

Brain versus Brawn: The Realization of Women's Comparative Advantage*

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ABSTRACT

This paper examines the evolution of women in the labor market, specifically their post-World War II employment, wages and education, by assessing the role of technology changing labor demand requirements, as a driving force. The empirical results in the United States data show that job requirements have shifted from more physical to more intellectual attributes. Moreover, women have always worked in occupations with relatively low physical requirements and, traditionally, also worked in occupations with lower intellectual requirements than men. However, the later trend has been reversed over time with women overtaking men in college education by the mid 1980s. This paper uses a model in which agents make work and education decisions to account for the importance of technological shifts in women's labor market experience. The key feature of the model is that individuals are heterogenous in their innate brain and brawn abilities, and women have on average less brawn than men. This is the main source for the employment, wage and education gaps in the 1950s between men and women. The general equilibrium model is simulated to account for the quantitative implications of brain biased technical change (BBTC), which is modeled as a rise in the share parameter on the brain factor in a CES production function, from 1950 to 2005. In particular, as BBTC favors women's comparative advantage in brain over brawn, the model is able to generate a large rise in female participation, closing gender wage and education gaps, in addition to a rising college premium. These results suggest that labor demand changes and multidimensional skill attributes are important in explaining the radical evolution of women's labor market participation, wages and education.

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Email: mrendall@iew.uzh.ch. All errors are mine.

1 Introduction

One of the greatest phenomena of the 20th century has been the rise in female labor force participation. Using evidence from United States data, this study develops a general equilibrium model based on the following three facts of women’s labor market experience since World War II:

1. *Female labor force participation:* Married women’s labor force participation, aged 25 to 64, rose from 25 percent in 1950 to 70 percent in 2005. Single women’s labor force participation rose by roughly 15 percent, while men’s labor force participation stayed fairly steady.
2. *Gender wage gap:* The gender wage gap, defined as average female to male wages, remained fairly steady around 60 percent until the mid 1970s, but by 2005 it had reached about 77 percent (figure 2(a), left panel).
3. *Education:* The fraction of women with a college degree rose substantially during this time period. In the 1950s, for every college-educated women, there were about two college-educated men. Women started to catch up with men in the mid 1960s and by the late 1980s the gap had disappeared (figure 2(b), right panel). Today, the gap is reversed with more women graduating from college than men.

While macroeconomists have extensively studied the rise in female labor force participation,¹ and the education gap² the evolution of female wages has remained largely unexplored. Since wage divergences and increases over the last decades are explained by emphasizing education, human capital and intellectual abilities,³ the goal of this paper is to explore how much of these three phenomena can be explained by a simple demand side story focusing on human capital and physical labor.

Empirical economic studies have found various important factors shaping women’s labor market experience, such as changes in women’s work experience, education, and occupational mix (see, for example, [Black and Juhn, 2000](#); [Blau, 1998](#); [Mulligan and Rubinstein, 2005](#)). In addition, the rise of female labor force participation has been the focus of recent macroeconomic research. Complementary to the theory presented in this paper, studies argue that improvements in home technology, such as the invention and marketization of household appliances (see, for example, [Greenwood, Seshadri, and Yorukoglu, 2002](#), and references therein), or the improvements in baby formulas (see [Albanesi and Olivetti, 2006](#)), enabled women to enter the labor market. While improvements in home technology freed women from time-consuming household chores, theories only focused on home technology improvements do not effectively address the evolution of the gender wage and education gap. Two recent studies focus on the effects of cultural, social, and intergenerational learning on female labor supply (see [Fernández, 2007](#); [Fogli and Veldkamp, 2007](#)). As before, these models are successful in explaining part of the rise in female labor force participation. In addition, [Fogli and Veldkamp \(2007\)](#) extend their theory to explain the evolution of wages through women’s self-selection bias, i.e., the characteristics of working women changed in the 20th century. However, this model is unable to match the complete wage evolution, only matching either the initial stagnation or the later rise in relative female to male wages.

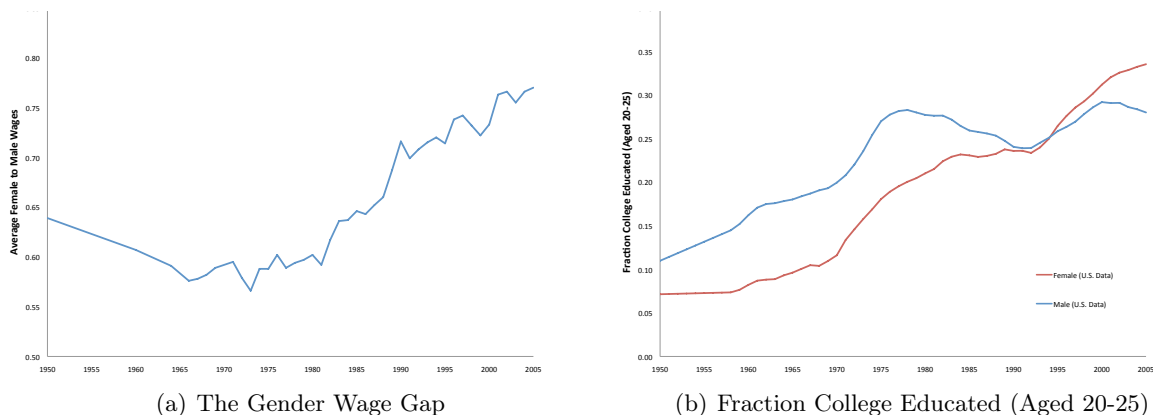
Nonetheless, the hypothesis of changing technological progress and closing of the gender wage gap has been analyzed in some econometric studies with varying conclusions. [Wong \(2006\)](#) finds

¹See for example [Albanesi and Olivetti \(2006\)](#); [Fernández \(2007\)](#); [Fogli and Veldkamp \(2007\)](#); [Greenwood and Guner \(2009\)](#); [Jones, Manuelli, and McGrattan \(2003\)](#); [Olivetti \(2006\)](#).

²See for example [Rios-Rull and Sanchez-Marcos \(2002\)](#); [Yang and Ge \(2008\)](#).

³See for example [Becker \(1994\)](#); [Juhn, Murphy, and Pierce \(1993\)](#).

Figure 1: *Women's Market Trends*



that skill-biased technical change had a similar impact on men's and women's wages and, therefore, cannot explain the closing wage gap. [Black and Spitz-Oener \(2010\)](#) quantify the contribution of changes in specific job tasks on the closing wage gap from 1979 to 1999 for West Germany. The authors find that skill-biased technical change in West Germany, especially through the adoption of computers, can explain about 41 percent of the closing wage gap. While these two studies estimate the effects of relative labor demand changes on the wage gap, both assume an inelastic labor supply. Consequently, they cannot address the non-linear path of average female to male wages stemming from women's self-selection bias into the labor market and changing education choices over time.

To summarize, while previous studies have been successful in explaining the rise in female labor force, they say little about the closing gender wage gap or the reversal in education trends. One unexplored fact, changing labor demand requirements due to technological progress, could potentially explain some of the observed wage and education trends. More specifically, in this paper, evidence from the United States points to labor requirement trends supporting an explanation of technological change favoring women. Thus, a simple general equilibrium model is developed, where women's improved labor market experience is driven by labor demand changes. The main factor in improving women's labor market opportunities, their potential wages, and their returns to education is the shift in labor factors away from brawn, as similarly suggested by [Galor and Weil \(1996\)](#) and supported by data evidence (see for example [Bacolod and Blum, 2005](#)). In doing so, the goal is to provide a driving force for observed labor market changes that previously have been taken as given in the study of female labor force participation. For example, [Jones, Manuelli, and McGrattan \(2003\)](#) successfully explain a large rise in participation by an exogenously closing wage gap and [Olivetti \(2006\)](#) does so with an exogenous increase in returns to experience for women. The shift in labor requirements is modeled here by a linear exogenous "skill-biased" technical change, where skill is defined as intellectual abilities ("brain"). This contrasts to the traditional literature on SBTC which distinguishes skill on an educational dimension (e.g., high or low, college or non-college). A similar alternative explanation of SBTC has previously been used by [Guvener and Kuruscu \(2009\)](#) in explaining the rise between and within wage inequality in the United States since 1970. The authors focus on men and on the human capital accumulation decision over the life-cycle. This paper abstracts from a life-cycle approach and only allows individuals to educate when young. I make this simplification since the focus here is in explaining gender differences in participation, college degrees and average wages, and does not focus on inequality across and within groups. The deviation from the traditional education-based skill classification and the introduction of a second dimension of

skill, brawn, is a key feature of the model in creating different labor market opportunities for men and women. Agents are heterogeneous in innate brain and brawn levels, and therefore, depending on current wages, differ in their willingness to work in the labor market or spend time on home production. Moreover, when young and deciding on obtaining a college education, individuals take into account future expected wages, which depend on the market price of brain and brawn. It is assumed that the only cost of obtaining higher education is a utility cost, which depends on an agent's innate brain with higher ability individuals facing a lower cost. Since, most of the rise in female labor force participation has been for married women, as in [Jones, Manuelli, and McGrattan \(2003\)](#), agents are randomly assigned to married or single life after the education decision has taken place. In addition, as previous research has found (see for example [Rios-Rull and Sanchez-Marcos, 2002](#)), women acquire additional education even if labor market returns are low in order to find a more educated (high earning) spouse. In this paper, the random marriage assignment allows for assortative matching on education, giving women an additional incentive to become educated.

In this model, women with a high earning spouse tend to stay out of the labor market when the returns to brawn are relative high (or the returns to brain are low), thus obtaining *ceteris paribus* less education (assortative matching might induce more education). Moreover, if the difference in brawn between men and women is sufficiently large, women who do work will, in general, have lower wages. With a fall in the returns to brawn and a rise in the returns to brain, women will be favored by their comparative advantage in brain, and will catch up in employment levels, wages and education with men. Moreover, women might surpass men in educational attainment, given their comparative advantage or greater dependence on brain for higher wages.

On the demand side, production is modeled as an aggregate constant elasticity of substitution (CES) production function as in [Güvener and Kuruscu \(2009\)](#), where brain and brawn are the inputs. Brain-biased technical change (BBTC) occurs starting in the 1950s, given the empirical evidence presented in section 2, unlike the traditional literature on SBTC which usually starts around the 1970s. I present results where agents are myopic and have perfect foresight, but in neither case do they anticipate BBTC until after 1950. Again, this CES specification, rather than the traditional low and high education specification, is important to model the wedge between men and women. Since individuals always supply both types of skills, everyone gains from the rise in brain wages and everyone loses from the rise in brawn prices. However, high brain people gain more from the rising brain price, while high brawn people (mostly men) lose more from the fall in brawn wages. This feature produces a rich set of dynamics, that are able to generate not only a convergence in female and male labor market outcomes because of a rise in female wages, but also a fall in uneducated male wages as observed in the U.S. since the mid 1970s.

The model is calibrated to the match 1950 U.S. data moments. The BBTC mechanism developed in this paper is able to explain: (1) a large rise in female labor force participation, (2) first a stagnation in average female to male wages, (3) a closing wage gap starting in the mid 1970s, and (4) a similar reversal in the education gap as depicted in figure [2\(b\)](#), right panel.

While the empirical results are specific to the United States, the model developed could also be used to study cross-country differences in women's labor market participation. [Rogerson \(2005\)](#) notes that the change in relative employment of women and the aggregate service share (a brain-intensive sector given data evidence) between 1985 and 2000 are highly correlated at 0.82, concluding that countries which added the most jobs to the service sector also closed the employment gap the most.

As labor demand changes are the key motivation for this study, Section 2 provides further evidence for the changing labor market, focusing on the evolution of physical and intellectual job

requirements in the United States and women’s self-selection into low-strength jobs. The general equilibrium model is outlined in Section 3, and Section 4 provides analytical results of BBTC on labor demand, labor supply, wages, and education. Section 5 discusses the estimation and calibration procedure, and Section 6 presents labor market trends resulting from a linear exogenous BBTC starting in the 1950s. Lastly, Section 7 concludes.

2 United States Labor Facts

To explore the relationship between the rise in female labor force participation and changes in labor demand, this study focuses on the relative demand and supply of two types of labor inputs: brains and brawn. This study starts from the premise that women have, on average, less brawn than men. Accepting that women and men have similar levels of brain, men have a comparative advantage in brawn-intensive occupations. However, technological change shifts labor demand toward low-brawn occupations diminishing men’s comparative advantage in the labor market.

Using factor analysis, I obtain brain and brawn estimates by United States census occupation and industry classifications from the 1977 Dictionary of Occupational Title (DOT).⁴ The 1977 DOT reports 38 job characteristics for over 12,000 occupations, documenting (1) general educational development, (2) specific vocational training, (3) aptitudes required of a worker, (4) temperaments or adaptability requirements, (5) physical strength requirements, and (6) environmental conditions. For example, general educational development measures the formal and informal educational attainment required to preform a job effectively by rating reasoning, language and mathematical development. Each reported level is primarily based on curricula taught in the United States, where the highest mathematical level is advanced calculus, and the lowest level only requires basic operations, such as adding and subtracting two-digit numbers. Specific vocational preparation is measured in the number of years a typical employee requires to learn the job tasks essential to perform at an average level. Eleven aptitudes required of a worker (e.g., general intelligence, motor coordination, numerical ability) are rated on a five point scale, with the first level being the top ten percent of the population and the fifth level comprising the bottom ten percent of the population. Ten temperaments required of a worker are reported in the 1977 DOT, where the temperament type is reported without any numerical rating. An example of a temperament is the ability to influence people in their opinions or judgments. Physical requirements include a measure of strength required on the job, rated on a five point scale from sedentary to very heavy, and the presence or absence of tasks such as climbing, reaching, or kneeling. Lastly, environmental conditions measure occupational exposure (presence or absence) to environmental conditions, such as extreme heat, cold, and noise. I use factor analysis similarly to [Ingram and Neumann \(2006\)](#) to reduce the dimensionality of DOT job characteristics. Using factor analysis, a linear relationship between normally distributed broad skill categories (e.g., brain, brawn, motor coordination) and the 38 DOT characteristics is estimated from the associated 38 variable correlation matrix. For a detailed explanation of the estimation procedure see Appendix A.

Using maximum likelihood estimation methods, three factors are determined sufficient in capturing the information contained in the 38 DOT characteristics. Given the estimated coefficients

⁴I also estimate factors using the 1991 DOT code, with very similar results. Unfortunately, the nature of factor analysis makes a direct comparison between the two surveys unfeasible, given that occupational requirements are only compared to the average in the sample.

(factor loadings) I term these factors: brain, brawn, and motor coordination (see Appendix A Table A.1). These factors are merged with the 1950 and 1960 United States Census data and the 1968 to 2005 Current Population Survey (CPS) data to compute trends over time.⁵ Note that for illustrative purposes and estimation in later sections, brain and brawn factors are normalized from zero to one (lowest to highest), i.e., we can think of occupations with zero as using zero percent of a factor and one as using 100 percent of a factor. Figure 2, which plots occupational brain and brawn combinations weighted by 1950 Census employed population, clearly depicts the difference in brain and brawn requirements across the economy. While, figure 3 shows the 2005 CPS weighted occupational brain and brawn requirements, which are shifted to the right (more brain) and the bottom (less brawn) compared to 1950.

To compute aggregate factor demand changes in the United States over time, 1977 occupation-industry factor estimates are aggregated using United States Census and CPS civilian labor force weights. Figure 4 depicts aggregate factor standard deviations. While motor coordination remains fairly constant over time, the brain supply steadily increases and the brawn supply steadily decreases. This rising trend in the supply of brain versus the falling trend in the supply of brawn is what I term BBTC. These trends are not specific to the 1977 DOT, since I obtain similar results for 1991 DOT, as do Ingram and Neumann (2006) (see Figure 3 in the referenced paper). Note that using a single DOT survey to determine job requirements implies that the specific job factor requirements did not change over the last five decades. For example, a craftsman utilized the same brawn level in 1950 as in 2005. Ergo, all trends pictured are due to changes in the composition (mix) of occupations within the economy, and the rise in brain and fall in brawn requirements might possibly be greater than shown due to intra occupation skill-biased technical changes.

Figure 5 depicts brain and brawn standard deviations by gender over time, with the selection of women into low-brawn occupations clearly evident. Given women’s lower innate brawn levels, this bias toward low brawn occupations can be either due to employee self-selection or employer discrimination. Additionally, the total brain supply has risen continuously since the 1950s, with women’s brain supply surpassing men’s by the 1980s. This trend could possibly be linked with increased educational investment.

Given the above facts, I argue that beginning in the 1950s women entered the labor market and their average wages improved due to the rise of brain-intensive occupations, which complemented women’s comparative advantage.

⁵Census and CPS data is obtained from the IPUMS-USA (Ruggles, Sobek, Alexander, Fitch, Goeken, Hall, King, and Ronnander, 2004) and the IPUMS-CPS project (King, Ruggles, Alexander, Leicach, and Sobek, 2004). The IPUMS projects provide a consistent 1950 United States Census classification of occupations and industries over the years, which is used in merging 1977 DOT brain and brawn factors.

Figure 2: *Brain and Brawn Job Combinations from the 1977 DOT with 1950 Census Labor Shares*

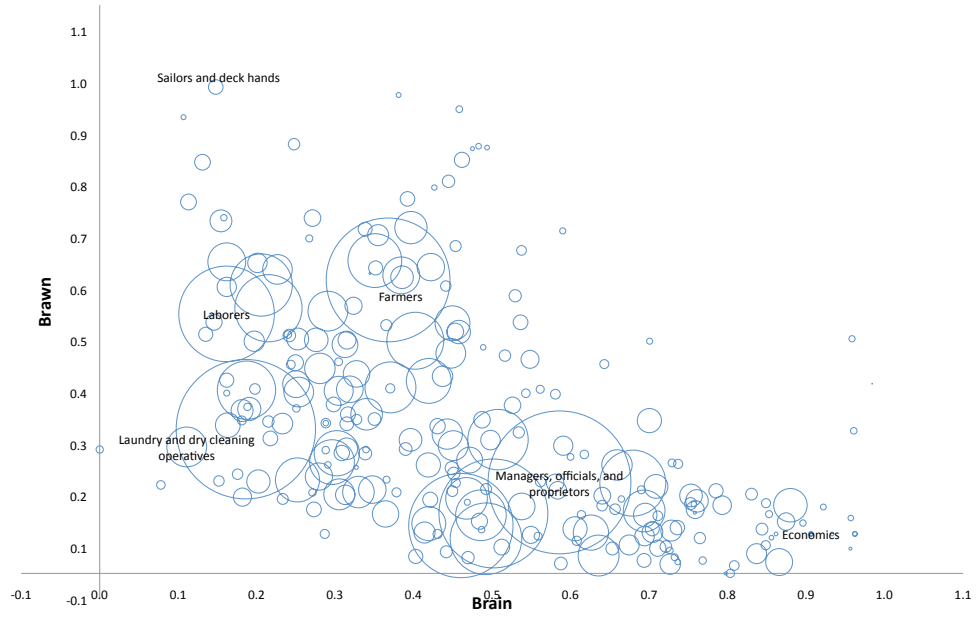


Figure 3: *Brain and Brawn Job Combinations from the 1977 DOT with 2005 Census Labor Shares*

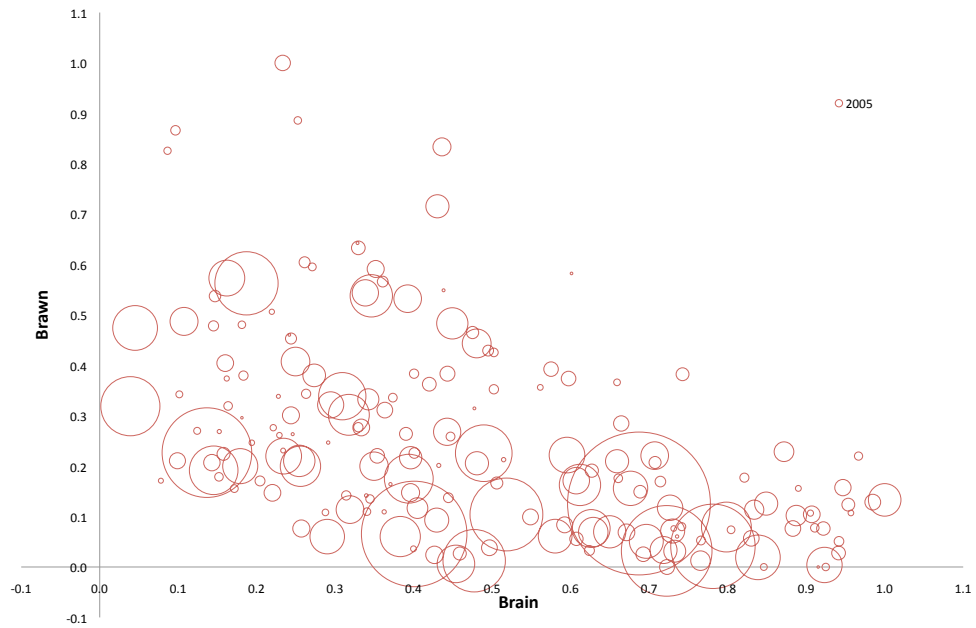


Figure 4: *Standard Deviations of Labor Input Supply Over Time*

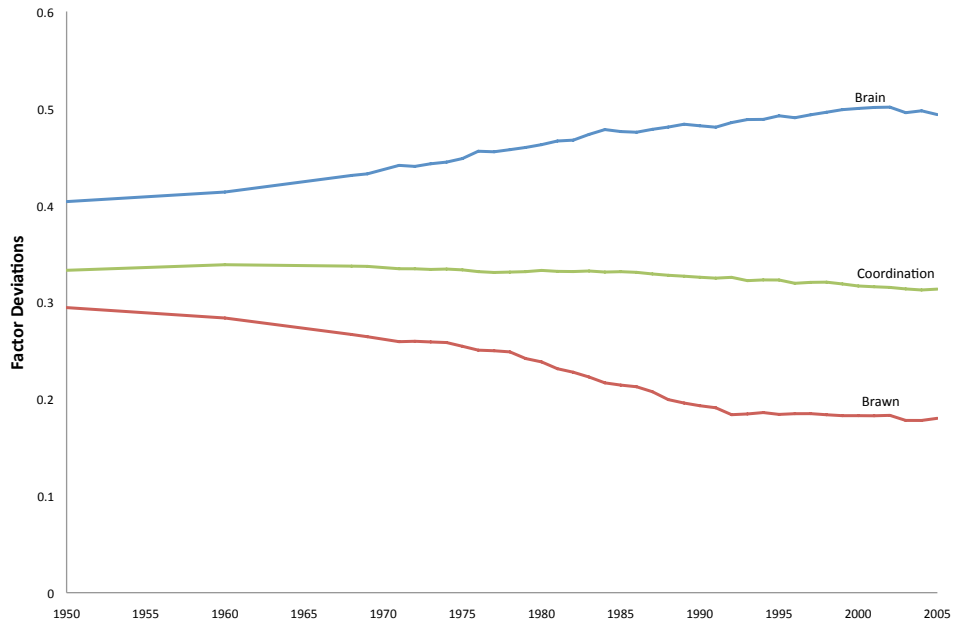
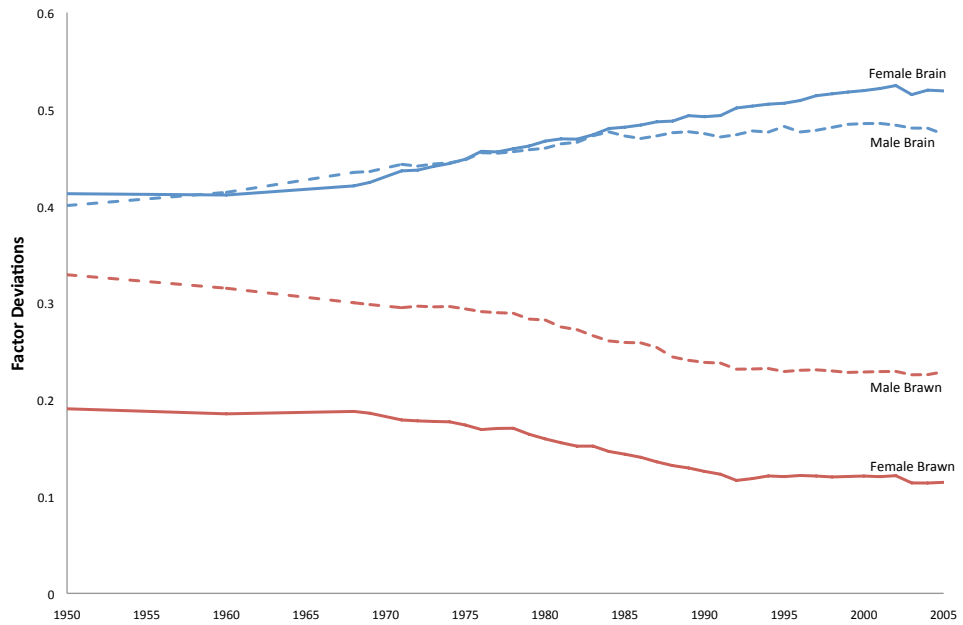


Figure 5: *Standard Deviation of Labor Input Supply by Gender*



3 Toy Model

The underlying forces of the model simulated in Section 6, are best demonstrated in a simplified partial equilibrium version. Assume there is a unit measure of men and women, who are all single and live for two periods. The one-period utility function is,

$$u(c_t) = c_t, \quad (1)$$

where $c_t = \max\{w_t, A_h\}$, that is agents choose between working and earning wage w_t or staying at home and consuming home production A_h . Wages are a function of an agents innate brain and brawn, that is $w_t = w_{b,t}b_i + w_{r,t}r_i$. Let all men have brawn r^m , and all women have brawn $r^f < r^m$, while brain is distributed identically for men and women by $b \sim U[b_h, b_l]$. Furthermore, assume that $w_{b,t}b_l + w_{r,t}r^m \geq A_h$, but $w_{b,t}b_l + w_{r,t}r^f < A_h$, that is, all men work, while women only work if,

$$b_i \geq \frac{A_h - w_{r,t}r^f}{w_{b,t}} \equiv \hat{b}_t^f. \quad (2)$$

3.1 Labor Supply and Wages

Given the above assumptions, men's labor force participation equals one and women's equals,

$$LFP_t^f = \int_{\hat{b}_t^f}^{b_h} dF(b) = \frac{b_h - \hat{b}_t^f}{b_h - b_l}. \quad (3)$$

Thinking of BBTC as a rise in $\frac{w_{b,t}}{w_{r,t}}$. Clearly, a rise in $w_{b,t}$ (a fall in $w_{r,t}$) will lead to a fall in \hat{b}_t^f and a rise in female employment given all previous assumptions still hold, i.e., BBTC leads to a closing employment gap.

Average female wages are then,

$$\bar{w}_t^f = w_{b,t}E(b^f) + w_{r,t}r^f = .5 \left(w_{b,t}b_h + w_{r,t}r^f + A_h \right), \quad (4)$$

where $E(b^f)$ is the average brain supply conditional on the working population,

$$E(b^f) = \frac{\int_{\hat{b}_t^f}^{b_h} b dF(b)}{LFP_t^f} = .5 \left(b_h + \hat{b}_t^f \right). \quad (5)$$

Similarly, male wages are,

$$\bar{w}_t^m = w_{b,t}E(b^m) + w_{r,t}r^m = .5w_{b,t}(b_h + b_l) + w_{r,t}r^m, \quad (6)$$

where $E(b^m) = .5(b_h + b_l)$ given that all men work.

3.2 Evolution of the Wage Gap with BBTC

Given wages and average brain supply there are two facts that will govern the evolution of the gender wage gap with BBTC. In addition, given that there has been a sharp rise in education, likely affecting brain supplies, an additional education effect arises when adding and education choice into the model. Therefore, the wage gap will evolve non-linearly over time, even if BBTC is rising linearly, given the interaction of,

1. The *Price Effect*: women benefit more (lose less) from falling brawn wages $w_{r,t}$
2. The *Supply Effect*: lower ability women will enter the market
3. The *Education Effect*: non-working women have no incentive to obtain education. However, once more women enter the labor market they will be more likely than men to obtain education, increasing average brain supplies disproportionately for women.

The price effect follows from taking the derivative of \bar{w}_t^g with respect to $w_{b,t}$ and $w_{r,t}$.

$$\frac{\partial \bar{w}_t^m}{\partial w_{r,t}} = r^m \quad \text{and} \quad \frac{\partial \bar{w}_t^f}{\partial w_{r,t}} = .5r^f \quad (7)$$

Since, $.5r^f < r^m$ a fall in $w_{r,t}$ will close the gender wage gap. Moreover,

$$\frac{\partial \bar{w}_t^m}{\partial w_{b,t}} = .5(b_h + b_l) > 0, \quad \text{and} \quad \frac{\partial \bar{w}_t^f}{\partial w_{b,t}} = .5b_h > 0, \quad (8)$$

both men and women's wages grow with a rise in $w_{b,t}$, however, the rise is smaller for men and than, because of the supply effect

$$\frac{\partial E(b^m)}{\partial w_{b,t}} = 0, \quad \text{and} \quad \frac{\partial E(b^f)}{\partial w_{b,t}} = -.5 \frac{\hat{b}_t^f}{w_{b,t}} < 0, \quad (9)$$

that is, lower ability women (lower brain) enter the market with a rise in brain returns. To summarize, the supply effect will lead to a widening gender wage gap with BBTC, while the price effect will close the gender wage gap. Lastly, to analyze the education effect we need to introduce the education decision, benefit and cost. Assume that individuals decide to attend college to increase their innate brain ability by a factor e in the first period. Attending college results in a utility cost η and college students cannot work in the labor market. The college education decision for a male i , assuming discounting of β and constant wage rates, is,

$$\beta(w_b b_i e + w_r r^m) - \eta \geq (1 + \beta)(w_b b_i + w_r r^m), \quad (10)$$

or men educated if and only if,

$$b_i \geq \frac{\eta + w_r r^m}{w_b(\beta(e - 1) - 1)} \equiv b^e(r^m). \quad (11)$$

Men become educated if the increased wages in the second period due to e , can compensate for the lost wages when young and the utility cost. If all women worked women would have a similar cut-off

$$b^e(r^f) < b^e(r^m),$$

since forgone earnings are smaller, $r^f < r^m$. However, since some women remain home if r^f is low enough

$$\tilde{b}^e(r^f, A_h) > b^e(r^m),$$

less women than men will obtain education in an economy that uses mostly brawn in production. Therefore, a fall in brawn returns or a rise in brain returns will eventually lead to a *reversal* in the

education gap between gender, leading ultimately to reverse the supply effect and a closing wage gap.

In summary, the price and education effect will close the wage gap, while the supply effect will widen the wage gap. The supply effect will dominate when women's labor force participation is considerably lower than men's and education levels are low, but will slowly disappear as labor force participation and education rates converge. Therefore, the natural evolution of these effects will initially cause a fall, or stagnation, of average female to male wages, which will close as the price and education effect begin to dominate.

These analytical results suggest that a model differentiating between brain and brawn labor requirements should replicate the initial United States employment, education, and wage differences across gender. Moreover, it should reproduce the subsequent evolution of the female labor force participation rate, college attainment, and the gender wage gap, including some initial stagnation in average female wages as observed during the 1960s and 1970s and a reversal of education attainments as in the mid 1980s.

4 General Equilibrium Model

The simulated economy consists of a unit measure of males and females, who at age 1 either wed or remain single forever. Agents, are endowed with brain and brawn, which they supply jointly to the labor market. These two labor inputs are aggregated to a final market good, which is consumed by households. Agents live and work until age N . At age 0, individuals can choose to attend college and, therefore, increase their innate brain ability. During college enrollement agents forgo earnings and pay a one time utility cost. Lastly, agents choose to work in the labor market or the home, and substitute consumption between market and home produced goods.

4.1 Marriage

After the education decision has been made (at age 1), individuals are either married or remain single until age N . Marriage is determined by chance, but varies with educational attainment. That is, age 1 educated women at time t marry with probability $p_{e,t}^f$ and uneducated women marry with probability $p_{u,t}^f$. Moreover, there is assortative matching in education, i.e., the probability that an educated woman marries an educated man, is strictly greater than marrying an uneducated man, $p_{e,e^*,t}^f > p_{e,u^*,t}^f$ and $p_{e,e^*,t}^f + p_{e,u^*,t}^f = 1$. Similarly, for uneducated women probabilities satisfy $p_{u,e^*,t}^f < p_{u,u^*,t}^f$ and $p_{u,e^*,t}^f + p_{u,u^*,t}^f = 1$. If a fraction λ_t^f of age 1 women and λ_t^m of men is educated at time t for a consistent equilibrium. Males marriage probabilities are,

$$\begin{aligned}
 p_{e,t}^m &= \frac{\lambda_t^f p_{e,t}^f p_{e,e^*,t}^f + (1 - \lambda_t^f) p_u^f p_{u,e^*,t}^f}{\lambda_t^m}, \\
 p_{u,t}^m &= \frac{\lambda_t^f p_{e,t}^f p_{e,u^*,t}^f + (1 - \lambda_t^f) p_u^f p_{u,u^*,t}^f}{1 - \lambda_t^m}, \\
 p_{e,e^*,t}^m &= \frac{\lambda_t^f p_{e,t}^f p_{e,e^*,t}^f}{\lambda_t^f p_{e,t}^f p_{e,e^*,t}^f + (1 - \lambda_t^f) p_u^f p_{u,e^*,t}^f}, \\
 p_{u,e^*,t}^m &= \frac{\lambda_t^f p_{e,t}^f p_{u,e^*,t}^f}{\lambda_t^f p_{e,t}^f p_{u,e^*,t}^f + (1 - \lambda_t^f) p_u^f p_{u,u^*,t}^f}, \\
 p_{e,u^*,t}^m &= 1 - p_{e,e^*,t}^m \quad \text{and} \quad p_{u,u^*,t}^m = 1 - p_{u,e^*,t}^m.
 \end{aligned}$$

4.2 Preferences

Given evidence on the intensive and extensive margin of labor supply,⁶ it is assumed that agents can either work full-time in the labor market or not at all, $\ell_i = \{0, 1\}$ for agent i . Moreover, given evidence on household consumption⁷ it is assumed that market and home produced goods are imperfect substitutes. To summarize a married household maximizes is,

$$U_p(c_t, h_t) = \left(\left(\frac{c_t - \bar{c}}{\phi} \right)^\zeta + \left(\frac{h_t}{\phi} \right)^\zeta \right)^{1/\zeta}. \quad (12)$$

subject to a standard budget constraint, the home production technology, and a time constraint,

$$c_t \leq \omega_t^m \ell_t^m + \omega_t^f \ell_t^f \quad (13)$$

$$h_t = A_h \left(1 - \ell_t^m + 1 - \ell_t^f \right) \quad (14)$$

$$0 \leq \ell_t^m, \ell_t^f \leq 1, \quad (15)$$

where the superscripts m, f stand for male, female, $\frac{1}{1-\zeta}$ is the elasticity of substitution between market and home goods, $0 < \phi < 2$ is an economy of scale parameter in marriage, and \bar{c} is a consumption subsistence level. The consumptions subsistence level, is necessary to account for the fact that in 1950 married educated and uneducated women had similar labor supply.⁸ Note, agent i earns the wage $\omega_{i,t} = \psi(b_{i,t}, r_{i,t}, e_i)$, a function of his/her innate brain and brawn abilities and their educational attainment e_i . By assumption men and women are perfect substitutes in home production, and, therefore, spouses specialize with the higher wage earner entering the labor market first. Moreover, given a positive subsistence level $\bar{c} > 0$, the primary earner will always work, while the secondary work works if,

$$w_t^2 > \left((w_t^1)^\zeta + A_h^\zeta \right)^{\frac{1}{\zeta}} - w_t^1 \quad (16)$$

or

$$w_t^1 < \bar{c}. \quad (17)$$

Single agents solve a similar maximization problem. However, since given the subsistence requirements and the discrete labor decision, single agents would always have to work in this model. To make the model consistent with the data, it is therefore, assumed that with probability $p_s \geq 0$ some agents have the option of staying at home. We can think of this, as the government providing benefits equal to the subsistence requirements for a random fraction of agents or some singles having other means of covering the subsistence requirements by, e.g., living with their parents, etc. Therefore, a fraction $1 - p_s$ of single agents has to work, while a fraction p_s works if and only if

$$w_{i,t}^j - \frac{\bar{c}}{\phi} \geq A_h \text{ for all } j = \{m, f\}, \quad (18)$$

⁶Single employed women worked nearly 40 hours per week in 1950 and slightly less than 40 hours per week in 2005, while married women worked about 38 hours per week both in 1950 and 2005.

⁷See for example McGrattan, Rogerson, and Wright (1997); Rupert, Rogerson, and Wright (1995); Aguiar and Hurst (2006); Chang and Schorfheide (2003); Rupert, Rogerson, and Wright (1995).

⁸About 37 percent of married educated women and 25 percent of married uneducated women worked in 1950.

where the subsistence requirements is adjusted for the economies of scales, ϕ . That is, single households have to cover less expenditure than a married household, but not necessarily half of the amount, given economies of scale in marriage. The one-period utility of a single household i is,

$$U_s(c_t, h_t) = (1 - p_s) \max \left\{ w_{i,t}^j - \frac{\bar{c}}{\phi}, A_h \right\} + p_s \left(w_{i,t}^j - \frac{\bar{c}}{\phi} \right). \quad (19)$$

4.3 Education

Assume that individuals choose education when young (at age 0) and single. Education requires to forgo earnings at age 0 and a utility cost $\eta_1 b_i^{\eta_2}$, but increases innate brain ability by a factor of e , i.e., $b_{i,e} = b_i e$. The utility cost is decreasing in an individual's innate brain, b_i , i.e., $\eta_2 < 0$ and $\eta_1 > 0$, making it more difficult to enter college the lower one's innate brain. Suppressing subscripts for heterogeneity in brain and brawn, let $V_{p,e,e^*}^{m,1}$ ($V_{p,e,u^*}^{m,1}$) denote the value function of a educated man married to an educated (uneducated) woman at age 1. While, $V_{s,j}^{g,1}$ is the value function when single for an individual i of gender $g = \{f, m\}$ and education $j = \{e, u\}$ at age 1. Other variables are defined analogously. Note the value functions at age 1 are the discounted present value of one-period utilities given wages over time until age N . The value functions at age zero are,

- Uneducated Agent:

$$V_{s,u}^{g,0} = U_s(c_t, h_t) + \beta E \left\{ p_{u,t+1}^g \left[p_{u,e^*,t+1}^g V_{p,u,e^*}^{g,1} + p_{u,u^*,t+1}^g V_{p,u,u^*}^{g,1} \right] + (1 - p_{u,t+1}^g) V_{s,u}^{g,1} \right\} \quad (20)$$

- Educated Agent:

$$V_{s,e}^{g,0} = -\eta_1 b_i^{\eta_2} + \beta E \left\{ p_{e,t+1}^g \left[p_{e,e^*,t+1}^g V_{p,e,e^*}^{g,1} + p_{e,u^*,t+1}^g V_{p,e,u^*}^{g,1} \right] + (1 - p_{e,t+1}^g) V_{s,e}^{g,1} \right\} \quad (21)$$

Therefore, singles educated if and only if:

$$\beta V_{s,e}^{g,0} \geq \beta V_{s,u}^{g,0}. \quad (22)$$

Note, there are two benefits from education, (1) higher wages in future periods, (2) assortative matching in marriage, $p_{e,e^*,t+1}^g > p_{e,u^*,t+1}^g$.

4.4 Production

Agents supply two labor inputs, brain and brawn to the labor market. The aggregate production function has constant elasticity of substitution in the two inputs, B_t and R_t (the aggregate labor supplies of brain and brawn),

$$Y_t = Z_t \left(\gamma_t B_t^\phi + (1 - \gamma_t) R_t^\phi \right)^{1/\phi}, \quad (23)$$

where Z_t is aggregate productivity, γ_t is the share parameter on brain and $\epsilon_\phi = \frac{1}{1-\phi}$ is the elasticity of substitution between the two inputs. A rise in γ_t over time represents the exogenous BBTC.

The relative wage rate for brain and brawn follows from the cost minimization of the final good,

$$w_t = \frac{w_{b,t}}{w_{r,t}} = \frac{\gamma_t}{1 - \gamma_t} \left(\frac{B_t}{R_t} \right)^{\phi-1}. \quad (24)$$

The relative wage is a function of relative factor productivity as well as relative quantities supplied. Using equation (24), and the aggregate production function (23), and normalizing, the final goods price to one, wage rates are,

$$w_{b,t} = Z_t \left[\gamma_t^{\epsilon_\phi - 1} + \left(\frac{1 - \gamma_t}{w_t} \right)^{\epsilon_\phi - 1} \right]^{1/(\epsilon_\phi - 1)}, \quad (25)$$

and

$$w_{r,t} = Z_t \left[\left(\frac{\gamma_t}{w_t} \right)^{\epsilon_\phi - 1} + (1 - \gamma_t)^{\epsilon_\phi - 1} \right]^{1/(\epsilon_\phi - 1)}. \quad (26)$$

Any technical change, defined as a change in γ_t , mimicking the movement from brawn-intensive to brain-intensive production must increase the relative demand for the brain-intensive efficiency units of labor. From (24) relative demand is,

$$\frac{B_t}{R_t} = \left(\frac{\lambda_t w_{r,t}}{(1 - \lambda_t) w_{b,t}} \right)^{\epsilon_\phi - 1}. \quad (27)$$

Therefore, a rise in γ_t leads to the following proposition, a rise in relative brain demand if and only if $\epsilon_\phi > 1$, implying the two inputs are substitutes in the aggregate production process, since

$$\frac{\partial \frac{B}{R}}{\partial \gamma} = (\epsilon_\phi - 1)X, \quad (28)$$

where $X > 0$. Thus, the relative quantity of brain to brawn-intensive labor efficiency units at any given wage ratio increases, and, as a consequence, the equilibrium wage $\frac{w_{b,t}}{w_{r,t}}$ rises as long as an outward shift in labor supply does not offset the increase in labor demand. The relative wage equation (24) shows that a rise in $\frac{B_t}{R_t}$ will offset relative demand increases since $(\phi - 1) < 0$.

4.5 Wages and the Distribution of Brain and Brawn

We can now explicitly state an agent's wage, $\omega_{i,t}$, which is determined by his/her innate brain and brawn ability. From the firm's problem it follows that $\omega_{i,t} = \{w_b b_{i,t} e_i, w_r r_{i,t}\}$ for $e_i = \{0, e\}$. Moreover, brain and brawn are jointly distributed $(b_i, r_i) \sim \mathcal{A}_g(b, r)$ with differing distributions by gender. Since the premise of this study is the lack of women's brawn, the two gender distributions, $\mathcal{A}_m(b, r)$ and $\mathcal{A}_f(b, r)$, only differ in their distribution of brawn, r_g . Consequently, the distribution of brain, \mathcal{B} , and the correlation of brain and brawn, ρ , are identical for men and women.

4.6 Decentralized Equilibrium

An equilibrium, given wages $\{w_b, w_r\}$, exists and is defined by:

1. The demand for market goods, c_i , the production of household goods, h_i , and the supply of labor, ℓ_i^g , that maximizes household utility;
2. The demand for labor inputs, B and R , that minimizes the final good's cost function; and
3. Factor returns, $\{w_b, w_r\}$ that clear,
 - (a) The labor market, $B_{hh} = B$ and $R_{hh} = R$; and
 - (b) The goods market, $C_{hh} = Y$,
 where B_{hh} , R_{hh} , and C_{hh} are aggregate household supply and demand levels obtained by integrating labor demand and market consumption of individuals over the brain and brawn distribution of all working agents.

5 Calibration

Simulating the model over time requires the calibration of individuals' brain and brawn distributions, and several household and production parameters. Given the pronounced hump-shape in the wage gap between 1940 and 1950, possibly due to the effects of World War II, the model is matched to various 1950 data targets.

5.1 Production Parameter Estimation

To determine the substitution parameter, ϕ , the regression of [Katz and Murphy \(1992, pg. 69\)](#) is re-estimated, where skilled labor is defined as brain labor and unskilled labor is defined as brawn labor. To determine brain and brawn labor inputs, assumptions regarding the matching of workers to job, given that the DOT only provides information on job requirements (not workers) have to be made. I assume that on average workers match "efficiently" to jobs, that is the average occupation wage for occupation j is,

$$\bar{w}_{j,t} = w_{b,t}\bar{b}b_j + w_{r,t}\bar{r}r_j + \epsilon_{j,t} \text{ for all } t, \quad (29)$$

where \bar{b}, \bar{r} , are the total (largest) factor amount available, b_j, r_j are the brain and brawn factors estimated in section 2,⁹ and $\epsilon_{j,t}$ is an error term. Since, $w_{b,t}\bar{b}$ and $w_{r,t}\bar{r}$ are not sparsely identifiable, an assumption needs to be imposed to obtain brain and brawn returns over time. That is, (29) can either be estimate for a base year, assuming $w_{b,0} = w_{r,0} = 1$ to estimate \bar{b} and \bar{r} or the regression (29) can be estimated for all t , implicitly assuming that average returns equal one. The two assumptions produce very similar results and, therefore, only the results to the second are provided. Note also that \bar{b} and \bar{r} are constant over time, similar to the data, i.e., it is conceivable that \bar{b} has grown over time, but given data availability it is impossible to verify or estimate. Figure 6, plots the estimates relative brain to brawn wage rates.

Note the wage rates are similar in "from" to the United States college wage premium, where relative wages first fall until 1980 and than rise continuously. Since, it was assumed that on average individuals match properly to occupations, the average brain and brawn efficiency units can be computed as,

$$E_{b,t} = \sum_j \bar{b}b_j L_{j,t}, \quad (30)$$

where $L_{j,t}$ are employment shares of occupation j . In computing wage rates, we only take full-time-full-year workers,¹⁰ while $L_{j,t}$ includes all individuals with working hours of 260 per year. However, individual's are weighted by their Census/CPS weights and hours worked per year to compute total factor supply.

The estimate the substitution parameter ϕ , I assumes a log-linear skill-biased technical change over time,

$$\ln \left(\frac{\gamma_t}{1 - \gamma_t} \right)_t = \zeta_0 + \zeta_1 t + \eta_t, \quad (31)$$

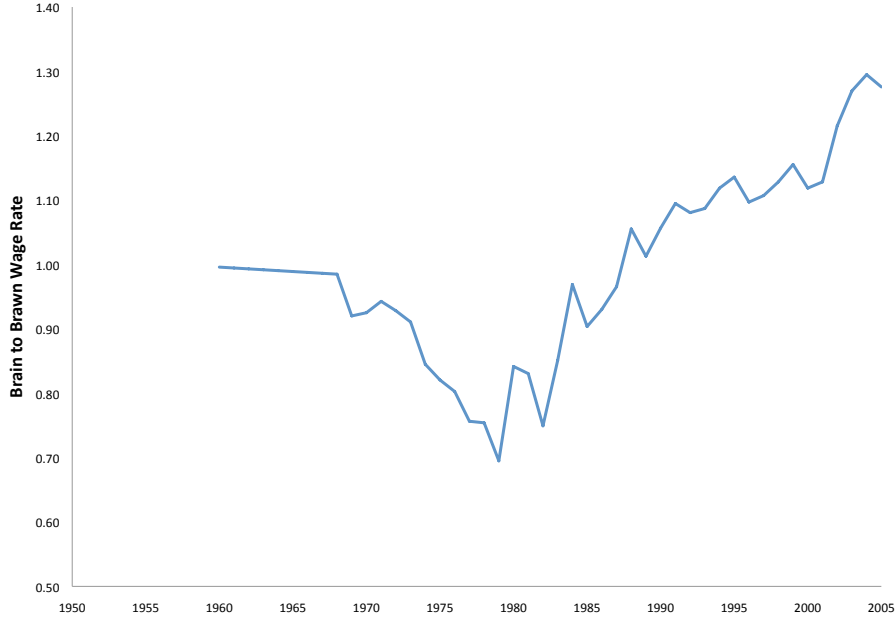
as in [Krusell, Ohanian, Rios-Rull, and Violante \(1997\)](#). Taking the natural logarithm of the relative wage equation (24), and inserting equation (31), leads to the following regression estimation,

$$\ln \left(\frac{w_b}{w_r} \right)_t = a_1 t + a_2 \ln \left(\frac{E_b}{E_r} \right)_t, \quad (32)$$

⁹Recall, factors are normalized from zero to one, with the interpretation that a factor of zero uses zero percent and a factor of 1 uses 100 percent of the largest possible factor available.

¹⁰Full-time-full-year workers are defined as working at least 1,400 hours per year (prior to 1976 only hours worked prior to the survey week are recorded in the data).

Figure 6: *Relative Brain to Brawn Wage Rate*



where $a_2 = \phi - 1$. Table 1 provides the regression estimates for the DOT 1977,¹¹

Table 1: *Baseline Parameters*

Production Parameters		
Substitution Parameter	ϕ	0.81818
95% Confidence Interval		0.73775 - 0.89861

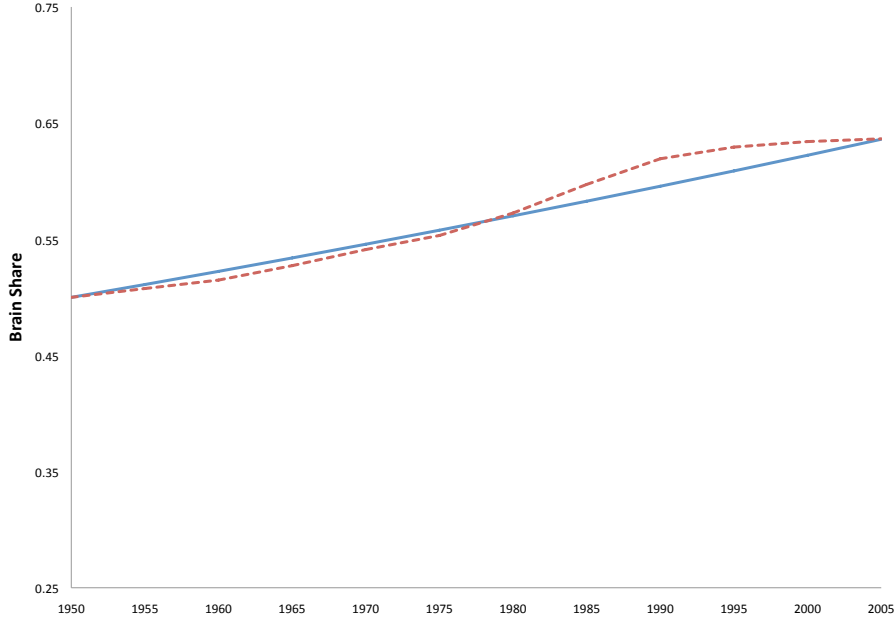
Additionally, for the calibration $\gamma_{1950} = .5$ is set for the base year. For the simulation purposes γ_t will evolve in two ways, either by a linear BBTC to match male college educated fraction in the United States in 2005 or directly taken from the DOT estimates (see figure 7). Note, the two trends are fairly similar, however, the DOT trend has a slight S-shape which will manifest itself in the labor force participation time trend of married women.

5.2 Household Parameters

The remaining parameters $\{A_h, p_s, \bar{c}, \rho, \mu_b, \mu_r^f, \mu_r^m, \sigma_b, \sigma_r\}$ and the education parameters $\{e, \eta_1, \eta_2\}$ are matched to the data targets listed in Table 2. Brain and brawn distributions are assumed to be jointly normal. In addition, rho is set to zero, and mu_b is normalized to one. As in [Güvenen](#)

¹¹Naturally, in using different base years, the estimate of ϕ varies depending on the year selected. However, the resulting estimates are within then range of the 95 percent confidence interval of the second method. In addition, redoing the estimation for the DOT 1991, generates similar results as well, with $\phi = 0.86479$ in the second method. All these additional results are available from the author upon requests.

Figure 7: *BBTC*



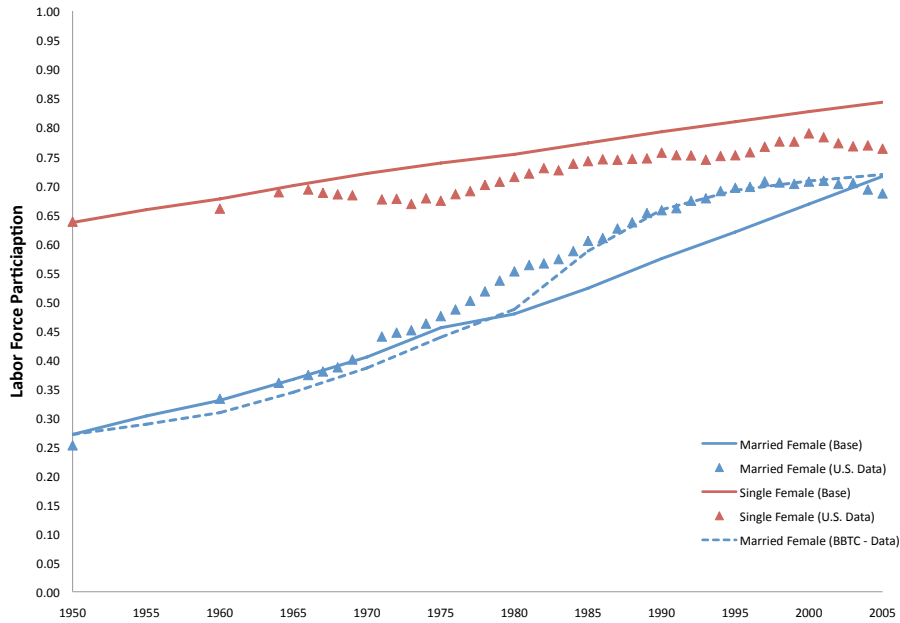
and Kuruscu (2009), the model abstracts from idiosyncratic shocks, which are present in the data. Therefore, instead of matching the log wage variance in the United States data in calibrating the standard deviation of brain and brawn, I follow Guvenen and Kuruscu (2009) in matching the residual variance of 0.104, define as variance less idiosyncratic shocks.

Table 2: *Moments and Parameter Estimates*

Moment	1950s Data	Model	Parameter Estimates
Single Female LFP	0.63950	0.63586	$A_h = 1.14583$
Male LFP	0.92477	0.98433	$p_s = 0.6396$
Married Uneducated Female LFP	0.25337	0.24195	$\bar{c} = 1.98750$
Female Relative Minimum Wage	0.20624	0.20549	$\mu_r^f = 1.45374$
Gender Wage Gap	0.63900	0.64695	$\mu_r^m = 3.51434$
Variance in Log Wages	0.10400	0.10294	$\sigma_r = 0.54387$
Married Female LFP	0.25126	0.26438	$\sigma_b = 0.13173$
Male College Premium	1.49446	1.65686	$e = 2.89371$
Young Male Fraction Educated	0.10987	0.11950	$\eta_1 = 2.57255$
Young Female Fraction Educated	0.07011	0.06554	$\eta_2 = 4.26093$

The model does well in matching most data targets, except the college wage premium for men, which is over estimated in the model. Moreover, note that men have about twice as much brawn than women, and the variance of brawn is considerable higher than the variance of brain. Lastly, the model is able to generate a large difference in average female to male wages, where women earn about 65 percent of men's wages.

Figure 8: *Labor Force Participation of Women*



Parameters are obtained from performing simulated annealing. To check the robustness of the estimates, the calibration is repeated numerous times with different initial parameter values chosen randomly from a grid of plausible values. The labor market trends discussed below are robust to all calibrations.

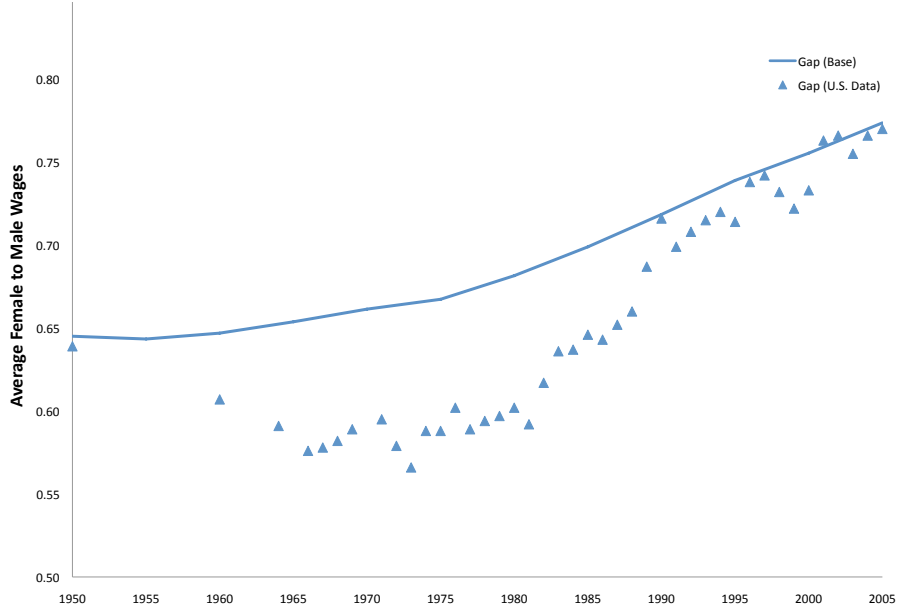
6 Main Results

Assuming that average wages grow at 1 percent per year, in order for the model to generate a large rise in female employment, I assume similar to θ , that home production grows at .04 percent slower. If home production grows at the same rate, the model only generates about half of the observed rise in employment. Since, this study is mainly interested in wage and education trends it is important to match the employment evolution. Moreover, a story of technological improvement in the home as mentioned in the introduction, would be consistent with such trends. The results presented in this section show that the mechanism highlighted in this study does well in matching the market outcomes of women in the United States. In contrast, a counterfactual model without education is unable to match the wage gap evolution beyond the period of stagnant average female to male wages.

6.1 Simulated Employment and Wage Gap Trends

This model generates a linear rise in female labor force participation. The trend for married women is S-shaped when using the BBTC of the DOT 1977 from Section 2 (labeled BBTC - Data). The results of figure 12 are similar for all specifications, including the counterfactual without education. Therefore, the rise in labor force participation is mainly due to the rise in the returns

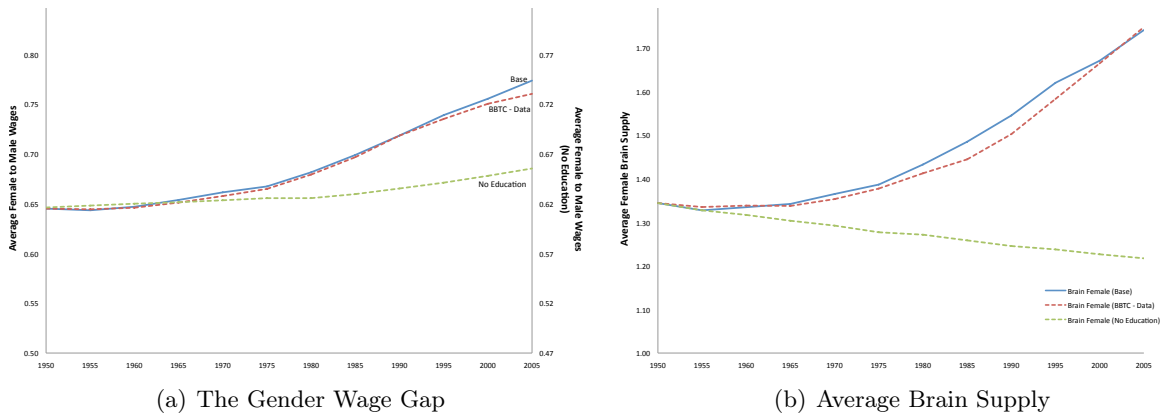
Figure 9: *Simulated Wage Gap*



to brain, rather than the modeled educational choice. Men’s labor force participation is close to full-employment in the model economy, but does decrease slight for married men over the time period, similar as in the United States where male employment fell from 92 percent and 87 percent in 1950 and 2005, respectively.

The model does also well in matching the wage gap evolution. While, the model does not generate large fall in wages, it does have a considerable period of stagnated female wages (supply effect dominates), before starting to close faster from the late 1970s onward (price and education effect dominate).

Figure 10: *Wage Gap Comparison*



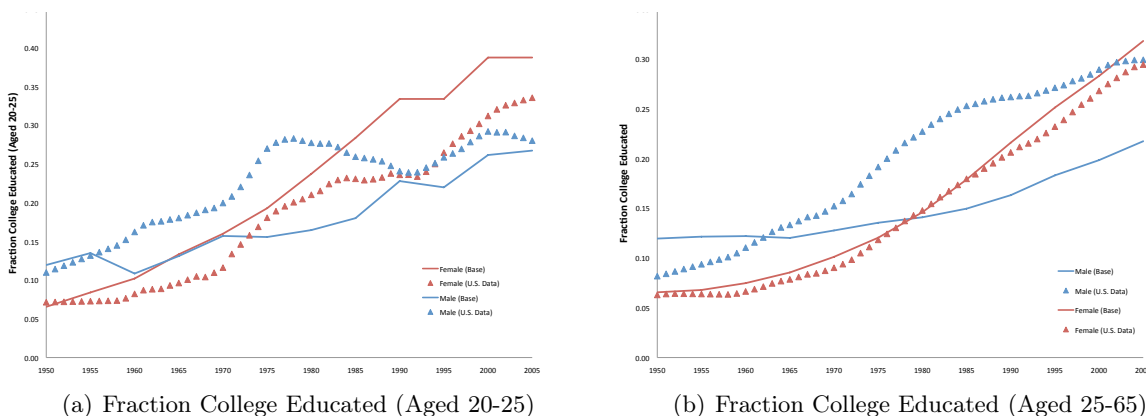
Moreover, while the technical change from the data generates a similar wage gap evolution the counterfactual model without education fails in closing the gap between men and women. In the counterfactual model the supply effect dominates throughout most of the period, and the price effect

does not generate a sufficient positive effect failing to match the closing wage gap in the United States.¹²

A large fraction of the stagnant wage gap in the counterfactual model is driven by the fact that women’s average brain supply do not rise but rather fall over time. Figure 11(b) shows female average brain supplies across the three models. Although women’s brain supply eventually exceeds that of men in the data, the model without education is unable to generate this effect.

The model does also fairly well in matching the overall education trends, and the reversal of the education gap over time. However, the results are slight over stated for young women and understated for young men (see figure 12(a), left panel). Moreover, in the model the gap closes much sooner than in the data. Clearly, the model being unable to generate the large rise in male education (possibly due to Vietnam war) it also fails in matching the education trend for the whole population (see figure 12(b), right panel). However, when comparing the education trends across the two possible BBTC simulations, the model following the DOT 1977 estimates does generate a somewhat larger result for men, while remaining fairly constant for women (see figure ??).

Figure 11: *Education Trends*



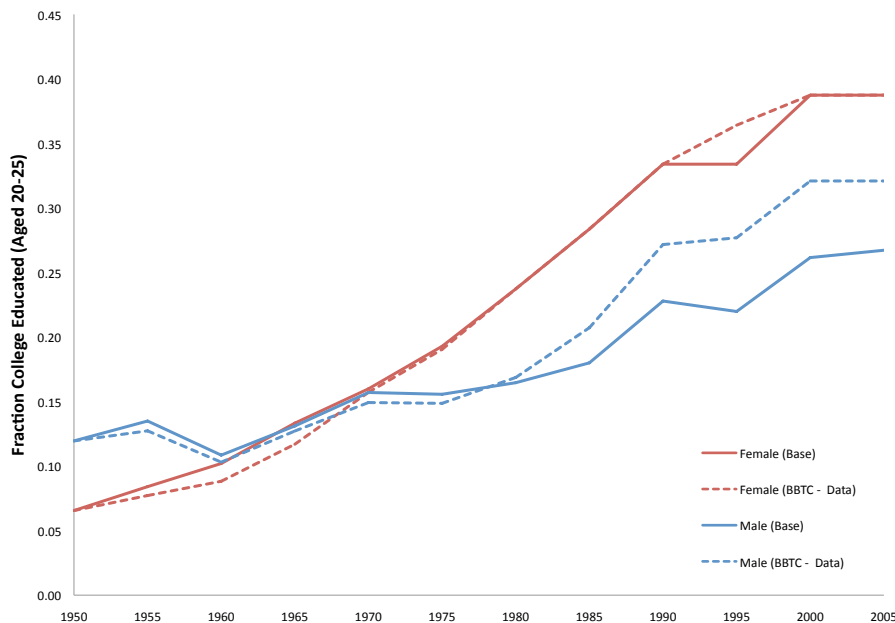
To conclude the simulation results, the model also has prediction of variance of log wages over time. While the model does generate some rise in the variance, it generates only about half of what we observe in the data. Therefore, the current model specification still leaves a large component of wage inequality unexplained.

7 Conclusion

The purpose of this study is to assess the importance of labor demand changes on women’s labor force participation and wages. For proper policy development, it is necessary to establish the extent to which the female labor market experience has been shaped by discrimination or other factors. This study focuses on the changes in occupational brain and brawn input requirements, and their effect on women’s labor force participation, education and average wages. A considerable rise in brain and fall in brawn requirements is estimated from the 1977 DOT. The simulation of the general equilibrium model provides further insight into the dynamics of these labor demand changes, and their quantitative impact on women’s labor force participation, the reversal of the education gap,

¹²The model without education was not recalibrated and, therefore, the initial wage gap is slightly lower at 62 percentage points. For illustration purposes of the trends across models, it is graphed on the secondary y-axis.

Figure 12: *College Education Trend Comparison (Aged 20-25)*



and the closing wage gap. Calibrating the model to the 1950s United States economy shows that BBTC is able to replicate the rise in female labor force participation. While the model without education is unable to generate a closing wage gap, the base model with an educational choice is able to generate a similar trend as observed in the data. This model generates both the initial stagnation and later rise of the post-World War II United States wage gap. Moreover, education trends generated in the model are consistent with United States trends for both men and women.

Clearly, the simple model presented in this paper, abstracting from many other potential factors influencing men's and women's labor market experiences, is unable to explain the complete evolution of the labor market over the last five decades. Nonetheless, this model is successful in explaining a significant portion of the changes in women's labor market experience.

Some questions remain for future research. The model has made some simplifying assumptions, such as modeling skill-biased technical change as an exogenous process. The next research step is endogenizing this process by developing a model where the entrance of women into the labor force possibly spurs the skill-biased technological change observed in the data. Lastly, a cross-country comparison to further test the validity of the BBTC hypothesis would be valuable in determining the importance of labor demand requirements in policies fostering women's well-being.

References

- AGUIAR, M., AND E. HURST (2006): “Measuring trends in leisure: the allocation of time over five decades,” Working Papers 06-2, Federal Reserve Bank of Boston.
- ALBANESI, S., AND C. OLIVETTI (2006): “Gender Roles and Technological Progress,” 2006 Meeting Papers 411, Society for Economic Dynamics.
- BACOLOD, M., AND B. S. BLUM (2005): “Two Sides of the Same Coin: U.S. "Residual" Inequality and the Gender Gap,” Working Papers 050617, University of California-Irvine, Department of Economics.
- BECKER, G. S. (1994): *Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education (3rd Edition)*, no. beck94-1 in NBER Books. National Bureau of Economic Research, Inc.
- BLACK, S. E., AND C. JUHN (2000): “The Rise of Female Professionals: Are Women Responding to Skill Demand?,” *American Economic Review*, 90(2), 450–455.
- BLACK, S. E., AND A. SPITZ-OENER (2010): “Explaining Women’s Success: Technological Change and the Skill Content of Women’s Work,” *The Review of Economics and Statistics*, 92(1), 187–194.
- BLAU, F. (1998): “Trends in the Well-Being of American Women, 1970-1995,” *Journal of Economic Literature*, 36(1), 112–165.
- CHANG, Y., AND F. SCHORFHEIDE (2003): “Labor shifts and economic fluctuations,” Working Paper 03-07, Federal Reserve Bank of Richmond.
- FERNÁNDEZ, R. (2007): “Culture as Learning: The Evolution of Female Labor Force Participation over a Century,” NBER Working Papers 13373, National Bureau of Economic Research, Inc.
- FOGLI, A., AND L. VELDKAMP (2007): “Nature or Nurture? Learning and Female Labour Force Dynamics,” CEPR Discussion Papers 6324, C.E.P.R. Discussion Papers.
- GALOR, O., AND D. N. WEIL (1996): “The Gender Gap, Fertility, and Growth,” *American Economic Review*, 86(3), 374–87.
- GREENWOOD, J., AND N. GUNER (2009): “Marriage and Divorce since World War II: Analyzing the Role of Technological Progress on the Formation of Households,” in *NBER Macroeconomics Annual 2008*, NBER Chapters. National Bureau of Economic Research, Inc.
- GREENWOOD, J., A. SESHADRI, AND M. YORUKOGLU (2002): “Engines of liberation,” Working papers 1, Wisconsin Madison - Social Systems.
- GUVENEN, F., AND B. KURUSCU (2009): “A quantitative analysis of the evolution of the U.S. wage distribution, 1970-2000,” Staff Report 427, Federal Reserve Bank of Minneapolis.
- HANSEN, G. (1993): “The Cyclical and Secular Behaviour of the Labour Input: Comparing Efficiency Units and Hours Worked,” *Journal of Applied Econometrics*, 8(1), 71–80.
- INGRAM, B. F., AND G. R. NEUMANN (2006): “The Returns to Skill,” *Labour Economics*, 13(1), 35–59.

- JONES, L. E., R. E. MANUELLI, AND E. R. MCGRATTAN (2003): “Why are married women working so much?” Staff Report 317, Federal Reserve Bank of Minneapolis.
- JUHN, C., K. M. MURPHY, AND B. PIERCE (1993): “Wage Inequality and the Rise in Returns to Skill,” *Journal of Political Economy*, 101(3), 410–42.
- KATZ, L. F., AND K. M. MURPHY (1992): “Changes in Relative Wages, 1963-1987: Supply and Demand Factors,” *The Quarterly Journal of Economics*, 107(1), 35–78.
- KING, M., S. RUGGLES, T. ALEXANDER, D. LEICACH, AND M. SOBEK (2004): *Integrated Public Use Microdata Series, Current Population Survey: Version 2.0. [Machine-readable database]*. Minneapolis, MN: Minnesota Population Center [producer and distributor].
- KRUSELL, P., L. E. OHANIAN, J.-V. RIOS-RULL, AND G. L. VIOLANTE (1997): “Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis,” Staff Report 239, Federal Reserve Bank of Minneapolis.
- MCGRATTAN, E. R., R. ROGERSON, AND R. WRIGHT (1997): “An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy,” *International Economic Review*, 38(2), 267–290.
- MULLIGAN, C. B., AND Y. RUBINSTEIN (2005): “Selection, Investment, and Women’s Relative Wages Since 1975,” NBER Working Papers 11159, National Bureau of Economic Research, Inc.
- OLIVETTI, C. (2006): “Changes in Women’s Hours of Market Work: The Role of Returns to Experience,” *Review of Economic Dynamics*, 9(4), 557–587.
- RIOS-RULL, J.-V., AND V. SANCHEZ-MARCOS (2002): “College Attainment of Women,” *Review of Economic Dynamics*, 5(4), 965–998.
- ROGERSON, R. (2005): *Women at Work: An Economic Perspective* chap. What Policy Should Do: Comments, pp. 109–114. Oxford University Press, Oxford, edited by tito boeri, daniela del boca, and christopher pissarides edn.
- RUGGLES, S., M. SOBEK, T. ALEXANDER, C. A. FITCH, R. GOEKEN, P. K. HALL, M. KING, AND C. RONNANDER (2004): *Integrated Public Use Microdata Series: Version 3.0 [Machine-readable database]*. Minneapolis, MN: Minnesota Population Center [producer and distributor].
- RUPERT, P., R. ROGERSON, AND R. WRIGHT (1995): “Estimating Substitution Elasticities in Household Production Models,” *Economic Theory*, 6(1), 179–93.
- VIJVERBERG, W. P., AND J. HARTOG (2005): “On Simplifying the Structure of Labour Demand: An Analysis of the DOT Data Abstract,” IZA Discussion Papers 1809, Institute for the Study of Labor (IZA).
- WONG, L. Y. (2006): “Women’s Economic Progress and Inequality,” 2006 Meeting Papers 477, Society for Economic Dynamics.
- YANG, F. A., AND S. GE (2008): “Accounting for the Gender Gap in College Attainment,” Discussion Papers 08-02, University at Albany, SUNY, Department of Economics.

Appendix A: Factor Estimation

I estimate brain and brawn requirements for United States census occupation and industry classifications from the 1977 Dictionary of Occupational Title (DOT).¹³ This DOT survey set is particularly useful since, (1) it is readily available in an electronic format, (2) it has been merged with the 1971 Current Population Survey (CPS) allowing for civilian employment population weighted results, and (3) it lies mid-way through the period under study (the late 1970s). To estimate brain and brawn levels over time and gender I use factor analysis as in [Ingram and Neumann \(2006\)](#). Factor analysis is a technique to reduce a large number of variables, called characteristics, within a dataset to a few unobserved random variables, called factors. The 1977 DOT reports 38 job characteristics for over 12,000 occupations (see Section 2 for detail on these characteristics). These characteristics capture the heterogeneity across jobs and workers. While they measure different specific job requirements, they can be grouped into broader categories of skills in terms of their common underlying dimensions. This grouping reduces the dimensionality of heterogeneity allowing factor requirements to be matched in a simple general equilibrium model.

Factor analysis uses the correlation matrix of a set of dependent variables to uncover the functional form of some undefined independent variables. In the general specification the characteristics, C_i , are modeled as linear combinations of the independent variables or factors, f_i , plus an error term ϵ_i ,

$$C_i = \mu + \Lambda F_i + \epsilon_i \quad \text{for } i=1, \dots, N, \quad (\text{A.1})$$

where N equals the number of occupations; C_i is the vector of characteristics (38×1); μ is the vector of characteristic means (38×1); Λ is a vector of coefficients ($38 \times n_f$) called factor loadings; $F_i = (f_1, f_2, \dots, f_{n_f})'$ is a vector of the factors ($n_f \times 1$); and $\epsilon_i \sim N(0, \Sigma)$ is the uncorrelated error vector, with Σ being the diagonal variance covariance matrix.

To preform factor analysis certain variables of the DOT need to be rescaled, for example, the variable documenting a job's location is coded $I=indoors$, $O=outdoors$, and $B=both\ indoors\ and\ outdoors$. I follow [Vijverberg and Hartog \(2005\)](#) in rescaling all variables. Additionally, to obtain population representative estimates, the occupations in the DOT must be weighted. As the DOT itself does not record the number of workers for a given job, the 1971 CPS merge is used. In the 1977 DOT, the *Committee on Occupational Classification and Analysis of the National Academy of Sciences funded by the Department of Labor and the Equal Employment Opportunity Commission* merged the 12,431 1977 DOT jobs to 7,289 unique occupation-industry pairs from the 1970 United States Census providing 1971 CPS weights of the civilian labor force. The reduction from 12,431 to 7,289 is the result of more detailed occupational classifications in the DOT. For example, while there is only one "waiter/waitress" category in the census classification, the DOT contains multiple categories, such as "waiter/waitress formal", "waiter/waitress, head", "waiter/waitress, take out."

Since only information on the characteristics is available, this information is used to estimate both, Λ and F_i from

$$E \left(\hat{C} - \mu \right) \left(\hat{C} - \mu \right)' = \Lambda E \left(\hat{F} \hat{F}' \right) \Lambda' + \Sigma, \quad (\text{A.2})$$

that is, the covariance in the 38 characteristics can be explained by a reduced number of factors, where $\hat{C} = [C_1 C_2 \dots C_N]$ and $\hat{F} = [F_1 F_2 \dots F_N]$. It is clear that Λ , $E \left(\hat{F} \hat{F}' \right)$, and Σ are not separately identifiable from this expression. Therefore, factor analysis generally assumes factors to follow a standardized normal distribution, which allows for the identification of Σ . To separately

¹³Data, including documentation, is available from the Inter-university Consortium for Political and Social Research (ICPSR).

identify Λ and $E(\hat{F}\hat{F}')$ additional restrictions must be imposed. In standard factor analysis the covariance between factors is set to zero,

$$E(\hat{F}\hat{F}') = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ & & \ddots & \\ 0 & \cdots & & 1 \end{bmatrix}, \quad (\text{A.3})$$

allowing both Λ and Σ , which is diagonal by assumption, to be identified separately. In this specification each characteristic is a function of all factors. In practice, the first factor estimate will explain the maximum possible covariance between the characteristics. The second factor is estimated to explain the maximum covariance remaining, and so on. A maximum of 38 factors could be estimated, in which case 38 factors are necessary to explain the covariance between all characteristics. In this study three factors explain most of the characteristics' covariance structure (over 93 percent of the total covariance).¹⁴ After performing initial factor analysis as described above, the first factor is positively related to intellectual characteristics and negatively correlated with both motor coordination and physical characteristics, making it difficult to interpret the factor consistently. Therefore, I reestimate the factors assuming they are correlated, similarly to [Ingram and Neumann \(2006\)](#). However, for identification purposes, job characteristics that explain one factor are restricted and cannot explain another factor. For example, mathematical development only explains a job's intellectual requirements directly, while it is only informative on the job's physical requirements through the correlation of the aggregate brain and brawn factor. Table A.1 provides the classification of characteristics across factors as well as the factor loading coefficients, which are used to determine factor estimates for each occupation-industry combination present in the 1971 CPS. Given the grouping of characteristics and the estimates of factor loadings, I call the three factors brain, motor coordination, and brawn. Brain, brawn, and motor coordination trends over time (see Figure 4) are robust to either the standard identification restriction of uncorrelated factors or my reestimated identification of correlated factors.

¹⁴[Ingram and Neumann](#) use the 1991 DOT with over 53 characteristics, primarily expanded by detailing physical and environmental characteristics, to estimate a total of four factors: (1) intelligence, (2) clerical skill, (3) gross motor skill, and (4) ability to deal with physically and hazardous work.

Table A.1: *Factor Loading Estimates* (Λ)

Job Characteristic	Coefficient (Λ_i)
Brawn Factor	
Repetitive Work	0.30406
Climbing/Balancing	0.77651
Stooping/Kneeling/Crouching/Crawling	0.81000
Strength Requirement	0.88075
Environmental Exposure ¹⁵	0.77673
Indoor or Outdoor Work	0.68110
Brain Factor	
Reasoning Development	0.96668
Mathematical Development	0.89217
Language Development	0.95275
Specific Vocational Preparation	0.77567
General Intelligence	0.94685
Verbal Aptitude	0.94068
Numerical Aptitude	0.83968
Clerical Aptitude	0.70447
Talking and Hearing	0.57950
Performs Variety of Duties	0.24961
Directing/Controlling	0.61560
Interpreting Feelings/Ideas/Facts	0.18598
Influencing People	0.37265
Making Evaluations Based on Judgment	0.60055
Making Judgments/Decisions	0.43480
Dealing with People	0.49332
Motor Coordination Factor	
Seeing	0.77650
Spatial Aptitude	0.43418
Form Perception	0.71349
Motor Coordination	0.84869
Finger Dexterity	0.88302
Manual Dexterity	0.66313
Eye-Hand-Foot Coordination	0.07607
Color Discrimination	0.37763
Attaining Precise Tolerances	0.72865
Reaching/Handling/Fingering/Feeling	0.50627
Making Decisions based on Measurable Criteria	0.30894

Notes: Estimated using maximum-likelihood factor analysis.