) = 0.$$

FOC wrt w_t.

$$\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) = \gamma_{W,t}\frac{\varepsilon_W - 1}{\psi_W}\int_i y_{i,t}u'(c_{i,t})\ell(di).$$

FOC wrt L_t. Using the FOC wrt w_t:

$$-\int_i \omega_{i,t}\ell(di)v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right) = 0.$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta\mathbb{E}_t\left[(1+r_{t+1})\hat{\psi}_{i,t+1}\right].$$

D.4 The HA economy without τ_t^L and τ_t^E with τ_t^W

In this case, there is no obvious simplification and the program is:

$$\max_{\left(\tau_t^W,\tau_t^E,\tau_t^K,B_t,T_t,\pi_t^P,\pi_t^W,w_t,r_t,\Omega_t,\tilde{R}_t^N,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega(y_t^i)\left(u(c_t^i) - v(L_t)\right)\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t + (1+r_t)\int_i a_{i,t-1}\ell(di) + w_t L_t + T_t \leq \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t L_t + \int_i a_{i,t}\ell(di),$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1+r_t)a_{i,t-1} + w_t y_{i,t} L_t,$

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0,\ \nu_{i,t} \geq 0,$$

$$u'(c_{i,t}) = \beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t},$$

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}w_t\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)L_t + \beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W + 1)\Big],$$

$$\pi_t^P(1+\pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P}\Big(\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^W)} - 1\Big) + \beta\mathbb{E}_t\Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\Big),$$

$$(1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^W} = \frac{w_t}{1-\tau_t^W}(1+\pi_t^P),$$

while the corresponding Lagrangian becomes:

$$
\begin{aligned}
\mathcal{L} = {} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1+\pi_t^W) \\
& + \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P (1+\pi_t^P) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{w_t}{(1-\tau_t^W)} - Z_t \right) L_t \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big((1 - \frac{\psi_P}{2}(\pi_t^P)^2) Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t \Big) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1+\pi_t^W) \frac{w_{t-1}}{1-\tau_{t-1}^W} - \frac{w_t}{1-\tau_t^W}(1+\pi_t^P) \right)
\end{aligned}
$$

We now turn to the computation of the FOCs.

FOC wrt π_t^W.

$$
-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t \frac{w_{t-1}}{1-\tau_{t-1}^W} = 0.
$$

FOC wrt π_t^P.

$$
-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) - \mu_t \psi_P \pi_t^P - \frac{\Lambda_t}{Z_t L_t} \frac{w_t}{1-\tau_t^W} = 0.
$$

FOC wrt r_t.

$$
\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t})\ell(di) = 0.
$$

FOC wrt w_t. Using the FOC wrt to τ^W, we have:

$$
0 = \int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) - \gamma_{W,t}\frac{\varepsilon_W - 1}{\psi_W}\int_i y_{i,t} u'(c_{i,t})\ell(di).
$$

FOC wrt L_t. Using the FOC wrt w_t:

$$
\begin{aligned}
0 = {} & -\int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big) Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right) \\
& - (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P (1+\pi_t^P) Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(\frac{w_t}{(1-\tau_t^W)} - Z_t \right).
\end{aligned}
$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{i,t+1} \right].$$

FOC wrt τ_t^W. We derive wrt $\frac{1}{1-\tau_t^W}$ and obtain:

$$0 = \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} L_t - \Lambda_t (1 + \pi_t^P) + \beta \mathbb{E}_t \left[\Lambda_{t+1} (1 + \pi_{t+1}^W) \right].$$

D.5 The HA economy without time-varying τ_t^L and τ_t^W with τ_t^E

I consider $\tau_{ss}^L = \left(\frac{\varepsilon_W - 1}{\varepsilon_W}\right)^{-1}$ and $\tau_t^W = \tau_{ss}^W$

In this case, there is no obvious simplification and the program is:

$$\max_{\left(\tau_t^E, \tau_t^K, B_t, T_t, \pi_t^P, \pi_t^W, w_t, r_t, \Omega_t, \tilde{R}_t^N, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t \geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(L_t) \right) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right],$$

$$G_t + (1 + r_t) \int_i a_{i,t-1} \ell(di) + w_t L_t + T_t \leq \left(1 - \frac{\psi_P}{2} (\pi_t^P)^2 \right) Z_t L_t + \int_i a_{i,t} \ell(di),$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1 + r_t) a_{i,t-1} + w_t y_{i,t} L_t,$

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0, \ \nu_{i,t} \geq 0,$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \left[(1 + r_{t+1}) u'(c_{i,t+1}) \right] + \nu_{i,t},$$

$$\pi_t^W (\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W} \left(v'(L_t) - w_t \int_i y_{i,t} u'(c_{i,t}) \ell(di) \right) L_t + \beta \mathbb{E}_t \left[\pi_{t+1}^W (\pi_{t+1}^W + 1) \right],$$

$$\pi_t^P (1 + \pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P} \left(\frac{1}{Z_t} \frac{w_t}{(1 - \tau^W)(1 - \tau_t^E)(1 - \tau_{ss}^L)} - 1 \right) + \beta \mathbb{E}_t \left(\pi_{t+1}^P (1 + \pi_{t+1}^P) \frac{Z_{t+1} L_{t+1}}{Z_t L_t} \right),$$

$$(1 + \pi_t^W) w_{t-1} = w_t (1 + \pi_t^P),$$

while the corresponding Lagrangian becomes:

$$
\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\
- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1+\pi_t^W) \\
+ \frac{\varepsilon_W}{\psi_W}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t \\
- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P (1+\pi_t^P) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{w_t}{(1-\tau_{SS}^W)(1-\tau_{SS}^L)(1-\tau_t^E)} - Z_t \right) L_t \\
+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big((1 - \frac{\psi_P}{2}(\pi_t^P)^2) Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t \Big) \\
+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1+\pi_t^W) w_{t-1} - w_t (1+\pi_t^P) \right)
\end{aligned}
$$

We now turn to the computation of the FOCs.

FOC wrt τ_t^E. We derive wrt $\frac{1}{1-\tau_t^E}$ and obtain:

$$0 = \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\frac{w_t}{(1-\tau_{SS}^W)(1-\tau_{SS}^L)} L_t$$

or

$$\gamma_{P,t} = 0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) - \mu_t \psi_P \pi_t^P - \frac{\Lambda_t}{Z_t L_t} w_t = 0,$$

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t \frac{w_{t-1}}{1-\tau_{t-1}^W} = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t})\ell(di) = 0.$$

FOC wrt w_t.

$$0 = \int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) - \gamma_{W,t}\frac{\varepsilon_W}{\psi_W}\int_i y_{i,t} u'(c_{i,t})\ell(di).$$

FOC wrt L_t.

$$\begin{aligned}
\mathcal{L} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1+\pi_t^W) \\
&+ \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big(Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t \Big)
\end{aligned}$$

$$\begin{aligned}
0 = &- \int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right) \\
&+ w_t \left(\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) - \gamma_{W,t}\frac{\varepsilon_W}{\psi_W}\int_i y_{i,t}u'(c_{i,t})\ell(di) \right)
\end{aligned}$$

Using the FOC wrt w_t:

$$0 = - \int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right)$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1+r_{t+1})\hat{\psi}_{i,t+1} \right].$$