

# Financial Integration and Financial Instability\*

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## Abstract

In the presence of bank funding risks, unregulated issuance of safe short-term liabilities by financial intermediaries leads to excessive reliance on this form of financing, which increases losses associated with financial crises. This paper studies welfare implications of international financial integration in the presence of bank funding risks. First, I show that integration increases the severity of potential financial crises in the countries that receive capital inflows. As a result, integration may reduce welfare for these countries. Second, I show that if macroprudential regulation of the banking sector is chosen by each country in an uncoordinated way, the outcome can be Pareto inefficient so that there is a role for global coordination of such policies. This effect arises because the macroprudential regulation that limits the overissuance of safe liabilities changes the international interest rate. The regulation may have an additional benefit from manipulating the interest rate. Third, the desire to manipulate the interest rate when regulating the local banking sector creates incentives to use two regulatory tools: macroprudential regulation of the banking sector and capital controls.

Keywords: Financial integration, financial stability.

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# 1 Introduction

The large increase in cross-border banking during the past decade has renewed interest in the effects of fluctuations in capital flows. The creation of the Eurozone in 1999 is a case in point. Capital account liberalization as a prerequisite for admission played a role in the increase of the cross-border capital flows. The cross-border assets of the Eurozone banks in domestic currency increased from \$2 trillion in 1999 to \$10 trillion in 2008, and the liabilities went up in the same period from \$2 trillion to \$8 trillion.<sup>1</sup> However, these flows were unevenly distributed across Eurozone countries. Slow-growing central countries were investing in fast-growing peripheral countries. For example, the net foreign asset positions of Spain decreased from -40 percent as a share of its GDP in 1999 to -80 percent in 2008 and continued falling after that.<sup>2</sup> More than half of the decline was associated with the banking sector increase in net foreign liabilities. A large fraction of these Spanish liabilities were held by surplus countries such as Germany and France. At the same time, bank lending to the foreign non-banking sector in the Eurozone did not show the same level of integration.<sup>3</sup>

The ongoing global financial crisis, which has had especially serious consequences in the Eurozone periphery, raises the question of whether increased financial integration may have played a role in exacerbating the negative effects of the crisis. Pre-crisis conventional wisdom suggested that financial integration leads to more efficient risk sharing by smoothing country-specific shocks and to capital reallocation from capital-abundant countries to capital-poor countries. However, in the presence of market imperfections, the benefits of financial integration may be mitigated or offset by exacerbated financial frictions.<sup>4</sup>

In this paper I ask four questions. First, does the integration of bank short-term funding markets exacerbate financial crises? Second, can this lead to a decrease in social welfare? Third, what regulations should be put in place to neutralize the negative consequences that financial frictions have when funding markets are integrated? Finally, is it necessary for countries to coordinate to achieve optimal regulation?

I present a model of bank funding risk based on [Stein \(2012\)](#). Banks finance themselves

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<sup>1</sup>Data comes from BIS locational banking statistics, Table 5A. The BIS uses US dollars as the numeraire in its international banking statistics.

<sup>2</sup>The data comes from the International Financial Statistics Database.

<sup>3</sup>[ECB \(2012\)](#) presents the data on establishment and activity of foreign branches and subsidiaries across the euro area countries. The report concludes that the integration in cross-border retail banking market is limited.

<sup>4</sup>The argument that removing a distortion in an environment with other distortions may lead to a reduction in welfare goes back to at least [Lipsey and Lancaster \(1956\)](#). [Hart \(1975\)](#) presents an example in which adding a new market that does not make the market structure complete makes every agent in the economy worse off. [Newbery and Stiglitz \(1984\)](#) show that opening countries to international trade in goods can make agents worse off in participating economies in the absence of insurance markets.

by issuing risky and safe debt to invest in long-term risky projects. Entrepreneurs have liquidity preferences from holding safe debt. This makes safe debt a cheaper and therefore a preferable means of financing for banks in comparison to risky debt. Because there is more uncertainty in the long run, it is easier for banks to issue *short-term* safe debt. For short-term debt to be safe, the banks must have enough resources to honor their short-term liabilities in an adverse state. When outside funding is not available in the adverse state, banks have to sell their assets at a fire-sale price.<sup>5</sup> Therefore, banks cannot issue more safe debt than the value of their assets in the adverse state. This implies that banks face endogenous collateral constraints on the issuance of safe debt. The banks do not internalize the fact that their choices of safe debt affect the collateral constraints of the other banks. This externality leads the banks to issue too much safe debt.

I embed this model of funding risk into a setting with two regions: the center and the periphery. Each region has entrepreneurs and banks. The entrepreneurs in the periphery have more productive marginal investment opportunities compared to the entrepreneurs in the center. The difference in productivities of marginal investment opportunities in the two regions leads to different returns on safe debt before the integration. The peripheral entrepreneurs create more risky projects (relative to the center) for the peripheral banks to buy. The banks need more funding to buy these assets which leads to a bigger safe debt issuance. Because entrepreneurs' liquidity preferences from holding safe debt have diminishing returns to scale, the interest that banks have to pay the safe debt holders is higher in the periphery than in the center.

The integration of banks' short-term liabilities funding markets leads to capital flows from the center to the periphery. As a result, the return on safe debt decreases in the periphery which increases the banks incentives to issue safe debt. More safe debt will lead to a larger fire-sale discount in the adverse state of the world in the periphery. At the same time, the return on safe debt increases in the center which decreases the banks incentives to issue safe debt. This results in a smaller fire-sale discount in the adverse state in the center.

I show that the center always benefits from integration while the periphery loses under certain conditions. There are two effects of the integration: capital reallocation and a change in the severity of welfare losses due to overissuance of safe debt. Consider the periphery. The inflow of resources from the center is a benefit because the banks in the periphery can issue safe debt cheaply. However, more safe debt leads to a larger fire-sale discount in the adverse

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<sup>5</sup>There is a large literature on fire sales. See [Shleifer and Vishny \(1992\)](#), [Shleifer and Vishny \(1997\)](#), [Gromb and Vayanos \(2002\)](#), [Geanakoplos \(2001\)](#), [Brunnermeier and Pedersen \(2009\)](#) on theoretical models of fire sales. [Shleifer and Vishny \(2011\)](#) provide a recent survey of the literature.

state of the world which exacerbates the negative externality associated with overissuance of safe debt, leading to bigger welfare losses. I show that these welfare losses always dominate welfare benefits from having access to cheaper safe debt financing if the difference in the marginal productivities of investment opportunities across the two regions is not too large. However, the integration always increases welfare in the center. The banks in the center reduce issuance of safe debt, which decreases losses in the adverse state. In addition, agents in the center are able to invest their savings at a higher return in the periphery. Thus, both effects increase welfare in the center.

In a closed economy setting, a regulator wants to impose a tax on safe debt issuance to make banks internalize the social costs of fire sales. In the two-region model with two local regulators, I show that the regulators will choose inefficient tax rates on safe debt issuance. An increase in the tax level decreases the issuance of safe securities that in turn decreases the world equilibrium return on the securities. Because the periphery is a net supplier of safe debt, a decrease in the rate of return decreases the amount that bankers have to repay to the agents in the center. Hence, the regulator in this region chooses the level of taxes that is higher than needed to correct the externality in the banking sector. On the other hand, the regulator in the center wants to increase the international interest rate because the center is the net buyer of safe debt. The Nash equilibrium outcome of the regulators' game can be Pareto improved.

Finally, the desire to manipulate the international interest rate when regulating the local banking sector creates incentives to use two regulatory tools—prudential taxes on the banking sector and capital controls—instead of just using prudential taxation in the banking sector.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 studies the equilibrium properties. Section 4 analyses the welfare consequences of integration. Section 5 investigates how incentives to correct the externality changes with integration. Section 6 concludes. Formal proofs are presented in the Appendix.

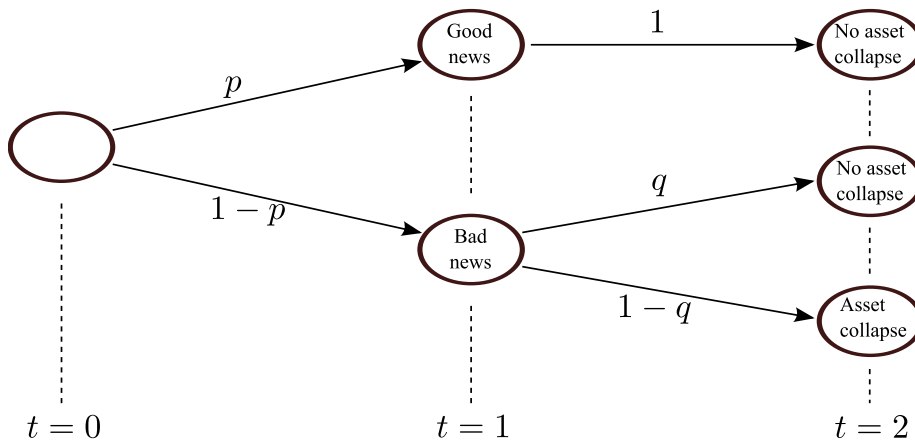
## 2 Model.

In this section, I describe a two-country model and derive agents' optimality conditions. I will use superscripts  $C$  (the center) and  $P$  (the periphery) to distinguish between country-specific variables. Each country is identical except for their marginal productivity of investment opportunities  $A^C < A^P$  (see the description below).

The economic environment is based on Stein (2012) but adds modifications to allow for

a two-country analysis. First, I assume that the liquidity preferences from holding safe securities have diminishing returns to scale. This assumption results in positive net capital flow after bank funding market integration of two asymmetric countries. Second, I assume that banks do not directly invest in the production of risky projects; instead, they buy the projects from entrepreneurs. This assumption allows me to consider the effects of the lending market integration at the end of the paper.

I will describe the model in terms of the periphery and then present a two-country equilibrium. The center description is identical. The economy goes on for three dates,  $t = 0, 1, 2$ , and there is a single consumption good that serves as the numeraire. The economy is populated by three types of agents: entrepreneurs, bankers, and outside investors. Each type of agent has measure 1. An entrepreneur has an endowment in period 0, and he chooses his consumption plan, portfolio allocation, and investments in risky projects that he immediately sells to the bankers in period 0.<sup>6</sup> A banker buys risky projects from the entrepreneurs and finances his purchases by issuing risky and safe debt to the entrepreneurs in period 0. The banker can sell his safe debt to entrepreneurs in both countries. The risky projects pay off in period 2. The uncertainty structure of the risky projects is presented in figure 1.



**Figure 1:** Aggregate uncertainty structure of risky projects. “No asset collapse” means that risky projects yield positive output in period 2 while “asset collapse” means that they are worthless.

In period  $t = 1$  news about the future payoff of the projects arrives. With probability  $p$  there is good news, called the good state and denoted  $s_1^P = G$ , where subscript 1 denotes period 1, which ensures that the risky projects will yield a positive amount of consumption

<sup>6</sup>I assume that the entrepreneurs can not insure that risky projects yield positive output at their completion. However, the bankers can guarantee that projects yield positive return in some future states if the bankers operate the risky projects.

good in period  $t = 2$ . The corresponding state in period 2 is denoted by  $s_2^P = G$ . With probability  $1 - p$  there is bad news, called the bad state and denoted  $s_1^P = B$ , informing that the risky projects will yield the same positive amount of consumption good in period 2 with probability  $q$ . I denote this state by  $s_2^P = Bnc$ , and 0 with probability  $1 - q$ . This state is denoted by  $s_2^P = Bc$ . The realizations of payoffs are common across different projects.

Bankers can sell their risky projects to outside investors in period 1. Outside investors have a fixed endowment of consumption goods in period 1, which they can invest in their late-arriving technology or the storage technology between period 1 and 2 or to buy bankers' assets. Only the outside investors have access to the storage technology.<sup>7</sup>

## 2.1 Entrepreneurs

An entrepreneur maximizes the following utility function

$$C_0^P + \beta \mathbb{E} C_2^P + v(D_d^P), \quad (1)$$

where  $C_0^P$  and  $C_2^P$  are consumption levels in period 0 and 2 respectively, which have to be non-negative.<sup>8</sup>  $v(D_d^P)$  represents the additional utility derived from holding safe claims on time 2 consumption,  $D_d^P$  is time 0 holdings of safe debt in units of period 2 consumption.<sup>9,10</sup>

The budget constraint of the entrepreneur at  $t = 0$  is

$$C_0^P + D_d^P P_D^P + \sum_{s_2^P} B^P(s_2^P) P_B(s_2^P) \leq Y + [P_0^P A^P F(I^P) - I^P], \quad (2)$$

where  $P_D$  is the price of safe debt (the return on the safe debt will be denoted  $R_D = 1/P_D$ ).  $\sum_{s_2^P} B^P(s_2^P) P_B^P(s_2^P)$  is the value of the entrepreneur's risky portfolio, where  $B^P(s_2^P)$

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<sup>7</sup>This assumption can be relaxed in two ways. I can allow all the agents to use the storage technology. In addition, I may allow the storage technology to operate between period 0 and 1. By allowing these additional opportunities, I would need to restrict my analysis to a specific range of parameters which guarantees that the bankers issue some amount of private safe debt.

<sup>8</sup>If I assume that discounting happens between period 0 and period 1 then the absence of consumption in period 1 is without loss of generality.

<sup>9</sup>Index  $d$  stands for demand. It will be useful later to differentiate it from supply of safe debt.

<sup>10</sup>The utility from holding safe securities captures the idea that safe securities provide transaction services. [Gorton and Pennacchi \(1990\)](#) and [Dang et al. \(2012\)](#) theoretically argue that private safe securities are useful for transactions because they eliminate the potential for adverse selection between transaction parties. Historically, demandable deposits were the main example of such securities. They pose a smaller threat to financial stability these days because demandable deposits are government-insured in most of the countries. Asset-backed securities (ABCP), repurchase agreements, short-term covered bonds are recent examples of private short-term safe securities, which, however, are not government insured. See, [Gorton and Metrick \(2012\)](#) and [Gorton \(2010\)](#) for a discussion of the pre-crisis developments in the unregulated banking in the U.S. [Sunderam \(2012\)](#) presents evidence that investors value safety of ABCP above their pecuniary return. [Krishnamurthy and Vissing-Jorgensen \(2012a\)](#) empirically show that investors value safety of US treasuries above and beyond their pecuniary returns. [Krishnamurthy and Vissing-Jorgensen \(2012b\)](#), using long-term US data, provide evidence that the supply of US treasuries is strongly negatively correlated with the amount of private safe securities outstanding, which is consistent with the idea that the two types of assets are substitutes.

is the repayment in state  $s_2^P$  and  $P_B^P(s_2^P)$  is the price of a security that pays off one unit of consumption good in period 2 in state  $s_2^P$ , i.e., this is an Arrow-Debreu security price. I assume that  $I^P$  units of investment immediately produce  $A^P F(I^P)$  units of risky projects that are sold to the bankers.  $P_0^P A^P F(I^P) - I^P$  is the profit from investing in the risky projects. I assume that  $A^P F(I^P)$  is increasing, strictly concave and twice continuously differentiable in  $I^P$ .

The budget constraint of the entrepreneur in period 2 is given by

$$C_2^P(s_2^P) \leq D_d^P + B^P(s_2^P). \quad (3)$$

The entrepreneur takes prices as given and chooses consumption plan  $C_0^P$ ,  $C_2^P(s_2^P)$ , amount of safe debt  $D_s^P$ , risky portfolio  $\{B^P(s_2^P)\}$ , and investment in the production of risky projects  $I^P$ . I assume that endowment  $Y$  is large enough so that non-negativity constraints on consumption do not bind.

The entrepreneur does not make any strategic decisions in period 1. The optimal interior choice of risky portfolio  $\{B(s_2)\}$  leads to

$$\begin{aligned} P_B^P(G) &= \beta p, \\ P_B^P(Bnc) &= \beta(1-p)q, \\ P_B^P(Bc) &= \beta(1-p)(1-q). \end{aligned}$$

This immediately implies that the return on any risky security bought in positive amount is given by

$$R_B = \frac{1}{\beta}. \quad (4)$$

The optimal interior choice of the amount safe debt by the entrepreneur implies

$$R_D = \frac{1}{\beta + v'(D_d^P)}. \quad (5)$$

It is immediate that  $R^P < R_B$  which represents the liquidity premium from holding safe debt.<sup>11</sup>

The optimal choice of investments in the production of risky projects implies

$$P_0^P A^P F'(I^P) = 1. \quad (6)$$

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<sup>11</sup> $R_D$  and  $R_B$  do not have country index. The return on safe debt is not indexed because the market for safe debt is common for the two countries. The return on any risky security is not indexed because it is determined by the discount factor which is common across agents in both countries.

## 2.2 Outside Investors

An outside investor has endowment  $W$  of consumption good in period 1 that he can invest in his late technology and in the storage technology in period 1. The outside investor can issue safe securities backed by the storage technology output because the storage technology is safe. The outside investor can use these securities to buy bankers' risky projects. The price of the bankers' risky projects is  $Q^P$  if bad news arrives. I will call this price a fire-sale price. The late technology yields  $g(x)$  units of consumption in case of success in period 2 and 0 in case of failure if  $x$  units of consumption are invested in period  $t = 1$ . Success and failure, which happens with probabilities  $\delta$  and  $1 - \delta$ , respectively, are common across the outside investors. This is aggregate uncertainty.<sup>12</sup> I assume that  $g(x)$  is increasing, strictly concave, twice continuously differentiable. I also assume that  $\delta g'(W) > 1$ . This assumption guarantees that the outside investor trades with the bankers only when bad news arrives. In addition, it guarantees that it is more profitable to invest in the late technology than in the storage technology. Imposing this assumption limits the number of uninteresting cases to consider.

If good news arrives the above assumption implies that the outside investor invests all his endowment in his late technology. If bad news arrives in period 1, the outside investor maximizes his revenue in period 2 from investing his endowment. This revenue equals his period 2 consumption. The problem in the bad state is

$$\begin{aligned} \max_{K_d^P, D_{OI}^P} \quad & qK_d^P + \delta g(W - D_{OI}^P) \\ \text{s.t.} \quad & Q^P K_d^P \leq D_{OI}^P, \end{aligned}$$

where  $D_{OI}^P$  is the amount of the endowment that the outside investor invests into the storage technology. The first term represents the expected payoff of the risky projects that the outside investor buys from the bankers. The second term represents the expected payoff of his investments in the late technology. The optimal choice implies

$$q = \delta Q^P g'(W - Q^P K_d^P) \tag{7}$$

Demand  $K_d^P$  decreases with  $Q^P$  because function  $g(\cdot)$  is strictly concave. Intuitively, each additional unit of the bankers assets bought by the outside investor has a marginal benefit which equals  $q$  while the marginal cost,  $\delta Q^P g'(W - Q^P K_d^P)$ , increases with the price and

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<sup>12</sup>This assumption will prevent the outside investors from issuing safe debt in period 1 backed by the proceeds of the late technology. This assumption is crucial to generate downward sloping demand curve for the bankers assets. Alternatively one can assume that  $\delta = 1$ , i.e., there is no uncertainty, but the outside investors cannot commit to keep their promises.

the amount of the risky projects being purchased. Hence, the optimal level of  $K_d^P$  decreases with  $Q^P$ .

Notice also that the elasticity of the outside investor assets demand with respect to price  $Q^P$  is greater than 1. The marginal cost of buying the bankers' assets is more sensitive to price change than to a change in the quantity bought. To understand the intuition consider a 1 percent change in price  $Q^P$ . Assume that the outside investor decreases the risky projects demand by 1 percent. This does not change the marginal value of an additional unit of resources invested in the late technology,  $g'(W - Q^P K_d^P)$ . However, it increases the marginal cost of investing in the risky projects,  $Q^P g'(W - Q^P K_d^P)$ , which must be constant according to optimality condition (7). Thus, the outside investor can decrease demand  $K_d$  by more than 1 percent.

Finally, the optimality condition (7) together with the assumption that  $\delta g'(W) > 1$  implies

$$Q^P < q \tag{8}$$

Intuitively, whenever the outside investor chooses to buy bankers risky projects the fire-sale price is less than the risky project's fundamental value  $q$ .

## 2.3 Bankers

A banker buys risky projects from the entrepreneurs and raises funding by issuing debt to maximize his period-2 profits, which equals his consumption. The banker prefers to issue safe debt because it earns a liquidity premium. Because there is a positive probability for risky projects to become worthless in period 2, safe debt cannot be made long-term. However, the banker can issue some amount of safe debt by promising potential holders to repay them early (in period 1) with riskless claims on period-2 consumption if the bad state occurs.

The banker can issue "risky debt" in addition to the safe debt. Such debt promises repayment of a fixed amount in period 2, and gives the holders of the debt the following rights: (i) a claim to any resources in the hands of the banker in period 2, after safe debt has been repaid, up to the amount promised to be repaid in period 2 (i.e., the claims of the risky debt holders are junior to those of the holders of the safe debt); (ii) a right to prevent the banker from undertaking any transactions in period 1 that would reduce the value of the risky debt except the early repayment on safe debt.<sup>13</sup>

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<sup>13</sup>Restricting risky funding to risky debt may be optimal from the point of view of the entrepreneurs. This may prevent the bankers from borrowing more and wasting money in period 1 when the good state occurs. See, [Hart \(1993\)](#) and [Hart and Moore \(1995\)](#).

If the bad state occurs, the banker has to obtain riskless claims on period-2 consumption to repay his safe debt holders. I assume that the severe debt overhang problem prevents the banker from issuing securities that can be attractive to potential investors (Myers, 1977).<sup>14</sup> Hence, the only way the banker can obtain riskless securities to fulfill the promise that he gave his safe debt holders is to sell some of his assets to the outside investors.

The banker's choice variables in period 0 are the quantity of risky projects to buy ( $Z^P$ ), the quantity of safe debt to issue (measured by the face value  $D_s^P$ ), and the quantity of the risky debt to issue (measured by the amount  $\bar{B}^P$  promised to repay in period 2). These three quantities determine the state-contingent payout to the holders of the risky debt, in each of the three possible states in period 2. There is a well-defined asset-pricing kernel (taken as given by an individual banker because the financial markets are competitive) that determines the market value in period 0 of any type of risky debt that might be issued; this determines the market value of the risky debt as a function of the three quantities ( $Z^P$ ,  $D_s^P$ ,  $\bar{B}^P$ ) chosen by the given banker.

To characterize the banker's problem, I first present his state-contingent profits. Denote the banker's state contingent profit as  $\pi_B^P(s_2^P)$ . In case of good news in period 1 there is no asset collapse in period 2, i.e.,  $s_2^P = G$ , the banker collects risky projects payoff  $Z^P$  and pays out the holders of his safe debt  $D_s^P$  and risky debt holders  $\bar{B}^P$ . Thus, his profit is  $\pi_B^P(G) = Z^P - D_s^P - \bar{B}^P$ . If there is bad news and no asset collapse, state  $s_2^P = Bnc$ , the banker has to sell part of his risky projects (denoted  $K_s^P$ ) to the outside investors in period 1 to make early repayment  $D_s^P$  to safe debt holders. The remainder of the risky projects  $Z^P - K_s^P$  pays out at  $t = 2$  and the banker repays risky debt holders. In this state, his profit is  $\pi_B^P(Bnc) = QK_s^P - D_s^P + Z^P - K_s^P - \min\{\bar{B}^P, Z^P - K_s^P\}$ . The last term takes into account the fact that the banker may end up having less output than the promised repayment on risky debt. Denote the last term as  $B_s(Bnc)$ . If there is bad news and assets collapse, state  $s_2^P = \{Bc\}$ , the banker has to sell  $K_s^P$  units of his risky projects in period 1, but then he gets nothing because his risky projects yield zero at  $t = 2$ . To summarize, the banker expected profits are

$$\begin{aligned} \mathbb{E}\pi_B^P = & p[Z^P - D_s^P - \bar{B}^P] \\ & + (1-p)q[QK_s^P - D_s^P + Z^P - K_s^P - \min\{\bar{B}^P, Z^P - K_s^P\}] \\ & + (1-p)(1-q)[Q^P K_s^P - D_s^P]. \end{aligned} \quad (9)$$

The value of the bankers' safe debt outstanding in period 0 is  $D_s^P/R_D$ , i.e., the face value of

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<sup>14</sup>New investors may not be willing to provide resources because the additional revenue that the banker gets from new funding will be paid off to senior investors.

the safe debt is discounted with riskless discount factor  $1/R_D$ . The value of the risky debt in period 0 equals  $V_B^P = P_B^P(G)\bar{B}^P + P_B^P(Bnc) \min\{\bar{B}^P, Z^P - D_s^P/Q^P\}$ . Hence, the banker period-0 budget constraint is

$$P_0^P Z^P \leq \frac{D_s^P}{R_D} + P_B^P(G)\bar{B}^P + P_B^P(Bnc) \min\{\bar{B}^P, Z^P - D_s^P/Q^P\}. \quad (10)$$

In addition to the budget constraint in period 0, the banker faces the resource constraint and the collateral constraint in period 1. The banker cannot sell more risky projects than he has on his balance sheet

$$K_s^P \leq Z^P. \quad (11)$$

For the safe debt to be safe, the value of the banker's assets in the bad state has to be more or equal to the value of safe debt

$$D_s^P \leq Q^P K_s^P \quad (12)$$

Let's now characterize the banker's optimality conditions. First, observe that constraint (12) is always binding. It is not optimal for the banker to sell more than it is required to service the safe debt. Thus, constraints (11) and (12) can be rewritten as a single constraint

$$D_s^P \leq Q^P Z^P, \quad (13)$$

which implies that the value of the safe debt cannot be greater than the value of all the assets on the banker's balance sheet in the bad state. Given the above analysis the banker maximizes (9) subject to (10) and (13) by choosing  $Z^P, D_s^P, \bar{B}^P$ .

It is easier to describe the banker's optimal behaviour after substituting out equilibrium prices. Thus, I turn to the definition of equilibrium.

### 3 Equilibrium

This section defines and characterizes the equilibrium of the model. I start by defining the equilibrium. Then I describe the bankers' optimality conditions in equilibrium which will allow me to characterize the equilibrium in closed and integrated worlds.

**Equilibrium.** An equilibrium in a two-country model is a collection of plans  $\{C_0^P, C_2^P(s_2), D_d^P, I^P, D_s^P, B^P(s_2), B_s^P(s_2), Z^P, K_s^P, K_d^P\}$  in the periphery and a collection of plans  $\{C_0^C, C_2^C(s_2), D_d^C, I^C, D_s^C, B^C(s_2), B_s^C(s_2), Z^C, K_s^C, K_d^C\}$  in the center and prices  $\{P_0^P, P_0^C, R_D, P_B^P(s_2), P_B^C(s_2)Q^P, Q^C\}$  such that all the agents solve their problems taking the prices as given and all the markets clear, i.e.,

1. markets for risky projects in period 0 in both countries:

$$Z^P = A^P F(I^P) \text{ and } Z^C = A^C F(I^C),$$

2. risky projects fire-sale markets in period 1 in both countries:

$$K_d^P = K_s^P \text{ and } K_d^C = K_s^C,$$

3. risky debt markets in both countries:

$$B^P(s_2) = B_s^P(s_2),^{15}$$

4. integrated safe debt market:

$$D_d^P + D_d^C = D_s^P + D_s^C \quad (14)$$

### 3.1 Bankers Behavior

In the previous section, I presented the banker's problem. I can now conveniently characterize the solution to this problem by taking into account the equilibrium prices. The following lemma, which is proved in the Appendix, presents the optimal conditions for a banker in the periphery.

**Lemma 1.** *The banker's optimal choice of the risky projects, the amount of safe debt and the face value of risky debt leads to the following conditions in equilibrium:*

$$[p + (1 - p)q] - R_B P_0^P + \theta^P = 0, \quad (15)$$

$$\frac{R_B}{R_D} - \left[ p + \frac{(1 - p)q}{Q^P} \right] - \frac{\theta^P}{Q^P} = 0, \quad (16)$$

where  $\theta^P \geq 0$  is a shadow value of a unit of risky projects. The face value of the risky debt

$$\bar{B}^P = \begin{cases} \frac{R_B}{p + (1 - p)q} \left[ P_0^P Z^P - \frac{D_s^P}{R_D} \right] & \text{if no default in } s_2 = Bnc, \\ \frac{R_B}{p} \left[ P_0^P Z^P - \frac{D_s^P}{R_D} \right] - \frac{(1 - p)q}{p} \left[ Z^P - \frac{D_s^P}{Q^P} \right] & \text{if default in } s_2 = Bnc. \end{cases} \quad (17)$$

Condition (15) states that the marginal return on a unit of risky projects equals the marginal cost when it is financed through risky debt. To see this, consider the following perturbation: the banker increases  $Z^P$  by one unit by increasing the issuance of the risky

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<sup>15</sup>In general, it would not be enough for the risky debt markets clearing to require that period 0 value of risky bonds supplied to be equal to the amount of resources that the entrepreneurs pay for this value, because this would allow the entrepreneurs to demand a portfolio with different state-contingent payoffs than the supply by the bankers.

debt such that period 0 value of the risky debt goes up by 1 unit while keeping  $D_s^P$  constant. A unit increase in  $Z^P$  delivers additional  $[p + (1 - p)q]$  units of period 2 consumption and relaxes the collateral constraint. A unit increase in value of the risky debt increases funding costs by  $R_B$  because the return on any risky security is  $R_B$  in equilibrium. This optimality condition makes clear that when constraint (13) binds, the banker wants to buy more risky projects relative to the case in which the constraint does not bind.

Condition (16) states that the banker is indifferent between risky and safe debt financing when he chooses his funding optimally. To see this, consider the following perturbation: the banker increases the face value of the safe debt by one unit but decreases period 0 value of the risky debt by  $1/R_D$ . This variation does not change the size of banker's balance sheet (it does not change the bankers budget constraint in period 0). However, it affects future repayments. First, it decreases the expected risky debt payments (the first term), which is a benefit for the banker. Second, it increases the expected payments on the safe debt (the second term), which is an additional cost to the banker. Third, it tightens the collateral constraint (13) (the third term), which is a loss to the banker if the constraint binds. This variation has no affect on profits when the banker optimizes.

Equation (17) determines the optimal face value of the risky debt. The first line presents the face value in equilibrium when default on risky debt only occurs in the asset collapse state  $s_2 = Bc$ . The second line presents the face value for the case when the banker defaults in  $s_2 = Bc$  and  $s_2 = Bnc$  states.

### 3.2 Closed Economy Equilibrium

In this section I describe equilibrium properties of the economy. I start by considering the equilibrium in the periphery conditional on  $D_s^C - D_d^C = 0$ . The assumption is equivalent to assuming that the economy is closed. This allows me to study comparative statics which will be useful when I consider an integrated equilibrium.

An equilibrium can be of two types: (i) the collateral constraint (13) does not bind and (ii) the collateral constraint binds. If the collateral constraint does not bind, the optimality condition (16) pins down price  $Q^P$  as a function of the endogenous return on safe debt  $R_D$

$$Q^P = \frac{1 - p}{\frac{R_B}{R_D} - p} q. \quad (18)$$

The banker's optimality condition (15) and the entrepreneur optimal choice of his investments in the risky projects production determine the level of investments  $I^P$

$$[p + (1 - p)q] A^P F'(I^P) = R_B. \quad (19)$$

The last two equations, the outside investor optimality condition (7) and the entrepreneur demand for safe debt (5), fully characterize the solution. The solution is unique.<sup>16</sup>

When the collateral constraint binds, I can combine the banker's optimality conditions (15) and (16) with the entrepreneur's optimal choice of investments (6) to get

$$\left[ \left( \frac{R_B}{R_D^P} - p \right) Q^P + p \right] A^P F' \left( F^{-1} \left( \frac{Z^P}{A^P} \right) \right) = R_B. \quad (20)$$

The equilibrium level of banker's risky project purchases  $Z^P$  depends on two endogenous variables: price  $Q^P$  and the return on safe debt  $R_D$ . To understand how  $Q^P$  affects the bankers consider the following intuition: an increase in  $Q^P$  raises the collateral value of  $Z^P$ , and it becomes more profitable to buy risky projects  $Z^P$ . This increases price  $P_0^P$  which increases the entrepreneurs incentives to invest in risky projects. As a result, investments in the risky projects  $I^P$  and production of risky projects  $Z^P$  go up. To understand how  $R_D$  affects the banker, consider the following intuition: an increase in  $R_D$  makes safe debt a less attractive mean of financing. This increases banker's financing costs and decreases the desire to buy risky projects  $Z^P$ . As a result, price  $P_0^P$  falls and the entrepreneur invests less in the risky technology. Thus,  $Z^P$  falls in equilibrium. Note also that the left-hand side of the above equation is an increasing function of  $A^P$ .<sup>17</sup>

Outside investor optimality condition (7) together with a market clearing condition  $K_s^P = K_d^P$  and the fact that the collateral constraint binds,  $D_s^P = Q^P Z^P$ , imply

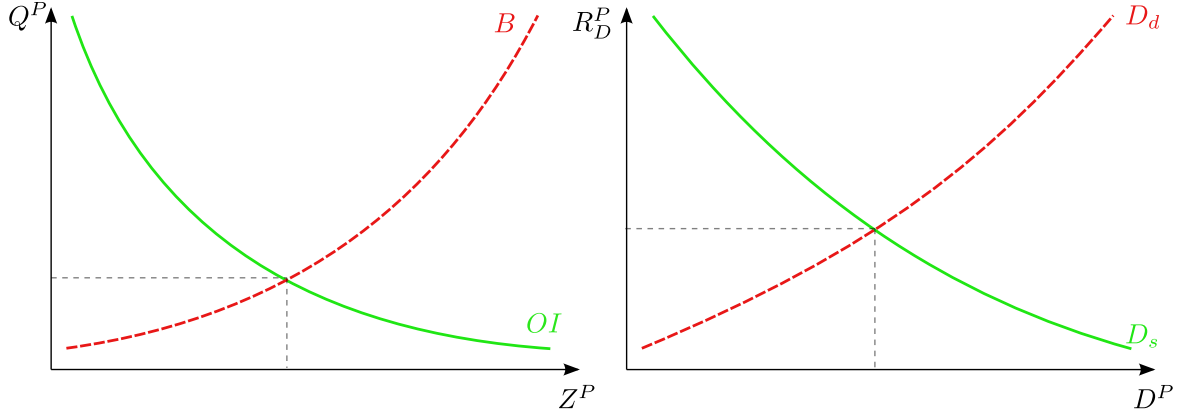
$$\delta g' (W - Q^P Z^P) = \frac{q}{Q^P}. \quad (21)$$

I can now solve equations (20) and (21) for  $Q^P = Q^P(R_D)$  and  $Z^P = Z^P(R_D)$  given  $R_D$ . The solution is graphically represented on the left panel of Figure 2. The line labeled as  $B$  corresponds to equilibrium condition (20). The line labeled as  $OI$  corresponds to equilibrium condition (21). The solution determines the supply of safe debt in the economy  $D_s^P(R_D) = Q^P(R_D)Z^P(R_D)$ .

To understand how the supply of safe debt changes with  $R_D$  consider an increase in  $R_D$ . This corresponds to a leftward shift in the  $B$  curve (see the intuition after equation (20)) and no shift in the  $OI$  curve. As a result,  $Q^P$  increases and  $Z^P$  decreases. Although,  $Q^P$  and  $Z^P$  change in the opposite directions we can still unambiguously determine the direction

<sup>16</sup>Solving the outside investor optimality condition (7) and equation (18) for  $D_s^P$  defines the safe debt supply function that depends negatively on  $R_D$ . The entrepreneur demand for safe debt (5) depends positively on  $R_D$ . The intersection of the supply and the demand determines uniquely  $R_D$  and  $D^P$ . Hence, the equilibrium is unique.

<sup>17</sup>This is because  $\frac{d[A^P F'(F^{-1}(\frac{Z^P}{A^P}))]}{dA^P} = F'(I^P) - \frac{F''(I^P)F(I^P)}{F'(I^P)} > 0$ .



**Figure 2:** Equilibrium determination in a closed economy when collateral constraints bind: risky projects fire-sale market (left), safe debt market (right).

of a change in their product  $Q^P Z^P = D^P$ . Product  $Q^P Z^P$  decreases because the elasticity of the outside investor demand for bankers assets with respect to price  $Q^P$  is greater than one. Thus, the supply of safe debt decrease with  $R_D$ , which is represented on the right panel of Figure 2 with downward sloping  $D_s$  curve. The upward sloping  $D_d$  curve represents the entrepreneur optimality condition (5). The intersection of these two curves determine the equilibrium level of  $D^P$  and  $R_D^P$ . The equilibrium level of  $R_D^P$  determines the position of  $B$ -curve on the left panel of the figure which in turn determines equilibrium  $Q^P$  and  $Z^P$ .

What happens to the equilibrium when the marginal productivity of investment opportunities  $A^P$  goes up? Given price  $P_0^P$  the entrepreneurs want to invest more  $I^P$  and sell more risky projects  $Z^P$  to the bankers. Price  $P_0^P$  will fall in equilibrium. The behavior of the rest of the equilibrium variables depends on whether the bankers collateral constraints bind or not. The following lemma, which is proved in the Appendix, summarizes the comparative statics.

**Lemma 2.** *There always exists  $\bar{A}$  such that for any  $A^P < \bar{A}$  the collateral constraint binds,  $\theta^P > 0$ , and for any  $A^P \geq \bar{A}$  the collateral constraint does not bind.*

- *If  $A^P < \bar{A}$  then investment in risky projects  $I^P$ , amount of risky projects  $Z^P$ , safe debt  $D^P$  and return on safe debt  $R_D^P$  go up while price of risky projects  $P_0^P$  in period 0 and fire-sale price  $Q^P$  in period 1 go down after an increase in  $A^P$ . In addition the shadow value of risky projects  $\theta^P$  strictly decreases.*
- *If  $A^P \geq \bar{A}$  then investment in risky projects  $I^P$ , amount of risky projects  $Z^P$  go up, price  $P_0^P$  goes down and all the other variables:  $D^P$ ,  $Q^P$ ,  $R_D^P$ ,  $\theta^P$  stay the same after an increase in  $A^P$ .*

The intuition behind this lemma is as follows. When  $A^P$  is sufficiently small, the amount of risky projects  $Z^P$  produced is small in equilibrium. Price  $Q^P$  is bounded by  $q$  from above. If the collateral constraint does not bind, then the amount of deposits  $D^P$  is smaller than  $Z^P Q^P$ . When the level of the safe debt is small,  $R_D^P$  is small. This creates strong incentives for the bankers to issue more safe debt. This eventually leads to a binding collateral constraint. When  $A^P$  is sufficiently high, this logic is reversed. Thus, the collateral constraints do not bind for high  $A^P$ . When the collateral constraints bind, an increase in marginal productivity  $A^P$  allows the entrepreneur to produce more and the bankers to buy more risky projects. To do that, the bankers increase their safe debt issuance. This leads to smaller price  $Q^P$  in the bad state and a higher return on safe debt. When the collateral constraints do not bind, the decision on the amount of the risky projects is decoupled from the safe debt issuance decision by the bankers because marginal projects don't serve as collateral in this case.

### 3.3 Open Economy Equilibrium

Now I remove assumption  $D_s^P - D_d^P = 0$  and study the properties of the integrated economy. Specifically, I compare how the equilibrium allocations and prices under integration differ from those under autarky.

The effects of integration will depend on the type of equilibrium in each country before the integration. As I discussed in the previous subsection, each country can have one of the two possible types of equilibrium before the integration. There are four possibilities to consider when integrating two countries. However, lemma 2 allows me to remove one possibility immediately. It is not possible for the collateral constraints to be binding in peripheral economy, with higher  $A^P$ , while the collateral constraints are not binding in the center, with smaller  $A^C$ . It would contradict the fact that the shadow value of the risky projects decreases with an increase in  $A$ . This leaves three possibilities to consider.

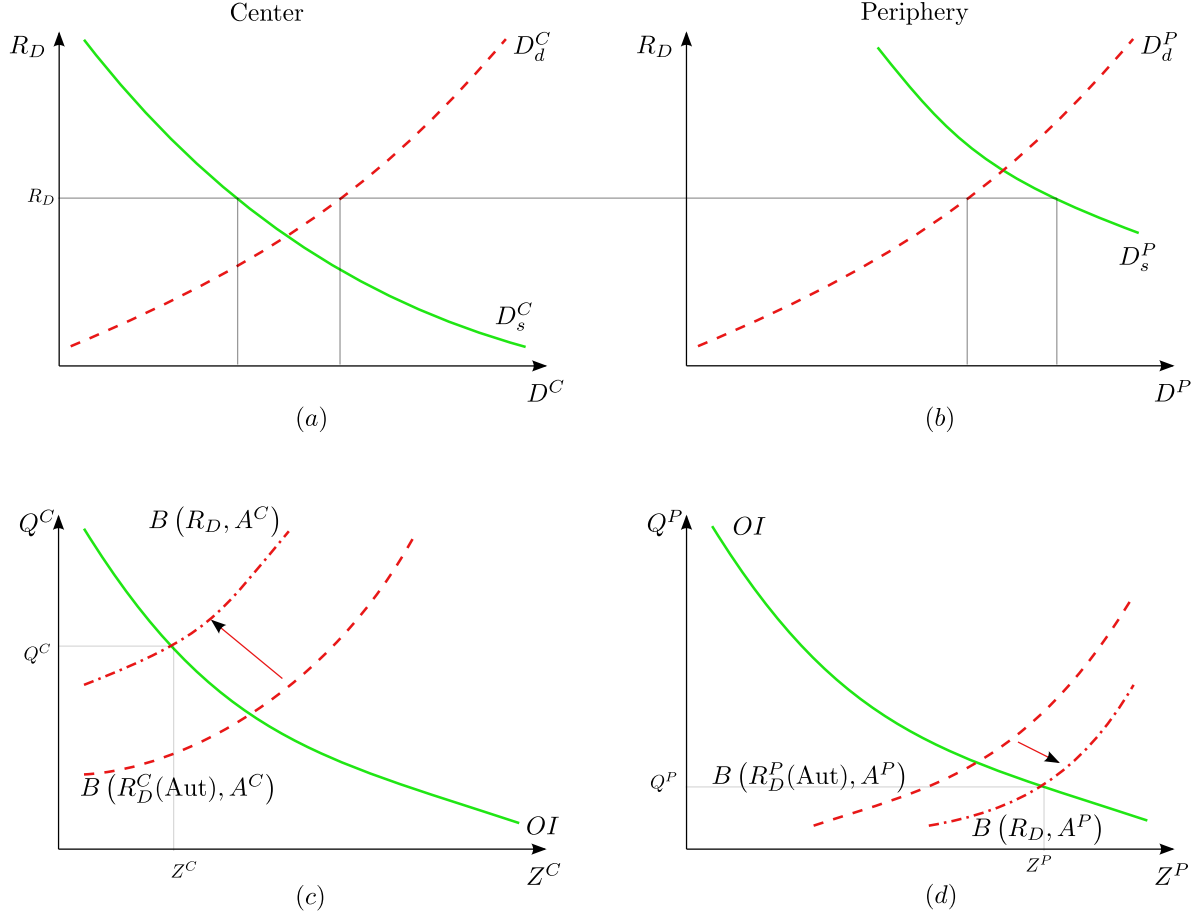
The first case is the situation in which the collateral constraints do not bind in both countries. According to lemma 2, the interest rate on safe debt is the same in both countries before the integration. This implies that opening up the two countries to trade in safe debt does not lead to changes in prices. Hence, none of the equilibrium variables change in both countries.

Consider the case in which the collateral constraints bind in both countries before the financial integration. This case is graphically illustrated in figure 3, which is an extension of figure 2 to a two-country model. The left column of plots represents the determination of equilibrium in the center; the right column presents the equilibrium in the periphery.

Let's first focus on autarky equilibria. Plot (c) of figure 3 presents the fire sale of risky projects equilibrium in period 1 in the center. The green solid line, denoted as  $OI$ , is the outside investors' demand for the risky projects. The red-dashed line, denoted as  $B(R_D^C(Aut), A^C)$ , is the combination of the bankers and the entrepreneurs optimality conditions. This curve can be interpreted as the supply of the risky projects in period 1.  $B(R_D^C(Aut), A^C)$  curve represents the supply conditional on the safe debt return  $R_D^C(Aut)$  in autarky equilibrium. Plot (d) similarly presents the equilibrium on the fire sale of risky projects market in period 1 in the periphery.  $B(R_D^P(Aut), A^P)$  line is shifted to the right on plot (d) relative to the respective line in plot (c). This is because of the difference in productivity of the risky projects' production,  $A^P > A^C$ , which makes the supply of the risky projects higher in the periphery (conditional on the same interest rate). Note that there is an opposing force: the interest rate on safe debt is higher in equilibrium in the periphery, which dampens the effect of the difference in productivity on the supply of the risky projects. However, the interest rate effect is always smaller (lemma 2). Because the supply of the risky projects in period 1 in the bad state is higher in the periphery for a given value of safe return  $R_D$ , the supply of safe debt in period 0 by bankers is higher in the periphery compared to the center. This fact is represented on plots (a) and (b) of figure 3:  $D_s^P$  curve is shifted relative to  $D_s^C$  curve. However, the demand for safe debt is the same in both countries. As a result, the interest rate is higher in the periphery relative to the interest rate in the center before the integration.

Let's now consider the effects of integration. Arbitrage forces equalize the returns on safe debt in both countries  $R_D^C = R_D^P$ . As a result, the return in the center rises compared to the autarky case, while the return in the periphery falls. This leads to a flow of resources from the center to the periphery. One can see on plots (a) and (b) that the periphery is a net supplier of safe debt while the center is a net buyer of safe debt at new world interest rate  $R_D$ . A decline in the safe debt return in the periphery increases the supply of the risky projects in the bad state in period 1. This is indicated by the shift in the supply curve from  $B(R_D^P(Aut), A^P)$  to  $B(R_D, A^P)$  in plot (d). Consequently, there is a decline in the risky project's fire-sale price  $Q^P$  and a rise in equilibrium amount of the risky projects  $Z^P$ . The center experiences the opposite effects. An increase in the safe interest rate decreases the supply of the risky projects in period 1. This is indicated by the shift from  $B(R_D^C(Aut), A^C)$  to  $B(R_D, A^C)$  in plot (c). As a result, price  $Q^C$  increases and amount of the risky projects produced  $Z^C$  falls.

The following proposition summarizes the above analysis



**Figure 3**

**Proposition 1.** *The financial integration of the center and the periphery, when the collateral constraints bind before and after integration in both countries, leads to*

1. *return on safe debt  $R_D^C$ , fire-sale price  $Q^C$ , purchases of safe debt  $D_d^C$  increase, investments in risky projects  $I^C$ , production of risky projects  $Z^C$ , supply of safe debt  $D_s^C$  decrease after the integration in the center;*
2. *return on safe debt  $R_D^P$ , fire-sale price  $Q^P$ , purchases of safe debt  $D_d^P$  decrease, investments in risky projects  $I^P$ , production of risky projects  $Z^P$ , supply of safe debt  $D_s^P$  increase after the integration in the periphery.*

The third case is a situation in which the bankers' collateral constraints do not bind in the periphery ( $\theta^P(Aut) = 0$ ); however, the constraints bind in the center ( $\theta^C(Aut) > 0$ ). According to lemma 2, the safe interest rate is higher in the periphery. Thus, financial integration leads to flows of resources from the center to the periphery. As a result, the equilibrium may have one of the following three types after integration: (i) the center collateral

constraints bind ( $\theta^C > 0$ ), while the periphery collateral constraints do not bind ( $\theta^P = 0$ ); (ii) the collateral constraints bind in both countries ( $\theta^P > 0$ ,  $\theta^C > 0$ ); and (iii) the collateral constraints do not bind ( $\theta^P = 0$ ,  $\theta^C = 0$ ). However, independent of a type of equilibrium of an integrated economy, the effect of integration on equilibrium variables is qualitatively similar to the previous case. I do not provide the graphical characterization of each case but only summarize the effects of integration in the following lemma.

**Lemma 3.** *The financial integration of two countries with a higher productivity of investment opportunities in the periphery than in the center ( $A^P > A^C$ ) and binding collateral constraints in the center but slack constraints in the periphery (before the integration) results in the following changes:*

1. *In the periphery, investment in the risky projects  $I^P$ , amount of risky projects  $Z^P$ , supply of safe debt  $D_s^P$  increase and interest rate  $R_D^P$ , risky projects price in period 1 in bad state  $Q^P$  and demand for safe debt  $D_d^P$  decrease;*
2. *In the center, investment in the risky projects  $I^C$ , production of risky projects  $Z^C$ , supply of safe debt  $D_s^C$  decrease and interest rate  $R_D^C$ , risky projects price in period 1 in bad state  $Q^C$  and demand for safe debt  $D_d^C$  increase.*

The results presented in this section are related to the recent literature on the global imbalances. [Bernanke \(2005\)](#) argued that the US capital inflows and a decrease in the real interest rate can be both explained by excessive savings in many emerging and oil-exporting countries (“global saving glut” hypothesis)<sup>18</sup>. [Caballero et al. \(2008\)](#) and [Mendoza et al. \(2009\)](#) proposed that emerging economies financial systems cannot produce enough assets that can be used for savings, hence, capital flows to the countries with better developed financial systems, capable of generating more of these assets. In this paper, I do not assume differences in financial development across countries. Instead, capital flows are driven by the difference in productivities of marginal investment opportunities. In addition, I explicitly consider the presence of financial sectors and a possibility of financial crises.<sup>19</sup> The equilibrium capital flows exacerbate potential crisis in the periphery while alleviating the consequences of a crisis in the center.

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<sup>18</sup>[Bernanke \(2011\)](#), [Shin \(2012\)](#) provide evidence on the heterogeneity of the rest of the world portfolio. Asian and oil-producing countries invest in US safe debt. However, there are large gross capital flows between the US and European countries. Many European banks raise funding in the US and then invest in the US assets. [Acharya and Schnabl \(2010\)](#) argue that difference in regulatory treatment of banks across countries may explain this behaviour.

<sup>19</sup>[Caballero and Krishnamurthy \(2009\)](#) study a model in which foreigners invests in the US safe debt issued by intermediaries, which leads to an increase in leverage of the financial system.

Gourinchas et al. (2010) present evidence that, in addition to large capital inflows to the US prior to the recent crisis, there were sizable wealth transfers from the U.S. to the rest of the world during the crisis. They interpret this observation as evidence that the US provides insurance to the rest of the world. They build a model in which US agents have a lower risk-aversion than agents in the rest of the world. Maggiori (2012) builds a model which rationalizes different attitudes to risk in the US and the rest of the world by assuming different levels of financial development. In this model, an arrival of bad news in the periphery results in a fire-sale of banks assets. Both peripheral and center entrepreneurs receive claims on the outside investors in the periphery. As results, there is no capital outflow from the periphery to the center. However, this counterfactual assumption goes away if I assume that the entrepreneurs can also use storage technology between period 1 and 2. In this case, the entrepreneurs in the center can directly accept consumption good in period 1 in the bad state instead of claims on the outside investors in the periphery.

## 4 Welfare Effects of Integration

In this section, I study the welfare effects of short-term liabilities funding markets integration between the two countries. The section presents the main welfare result, which shows that financial integration leads to welfare decline in the periphery under certain conditions.

All of the agents are risk-neutral with respect to their consumption. I will evaluate social welfare in each country by adding consumption levels of all of the agents in each period.<sup>20</sup> The following lemma, which is proved in the Appendix, expresses the social welfare in equilibrium in the periphery.

**Lemma 4.** *The expectation of the social welfare in the periphery in period 0 is*

$$\begin{aligned} \mathbb{E}U^P &= Y - I^P + \frac{D_s^P - D_d^P}{R_D} \\ &+ v(D_d^P) \\ &+ \beta[p + (1-p)q]A^P F(I^P) \\ &+ \beta \{p [\delta g(W) - (D_s^P - D_d^P)] + (1-p) [\delta g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)]\}. \end{aligned} \quad (22)$$

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<sup>20</sup>Alternatively I can assume that all three types of agents belong to the same large household with utility function similar to the entrepreneur utility function. To formally use this assumption I have to present the household problem rather than three separate problems. However, this does not change any of my conclusions. See Lucas (1990) on the exposition of the large family construct.

The first line represents the amount of goods available for consumption in period 0:  $Y$  is the initial endowment of the entrepreneurs.  $(D_s^P - D_d^P)/R_D$  is the amount of goods that the entrepreneurs in the center pay to peripheral bankers to obtain  $D_s^P - D_d^P$  units of safe debt. The second line represents the liquidity preferences from holding  $D_d^P$  units of safe debt. The third line represents the expected discounted output of the risky projects. The last line is the expected discounted revenue of the outside investors net of the repayments to the entrepreneurs in the center. Term  $[g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)]$  in the last line takes into account that in the bad state  $D_s^P$  units of the outside investors' endowment have to be invested in the storage technology rather than in the risky technology. The welfare in the center has the same form.

As demonstrated earlier, there are three possible types of equilibrium after integration. The first case, i.e., that the collateral constraints do not bind before the integration, is trivial. The integration has no effect on welfare. As I showed in the previous section, this result is due to the fact that the returns on safe debt are the same in both countries before the integration. The integration does not lead to a change in the safe debt return. Thus, the equilibrium allocations do not change. This implies that the social welfare is the same in both countries. The third case, i.e., the collateral constraints bind in the periphery but not in center (before integration), features similar effects that will be analyzed in the second case. I believe that the second case, i.e., the collateral constraints bind in both countries before integration, is the most interesting to analyze.

The next proposition summarizes the welfare effects of the integration.

**Proposition 2.** *The financial integration of the center and the periphery, with binding collateral constraints before financial integration, has the following effects on welfare:*

1. *The center always benefits from integration.*
2. *There always exists  $\hat{A}$  such that for all  $A^P \in (A^C, \hat{A})$  the periphery loses from integration.*

See the Appendix for the proof of this proposition. The first part of the proposition states that the center always benefits from integration. The second part states that the periphery loses from integration if the difference in productivities is not very large.

There are two welfare effects of the financial integration: efficient capital reallocation and changes in welfare losses associated with a negative externality. The first effect is an efficient capital reallocation. Both countries benefit. The entrepreneurs in the center invest their

resources in safe debt of the peripheral bankers and receive higher interest rate. Although this means that they invest less in their local banks, which implies smaller profits of the center entrepreneurs, the net effect is positive. The bankers in the periphery can fund themselves more cheaply after the integration. Although this effect is dampened by smaller holdings of safe debt by the periphery entrepreneurs, the net effect is positive.

Before explaining the source of welfare losses, I should comment on the nature of the externality. To simplify the exposition, I will focus on a closed economy equilibrium in which the collateral constraints bind. Consider the following perturbation: a banker decreases his issuance of safe debt by a small amount in period 0. Thus he will have to sell less risky projects in the bad state in period 1. This increases fire-sale price  $Q^P$  in a possible bad state. The marginal decrease in safe debt issuance has no effect on the banker profits because he optimizes in equilibrium. However, this perturbation has three effects through price  $Q^P$  that the banker does not internalize. First, an increase in  $Q^P$  is a benefit to the other bankers because they can get more on the fire-sale market for the same amount of risky projects. Second, an increase in  $Q^P$  is a loss to outside investors because they have to pay more for the same amount of the risky projects. Third, an increase in  $Q^P$  relaxes the collateral constraints of the other bankers in period 0. This allows them to issue more safe debt, which is a cheaper source of funding.

The first two effects cancel each other out from the perspective of the social welfare function used here. The fact that the bankers and the outside investors have the same marginal utility of consumption makes the transfer between them associated with an increase in  $Q^P$  welfare-neutral.<sup>21</sup> The third effect is a pure gain for the economy. However, because an individual banker does not internalize this positive effect from a smaller issuance of safe debt he overissues safe debt in equilibrium. In other words, there is a negative externality associated with binding collateral constraints.<sup>22</sup>

Next, I continue discussing the effects of financial integration. The second effect of financial integration is due to changes in welfare losses associated with the negative pecuniary externality. The bankers in the periphery start issuing more safe debt after the integration. Because there is a wedge between social and private returns on issuing safe debt, an increase

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<sup>21</sup>Note that the generic inefficiency result in environments with incomplete markets described in [Geanakoplos and Polemarchakis \(1985\)](#) and [Greenwald and Stiglitz \(1986\)](#) deals with cases in which marginal utilities of agents are not equalized in equilibrium. That is, their inefficiency result stems from transfers associated with price changes. See also [Lorenzoni \(2008\)](#) who builds a model in which financial frictions prevent equalization of the marginal utilities of agents which leads to welfare losses associated with the pecuniary externality.

<sup>22</sup>This externality has similar implications as the externality in [Bianchi and Mendoza \(2010\)](#). They present a model in which agents face borrowing constraints. However, in their model agents cannot borrow more than the current value of their collateral.

in issuance of safe debt increases losses because this wedge is multiplied by a larger amount of safe debt. Note that this is a first-order effect in the size of increase in the issuance of safe debt. On the other hand, because the center bankers issue smaller amount of safe debt, the loss becomes smaller because the wedge in the center now multiplies by a smaller amount of safe debt.

To formally see the influence of these two effects on the level of the social welfare in the periphery I can express the change in the welfare due to the integration as follows:

$$\begin{aligned}
X^P &= U^P(A^P, A^C; \text{integration}) - U^P(A^P, A^C; \text{autarky}) \\
&= U^P(A^P, A^C; \text{integration}) - U^P(A^P, A^P; \text{integration}) \\
&= - \int_{A^C}^{A^P} \frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A}. \tag{23}
\end{aligned}$$

$U^P(A^P, A^C; \text{integration})$  is a social welfare function where the first argument is the marginal productivity of investment opportunities in country  $P$ , the second argument is the marginal productivity of investment opportunities in country  $C$ , and the third argument is a dummy variable that indicates whether the two countries are integrated. In the proof of proposition 2, I show that

$$\frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} = \beta \frac{\theta^P A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tilde{A}} - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tilde{A}}$$

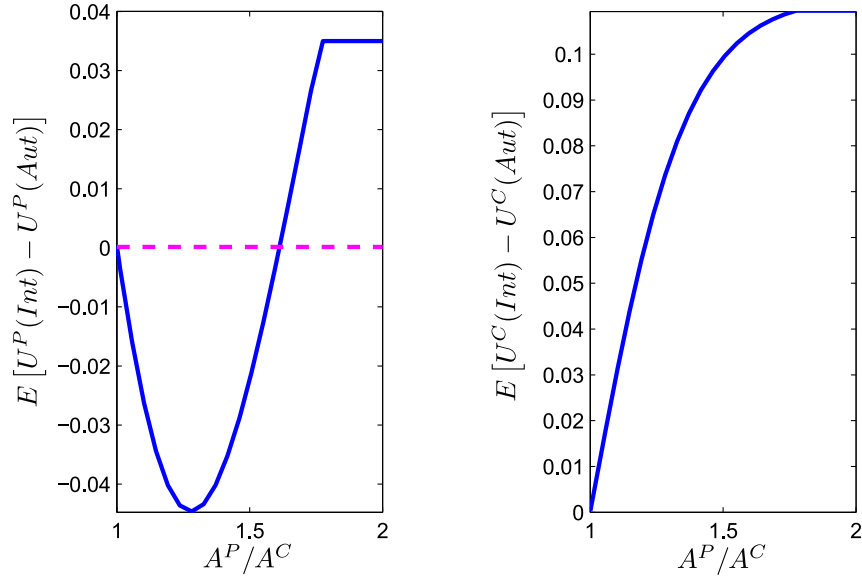
In the above formula, the first term is positive because  $dQ^P/d\tilde{A} > 0$ : an increase in  $\tilde{A}$  in the center leads to a larger supply of safe debt  $D_s^C$  which decreases the equilibrium issuance of safe debt in the periphery and increases price  $Q^P$ . When  $\tilde{A} < A^P$ , there is an inflow of resources to the periphery; hence,  $D_s^P - D_d^P > 0$ . Derivative  $dR_D/d\tilde{A}$  is positive because the increase in the issuance of safe debt in the center leads to higher holdings of safe debt in both countries, which increases safe debt return  $R_D$ . Thus the second term of the above formula is positive. We can now see that

$$X^P = \int_{A^C}^{A^P} \underbrace{\left( -\beta \frac{\theta^P A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tilde{A}} \right)}_{<0} d\tilde{A} + \int_{A^C}^{A^P} \underbrace{\left( -\frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tilde{A}} \right)}_{>0} d\tilde{A},$$

where the first term represents increased losses due to the negative pecuniary externality while the second term represents the efficient capital reallocation effect. In the proof of the proposition I show that  $dQ^P/\tilde{A}$  and  $dR_D/d\tilde{A}$  are bounded from zero for all  $\tilde{A}$ ; however, because the second term features the difference  $D_s^P - D_d^P$ , the value of this term can be arbitrarily close to zero. This observation implies that the efficient capital reallocation

benefit is smaller than the welfare losses associated with the pecuniary externality when the difference  $A^P - A^C$  is small.

The same reasoning may be applied the center to show that both effects go in the direction of increasing the social welfare.



**Figure 4:** Change in the social welfare in the periphery  $E[U^P(Int) - U^P(Aut)]$  and in the center  $E[U^C(Int) - U^C(Aut)]$  as a function of the ratio of productivities  $A^P/A^C$ . The utility function from holding debt has the following form  $v(D) = \gamma D^{\alpha_D}$ , risky projects production function  $F(I) = AI^{\alpha_F}$ , the late technology  $g(x) = Bx^{\alpha_G}$ . Parameters:  $\alpha_F = 0.8, \alpha_G = 0.65, \beta = 0.9, \gamma = 3, \alpha_M = 0.76, p = 0.8, q = 0.5, W = 5, A^C = 2, A^P \in [2, 4], B = 5, \delta = 0.99$ .

**Example.** Figure 4 presents a numerical example that shows the change in welfare for various values of  $A^P$  relative to  $A^C$ . First, observe when  $A^P/A^C = 1$  the countries are identical ex-ante which prevents net flows from one country to the other with integration. Thus there are no welfare changes. For  $A^P$  slightly larger than  $A^C$ , integration leads to a net inflow of resources into the periphery. This leads to a decline in the welfare in the periphery, negative  $\mathbb{E}[U^P(Int) - U^P(Aut)]$ , and an increase in welfare in the center, positive  $\mathbb{E}[U^C(Int) - U^C(Aut)]$ . For  $A^P/A^C > 1.64$ , the gain from efficient capital reallocation dominates the welfare loss due to the negative externality. Finally, when  $A^P/A^C > 1.78$ , productivity  $A^P$  is large enough so that the collateral constraints do not bind in the periphery in line with the results of lemma 2. When the collateral constraints do not bind before and after integration, the integration has only positive capital reallocation effect in the periphery.

The recent literature on global imbalances emphasized the welfare consequences of integration of countries with difference levels of financial development. Most closely related to this paper is [Mendoza et al. \(2007\)](#). They argue that financial flows that arise from different

levels of financial development lead to an increase in welfare in a more financially developed country, that experiences financial inflows, and a decrease in welfare in a less financially developed country after integration. [Eden \(2012\)](#) studies the welfare effects of financial integration in the presence of the working capital constraints in a less financially developed country. The author concludes that the more financially developed country that does not face working capital constraints and experiences capital inflows benefits, while the less financially developed country loses from financial integration. In contrast to this literature, the results of this section suggest that it is the source country that benefits from integration and the recipient country that loses from integration.

## 5 Regulation

This section studies policy. I first consider the optimal macroprudential taxes on the safe debt issuance. Then I show that both countries benefit from adding capital controls to their policy tools.

A number of recent papers suggested that a system of Pigouvian taxes can be used to bring financial sector incentives closer to social interests.<sup>23</sup> [Kashyap and Stein \(2012\)](#) and [Woodford \(2011\)](#) argue that such Pigouvian taxes can be implemented by the interest rate paid on reserves.<sup>24</sup> I start this section by studying the optimal policy in the presence of just one tool: safe debt taxes. Later, I investigate whether additional tools can help improve welfare.

### 5.1 Safe Debt Taxation and Interest Rate Manipulation

In this section, I consider the problem faced by a regulator in the periphery who maximizes the social welfare function in his country by choosing safe debt taxes given all of the equilibrium conditions and fixed behavior of the regulator in the other country. The regulator rebates the proceeds of the taxes to the entrepreneurs in a non-distortionary way.

I formally introduce safe debt taxes into the banker period 0 budget constraint as follows

$$P_0^P Z^P \leq V_B^P + \frac{D_s^P}{R_D} (1 - \tau^P),$$

---

<sup>23</sup>See, for example, [Jeanne and Korinek \(2010\)](#), [Perotti and Suarez \(2010\)](#).

<sup>24</sup>The effectiveness of this tool depends on the ability of the government to impose its reserve requirements on the issuance of assets that create systematic risk to the stability of financial system. For example, if the government can only impose reserve requirements on the traditional banking sector deposits, this may not be welfare increasing if deposits are already appropriately insured by the government.

where  $V_B^P$  is period 0 value of the risky debt. The optimal choice of safe debt funding  $D_s^P$  leads to

$$\frac{R_B}{R_D}(1 - \tau^P) - \left( p + \frac{(1-p)q}{Q^P} \right) - \frac{\theta^P}{Q^P} = 0. \quad (24)$$

$\tau^P$  reduces the benefit from using cheaper safe debt financing. The optimal choice of the risky projects purchases  $Z^P$  is given (15) because the proportional taxes on safe debt do not affect this choice directly.

The regulator maximizes the peripheral social welfare (22), i.e., the sum of all agents consumption, by choosing  $\tau^P$  subject to seventeen equilibrium conditions: bankers optimality conditions in the periphery (24) and (15), constraint on the issuance of safe debt (13), non-negativity of the Lagrange multiplier  $\theta^P$ , the complementarity slackness condition, outside investor optimality condition (7), entrepreneurs optimality condition with respect to safe debt holdings (5), and entrepreneurs optimality condition with respect to investments into the risky projects, eight similar equations for the center and the safe debt market clearing condition. The proof of lemma 5 states this problem explicitly and derives the first order necessary condition.

**Lemma 5.** *At the optimum of the periphery regulator problem, in which either  $\theta^P > 0$  or  $M_s^P < Q^P Z^P$ , the following condition must hold:*

$$\frac{dU^P}{d\tau^P} = \beta \frac{A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tau^P} \left( \theta^P - \tau^P \frac{R_B Q^P}{R_D \tilde{\epsilon}_g^P} \right) - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} = 0.$$

This lemma states that if the regulator chooses to impose taxes  $\tau^P$  on its bankers, then the above condition should hold for optimal choice of  $\tau^P$ .<sup>25</sup> The condition that either  $\theta^P > 0$  or  $M_s^P < Q^P Z^P$  holds in the optimum rules out the case with  $\theta^P = 0$  and  $M_s^P = Q^P Z^P$ . In this situation the welfare function derivative is not defined. If the left derivative of the welfare function is positive while the right derivative is negative, then the optimum is attained at this kink.

The first term of this optimality condition represents two effects. On the one hand, an increase in  $\tau^P$  has a positive effect because it mitigates the negative externality. Observe that this effect is only present when  $\theta^P > 0$ . On the other hand, an increase in  $\tau^P$  makes it more expensive for the bankers to fund themselves, which leads to a lower production of the risky projects that yield less consumption in period 2. The second term  $\frac{D_s^H - D_d^P}{R_D^2} \frac{dR_D}{d\tau^H}$

<sup>25</sup>Note that this condition is trivially satisfied if the regulator taxes out the issuance of safe debt in the periphery from existence. That is, the tax rate is sufficiently high so that the bankers do not issue any safe debt in the periphery. As a result, further changes in the tax rate can not alter the equilibrium variables, i.e.,  $dQ^P/d\tau^P = 0$  and  $dR_D/d\tau^P = 0$ .

is due to manipulation of the international interest rate. If the periphery experiences an inflow of resources directed to investments in safe debt, then a decrease in the interest rate will benefit the bankers in the periphery because they will have to repay less in period 2 to the entrepreneurs in the center. The policy maker decreases the interest rate by taxing his bankers more than he would without the manipulation motive.

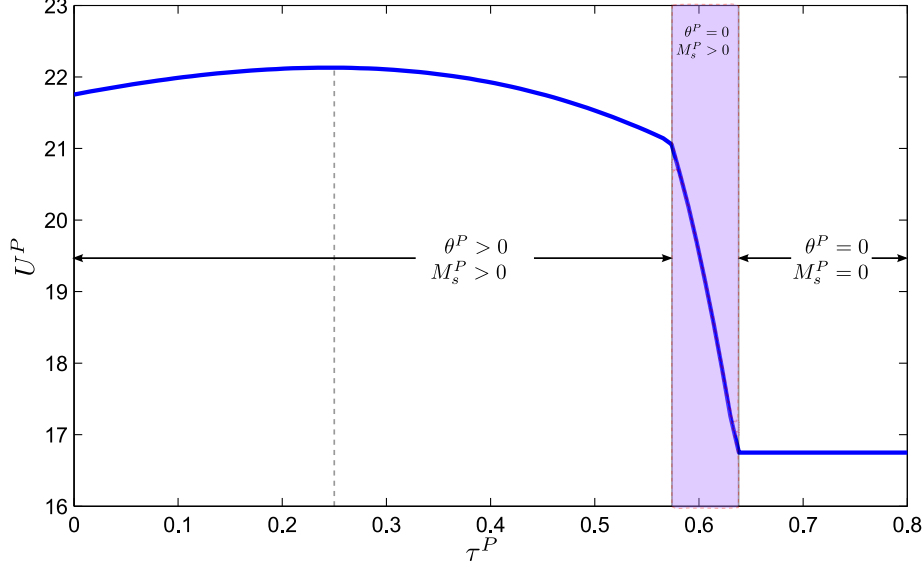
A symmetric condition holds for the regulator in the center

$$\frac{dU^C}{d\tau^C} = \beta \frac{A^C F(I^C)}{Q^C} \frac{dQ^C}{d\tau^C} \left( \theta^C - \tau^C \frac{R_B Q^C}{R_D \tilde{\epsilon}_g^C} \right) - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tau^C} = 0.$$

The only difference is that the last term is necessarily of the opposite sign relative to a similar term in the periphery. If the center experiences the outflow of resources  $D_s^C - D_d^C < 0$ , then the term is negative. This implies that the regulator sets lower taxes compared to the situation without the interest-manipulation motive.

**Example.** Figure 5 presents a numerical example that shows how the peripheral social welfare function depends on safe debt taxes. The parameters are chosen such that the periphery is a net issuer of safe debt when there is no regulation in this country which corresponds to the assumption that  $A^P > A^C$  and the collateral constraints bind at least in the center. Several observations can be made looking at this example. First, a small, positive debt taxes level benefits the periphery. This is a combination of the interest-rate-manipulation benefit and the externality-correction benefit. At  $\tau^P \approx 0.25$ , the social welfare in the periphery attains its maximum. This point corresponds to the condition in lemma 5. A further increase in the level of taxes makes losses from distortionary taxation dominate the benefits from the taxes. At  $\tau^P \approx 0.57$ , there is the first kink, i.e., the collateral constraints stop being binding in the periphery. It becomes costly enough for banks to issue safe debt that they decide to issue less safe debt than the value of their risky projects in the bad state. At  $\tau^P \approx 0.64$ , there is the second kink. It becomes extremely costly for bankers to issue safe debt, and they decide not to issue it at all.

**Welfare.** In the previous section I showed that the periphery may lose from financial integration because the negative effect associated with overissuance of safe debt may dominate the efficient capital reallocation effect. I now show that setting safe debt taxes optimally makes the integration beneficial if the center regulator is passive. Formally, I compare the welfare of the periphery before and after integration, assuming that the regulator in the periphery sets his taxes optimally. At the same time I assume that the regulator in the center is passive. That is, she does not change her taxes after the integration.



**Figure 5:** Ex ante social welfare in the periphery  $U^P$  as a function of the level of safe debt taxes  $\tau^P$ . The utility function from the holding debt has the following form  $v(D) = \gamma D^{\alpha_D}$ , risky projects production function  $F(I) = AI^{\alpha_F}$ , the late technology  $g(x) = Bx^{\alpha_G}$ . Parameters:  $\alpha_F = 0.8, \alpha_G = 0.65, \beta = 0.9, \gamma = 3, \alpha_M = 0.76, p = 0.8, q = 0.5, W = 5, A^C = 2, A^P = 2.3, B = 5, \delta = 0.99, \tau^C = 0$ .

**Proposition 3.** *If the regulator in the periphery (i) chooses the levels of safe debt taxation optimally before and after the integration; (ii) the collateral constraints bind in both countries before and after integration, (iii) the periphery is a net supplier of safe debt after the integration then both countries benefit from the integration.*

When choosing taxes optimally, the regulator in the periphery offsets the negative welfare effect of debt overissuance. In addition, the regulator increases the welfare by manipulating the interest rate. It is important for this result to assume that the regulator in the center is passive. If the regulator in the center chooses her taxes optimally then the interaction of the two regulators has to be taken into account.<sup>26</sup> I turn to this issue next.

## 5.2 Non-Cooperative Safe Debt Taxation

I will now solve for a Nash equilibrium of the regulation game. A regulator in each country chooses the optimal level of taxes by taking the behavior of the other regulator as given. I will only focus on the case in which regulators optimal choices can be described using the first order necessary conditions from lemma 5. In a Nash equilibrium, each regulator optimizes. Thus a marginal change in his policy has a second-order effect on the social welfare in his

<sup>26</sup>If the regulators choose their policies in uncoordinated way, the result in proposition 3 does not hold in general. There is a negative effect that the regulator in the center imposes on the welfare in the periphery which may lead to a decrease in welfare.

country. The next proposition shows that this marginal change leads to a first-order loss in the other country, which leaves the Nash equilibrium strictly inside the Pareto frontier.

**Proposition 4.** *A Nash equilibrium can be locally Pareto-improved if the periphery regulator decreases and the center regulator increases their taxes.*

To describe the effects at work, consider a marginal increase in taxes in the periphery and the corresponding reaction of the center social welfare:

$$\frac{dU^C}{d\tau^P} = \underbrace{\beta A^C \frac{F(I^C)}{Q^C} \frac{dQ^C}{d\tau^P} \theta^C}_{<0} - \underbrace{\beta \frac{A^C F(I^C)}{Q^C} \frac{dQ^C}{d\tau^P} \tau^C \frac{R_B Q^C}{R_D \tilde{\epsilon}_g^C}}_{<0} - \underbrace{\frac{D_s^C - D_d^C}{R_D^2}}_{<0} \underbrace{\frac{dR_D}{d\tau^P}}_{<0}$$

The first term is a loss due to the negative externality after a marginal increase in  $\tau^P$ . Note that this term is present only when the collateral constraints bind in the center. An increase in  $\tau^P$  decreases the supply of the safe debt in the periphery, which makes the world supply of safe debt smaller leading to an increase in the price (and decrease in returns) of safe debt  $1/R_D$ . As a result, the center bankers' incentives to issue safe debt go up. However, this leads to a more severe decline in the bankers' assets price in the bad state, i.e.,  $dQ^C/d\tau^P < 0$ , which has negative consequences for welfare. The second term shows that the losses associated with distortional taxes in the periphery become smaller. The last term represents the loss for the entrepreneurs in the center who now receive smaller return on their purchases of safe debt from the periphery.

When choosing his optimal level of taxes, the regulator in the periphery does not internalize that he has the above three effects on the center economy. The proposition states that in a Nash equilibrium the net effect is negative. In addition, the net effect of a marginal increase in taxes in the center on the welfare in the periphery is positive.

These results are related to the literature that studies the international terms of trade manipulation. [Obstfeld and Rogoff \(1996\)](#) in a two-period and recently [Costinot et al. \(2011\)](#) in a dynamic model study how the incentives to install capital controls may arise because of the desire to manipulate the intertemporal terms of trade. In my paper, a regulator who only intends to limit the scope of the negative externality in the banking sector will inevitably affect the international interest rate. This creates the desire to manipulate the interest rate.

### 5.3 Safe Debt Taxation and Capital Controls

The regulators have incentives to use macroprudential safe debt taxation to manipulate the international interest rate. Thus, it is logical to add another tool to their policy choice sets.

One such tool can be capital controls. By capital controls I mean a proportional tax or subsidy on capital flows. Consider the periphery. If the local interest rate on the safe debt equals  $R_D^P$ , then the agents in the center who invest in safe debt in the periphery will receive  $(1 - \tau_f^P)R_D^P$  units of consumption good in period 2 for each unit invested in period 0.<sup>27</sup> A symmetric definition is applied to the center.

In the next lemma, I solve for the first-order necessary conditions of the regulator problem in the periphery, assuming that the other regulator is passive. The full problem that the regulator solves is defined in the proof of the lemma.

**Proposition 5.** *At an optimum of the regulator problem, the following condition must hold*

$$\tau^P = \theta^P \frac{\tilde{c}_g^P R_D^P}{Q^P R_B}, \quad (25)$$

$$\tau_f^P = \frac{-\frac{R_D^P}{R_D^C} \frac{D_s^P - D_d^P}{R_D^C} \frac{dR_D^C(D_s^P - D_d^P)}{dD_d^P}}{1 - \frac{R_D^P}{R_D^C} \frac{D_s^P - D_d^P}{R_D^C} \frac{dR_D^C(D_s^P - D_d^P)}{dD_d^P}}. \quad (26)$$

It is easy to see that  $\tau^P, \tau_f^P \in [0, 1]$ . The first condition states that the regulator does not use safe debt taxes to manipulate the interest rate. The second condition states that the capital control tax is proportional to the regulator's effect on the interest rate in the center, i.e.,  $dR_D^C(D_s^P - D_d^P)/dD_d^P$ , and to the level of cross-border net safe debt  $D_s^P - D_d^P$ .

Jeanne and Korinek (2010) and Bianchi (2011) argue that negative externality associated with borrowing from abroad give rise to prudential capital controls.<sup>28</sup> In my paper, borrowing from abroad per se does not create inefficiencies. However, the incentives to manipulate the international interest rates, when regulating the local banking sector, will lead to the desire to use two tools—prudential taxes on banking sector and capital controls— instead of just using prudential taxation in the banking sector.

## 6 Conclusion

In this paper, I analyzed the effects of international financial integration in the presence of bank funding risk. The central feature of the analysis is the presence of negative pecuniary externality that bankers do not internalize. This leads to overissuance of safe debt that leads to inefficiently low price of bankers' assets in crises. The integration of the short-term safe

<sup>27</sup>Subscript  $f$  distinguishes this tool from the tax on safe debt.

<sup>28</sup>Martin and Taddei (2012) build a model in which adverse selection problems leads to inefficient borrowing from abroad.

funding markets leads to capital flows. As a result, the severity of possible financial crises increases in the region that experiences capital inflows, the periphery, but becomes smaller in the region that experiences capital outflows, the center. Thus, the integration leads to changes in the severity of this distortion.

I show that, in unregulated world, the periphery may lose from integration. The center always gains from the integration. There are two effects of the integration: efficient capital reallocation and changes in the welfare losses due to overissuance of safe debt. When the difference in the productivities of the marginal investment opportunities in the two regions are not large, the negative welfare effect always dominates efficient capital reallocation effect for the periphery. However, the two effects are positive for the center because the integration leads to less issuance of safe debt in the center.

A regulator in each country may want to correct the effects of the overissuance of safe debt by imposing macroprudential taxes on safe debt funding. In the integrated world, this macroprudential tool will have effect on the international price of safe debt. This creates incentives for the regulators to manipulate the interest rate. If the regulators set their policies in a non-cooperative manner then a resulting Nash equilibrium is not Pareto efficient. If the regulator in the periphery reduces his taxes while the regulator in the center increases her taxes the welfare of both countries can be Pareto improved.

Finally, I show that the regulators will have incentives to add capital controls to their policy tools. Using capital controls allows to correct the externality in the banking sector and to manipulate international interest rate more effectively.

The analysis in this paper was purely qualitative. It is important to quantify the effects discussed in the paper. I leave this for a future research.

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# A Appendix: Proofs

## A.1 Proof of Lemma 1.

A banker in the periphery solves the following problem

$$\begin{aligned} \max_{Z^P, D_s^P, \bar{B}^P} \mathbb{E}\pi_B^P &= [p + (1-p)q] Z^P - \left(p + \frac{(1-p)q}{Q^P}\right) D_s^P - \left[p\bar{B}^P + (1-p)q \min\{\bar{B}^P, Z^P - D_s^P/Q^P\}\right] \\ \text{s.t. } P_0^P Z^P &\leq \frac{D_s^P}{R_D} + P_B^P(G)\bar{B}^P + P_B^P(Bnc) \min\{\bar{B}^P, Z^P - D_s^P/Q^P\} \\ D_s^P &\leq Q^P Z^P \end{aligned}$$

Define  $\theta^P$  such that the Lagrange multiplier on the collateral constraint is  $\theta^P/Q^P$ , denote the Lagrange multiplier on the budget constraint as  $\eta$ .

I consider two different cases depending on whether the banker defaults or not in state  $s_2^P = Bnc$ . **Default.** If the banker defaults then  $\bar{B}^P > Z^P - D_s^P/Q^P$ . The optimal interior choice of  $Z^P$  leads to

$$p - \eta[P_0^P - P_B^P(Bnc)] + \theta^P = 0 \quad (\text{A.1})$$

It is clear that the banker chooses positive  $Z^P$  in equilibrium because otherwise  $P_0^P$  would be zero. This implies an infinite gain for the banker from choosing small positive  $Z^P$ . The optimal choice of  $\bar{B}^P$  leads to

$$-p + \eta P_B^P(G) = 0 \quad (\text{A.2})$$

If the banker chooses positive amount of safe debt financing this implies

$$\frac{R_B}{R_D} - \left[p + \frac{(1-p)q}{Q^P}\right] - \frac{\theta^P}{Q^P} + \eta \left[\frac{1}{R_D} - \frac{P_B^P(Bnc)}{Q^P}\right] = 0. \quad (\text{A.3})$$

In a closed economy the banker always chooses positive amount of safe debt in equilibrium. However, in open economy there can be parameter values that imply zero safe debt issuance in one of the countries. Because I am interested in analyzing situations in which the collateral constraints bind in both countries I assume here that  $D_s^P > 0$ .

In equilibrium the state prices equal  $P_B^P(G) = \beta p$ ,  $P_B^P(Bnc) = \beta(1-p)q$ ,  $P_B^P(Bc) = \beta(1-p)(1-q)$ . From (A.2) I get  $\eta = 1/\beta$ . Then (A.1) implies

$$[p + (1-p)q] - R_B P_0^P + \theta^P = 0. \quad (\text{A.4})$$

Finally, (A.3) implies

$$\frac{R_B}{R_D} - \left[p + \frac{(1-p)q}{Q^P}\right] - \frac{\theta^P}{Q^P} = 0. \quad (\text{A.5})$$

The budget constraint implies  $P_0^P Z^P = D_s^P/R_D + \beta p \bar{B}^P + \beta(1-p)q (Z^P - D_s^P/Q^P)$ . Thus, the face value of the risky debt is

$$\bar{B} = \frac{R_B}{p} \left[ Z^P P_0^P - \frac{D_s^P}{R_D} \right] - \frac{(1-p)q}{p} \left[ Z^P - \frac{D_s^P}{Q^P} \right].$$

Using this equation, the default condition  $\bar{B}^P > Z^P - D^P/Q^P$  can be rewritten as

$$Z^P(R_B P_0^P - (1-p)q - p) > D_s^P \left( \frac{R_B}{R_D} - \frac{(1-p)q}{Q} - \frac{p}{Q} \right).$$

**No default.** If the banker does not default then  $\bar{B}^P \leq Z^P - D^P/Q^P$ . The optimal interior choice of  $Z^P$  leads to

$$\eta[P_B^P(G) + P_B^P(Bnc) - P_0^P] + \theta^P = 0$$

The optimal choice of  $\bar{B}^P$  leads to

$$\eta[P_B^P(Bnc) + P_B^P(G)] - p - (1-p)q = 0$$

If the banker chooses positive amount of safe debt financing this implies

$$\frac{\eta}{R_D} - \left[ p + \frac{(1-p)q}{Q^P} \right] - \frac{\theta^P}{Q^P} = 0.$$

Taking into account the equilibrium prices I obtain  $\eta = 1/\beta$  and conditions identical to (A.4) and (A.5). The budget constraint implies  $P_0^P Z^P = D_s^P/R_D + \beta[p + (1-p)q]\bar{B}^P$ . Thus, the face value of the risky debt is

$$\bar{B} = \frac{R_B}{p + (1-p)q} \left[ Z^P P_0^P - \frac{D_s^P}{R_D} \right].$$

Using this equation, the default condition  $\bar{B}^P \leq Z^P - D^P/Q^P$  can be rewritten as

$$Z^P(R_B P_0^P - (1-p)q - p) \leq D_s^P \left( \frac{R_B}{R_D} - \frac{(1-p)q}{Q} - \frac{p}{Q} \right). \blacksquare$$

## A.2 Proof of Lemma 2.

**Step 1.** Denote the level of investment productivity by for which  $\theta^P = 0$  and  $D^P = Z^P Q^P$  by  $\bar{A}$ . Let's show that such a level exists. The bankers optimality condition in equilibrium with  $\theta^P = 0$  and the investors optimality condition when  $D^P = Z^P Q^P$

$$\frac{Q^P}{q} = \frac{1-p}{\frac{R_B}{R_D} - p} \quad \text{and} \quad \frac{q}{Q^P} = \delta g'(W - D^P)$$

imply

$$\delta g'(W - D^P) = \frac{\frac{R_B}{R_D} - p}{1-p}.$$

Using the fact that the entrepreneur optimal choice of safe debt  $D^P$  leads to  $1/R_D = \beta + v'(D^P)$ , I can rewrite the last equation

$$\delta g'(W - D^P) = \frac{1-p + v'(D^P)/\beta}{1-p}.$$

Because  $g(\cdot)$  is strictly concave, the left-hand side (LHS) of the last equation is increase in  $D^P$ . Because  $v(D^P)$  is strictly concave, the right-hand side (RHS) of the above equation is increasing

in  $D^P$ . If the value of the RHS is higher than the value of the LHS in  $D^P = 0$  then the equation always has a solution. If  $1 + v'(0)/[\beta(1-p)] > \delta g'(W)$  then the RHS is greater than the LHS at  $D^P = 0$ . This condition holds when  $v(\cdot)$  satisfies  $\lim_{D \rightarrow 0} v'(D) = \infty$ . Denote the solution to the last equation by  $\bar{D}$ .

Equilibrium condition  $\bar{D} = Q^P Z^P = q/[\delta g'(W - \bar{D})] A^P F(I^P)$  determines a negative relation between  $I^P$  and  $A^P$ ,  $I^P = \phi(A^P)$ .

The bankers optimal choice of  $Z^P$  and the entrepreneurs optimal choice of  $I^P$  when  $\theta^P = 0$  imply  $[p + (1-p)q] A^P F'(I^P) = R_B$ . This determines a positive relation between  $I^P$  and  $A^P$ ,  $I^P = \psi(A^P)$ .

Because  $F(\cdot)$  satisfied the Inada conditions the solution to equation  $\phi(A^P) = \psi(A^P)$  always exists, unique and equals  $\bar{A}$ . I will distinguish all the equilibrium endogenous variables corresponding to  $\bar{A}$  with a bar.

**Step 2.** Consider  $A < \bar{A}$ . Let's show by contradiction that  $\theta^P > 0$ . Assume that  $\theta^P = 0$  for this  $A$ . If  $D^P = Q^P Z^P$  then all the equilibrium variables should be equal to equilibrium variables under  $\bar{A}$ . This is not possible because  $A < \bar{A}$ . If  $D^P < Q^P Z^P$  then  $D^P(A) = \bar{D}$ , which is a result of a reasoning similar to the one in the beginning of Step 1. Fire-sale price  $Q^P = q/[\delta g'(W - D^P)] = q/[\delta g'(W - \bar{D})] = \bar{Q}$ . Because  $D^P < Q^P Z^P$  we have

$$\bar{A}F(\bar{I}) \frac{q}{\delta g'(W - \bar{D})} = \bar{D} = D^P < Z^P Q^P = AF(I^P) \frac{q}{\delta g'(W - D^P)} = AF(I^P) \frac{q}{\delta g'(W - \bar{D})}$$

Comparing the first and last terms in this equation I get  $\bar{A}F(\bar{I}) < AF(I^P)$ .  $A < \bar{A}$  implies  $I^P > \bar{I}$ .

The bankers optimal choice of  $Z^P$  and the entrepreneurs optimal choice of  $I^P$  when  $\theta^P = 0$  imply  $[p + (1-p)q] A^P F'(I^P) = R_B$ . Thus,  $AF'(I^P) = \bar{A}F'(\bar{I})$ . Because  $I^P > \bar{I}$  I can write

$$\bar{A}F'(\bar{I}) = AF'(I^P) < AF'(\bar{I}) < \bar{A}F'(\bar{I}),$$

which is a contradiction. Hence,  $\theta^P > 0$  for all  $A < \bar{A}$ .

**Step 3.** In this step I prove the comparative statics statements in the lemma for  $A < \bar{A}$ . From (20), the banker and entrepreneur optimal choices, I know that

$$\left[ \left( \frac{R_B}{R_D^P} - p \right) Q^P + p \right] A^P F' \left( F^{-1} \left( \frac{Z^P}{A^P} \right) \right) = R_B.$$

Given properties of  $F(\cdot)$

$$\frac{\partial \left[ A^P F' \left( F^{-1} \left( \frac{Z^P}{A^P} \right) \right) \right]}{\partial A^P} = F'(I^P) - \frac{F''(I^P)F(I^P)}{F'(I^P)} > 0.$$

This implies that for a given  $Z^P, R_D^P$  a marginal increase in  $A$  leads to a decrease in  $Q^P$ . This corresponds to a shift in the  $B$  curve to the right on the left panel of Figure 2. Conditional on  $R_D^P$  the equilibrium value of  $Z^P$  goes up. Because the elasticity of the outside investor demand in greater than 1 the supply of safe debt  $D^P = Q^P Z^P$  increases. This increases the amount of safe debt issued and the return on safe debt in equilibrium. An increase in  $R_D^P$  has the opposite effect on  $Z^P$  and  $Q^P$  relative to the direct effect of changes in  $A$ . However, the indirect effect is weaker than the direct effect.

The shadow value of risky projects for the bankers is

$$\theta^P = \left[ 1 - p + \frac{v'(D^P)}{\beta} \right] Q^P - (1 - p)q.$$

Because  $D^P$  increases and  $Q^P$  decreases as a result of increase in  $A$  it is clear that  $\theta^P$  falls.

**Step 4.** Consider  $A > \bar{A}$ . I need to show that  $\theta^P = 0$ . Assume that  $\theta^P > 0$ . Then, by Step 2 of this proof this implies that  $\theta^P(\bar{A}) > 0$  which is a contradiction. Thus,  $\theta^P = 0$  for all  $A > \bar{A}$ .

**Step 5.** For  $A > \bar{A}$   $D^P, Q^P, \theta^P, R_D^P$  are all determined independently of  $A$ . To see this observe that the optimal choice of  $D_s^P$  by the banker is decoupled from optimal choice of  $Z^P$  which depends on price  $P_0^P$  which in turn depends on  $A$ . The bankers optimal choices of  $Z^P$  and the entrepreneurs optimal investments  $I^P$  imply  $[p + (1 - p)q] A^P F'(I^P) = R_B$ . Hence,  $I^P$  and  $Z^P$  are negatively related to  $A$  in equilibrium. ■

### A.3 Proof of Lemma 4.

The social welfare function equals

$$U = C_0^P(E) + \beta \mathbb{E} [C_2^P(E) + C_2^P(B) + C_2^P(OI)] + v(D_d^P), \quad (\text{A.6})$$

where  $C_0^P(E)$  and  $C_2^P(E)$  represent consumption of the entrepreneurs in the periphery,  $C_2^P(B)$  is consumption of the bankers,  $C_2^P(OI)$  is consumption of the outside investors. I use agents' budget constraints to express consumption levels. The entrepreneurs and the bankers budget constraints in period 0 are

$$C_0^P(E) = Y + P_0^P A^P F(I^P) - I^P - \frac{D_d^P}{R_D} - \sum_{s_2^P} B(s_2^P) P_B^P(s_2^P),$$

$$P_0^P Z^P = V_B^P + \frac{D_s^P}{R_D}.$$

The market clearing conditions imply  $V_B^P = \sum_{s_2^P} B(s_2^P) P_B^P(s_2^P)$  and  $Z^P = A^P F(I^P)$ . Thus,

$$C_0^P(E) = Y + \frac{D_s^P - D_d^P}{R_D} - I^P.$$

Next, consider period 2. Because the bankers consume their profits and the outside investors consume their revenues, I can write

$$\begin{aligned} & \mathbb{E} [C_2^P(E) + C_2^P(B) + C_2^P(OI)] \\ &= \mathbb{E} [C_2^P(E) + \pi_B^P + \pi_{OI}^P] \\ &= D_d^P + \mathbb{E} \min\{\bar{B}^P, Z^P - \frac{D_s^P}{Q^P}\} && \text{entrepreneur} \\ &+ [p + (1 - p)q] Z^P - \left( p + \frac{(1 - p)q}{Q^P} \right) D_s^P - R_B \left( P_0^P Z^P - \frac{D_s^P}{R_D} \right) && \text{banker} \\ &+ p \delta g(W) + (1 - p) \left[ q \frac{D_s^P}{Q^P} + \delta g(W - D_s^P) \right] && \text{outside investor} \end{aligned}$$

Note that in equilibrium  $\mathbb{E} \min\{\bar{B}^P, Z^P - \frac{D_s^P}{Q^P}\} = R_B \left( P_0^P Z^P - \frac{D_s^P}{R_D} \right)$ . Thus,

$$\begin{aligned} & \mathbb{E} [C_2^P(E) + C_2^P(B) + C_2^P(OI)] \\ &= [p + (1-p)q] Z^P + p [g(W) - (D_s^P - D_d^P)] + (1-p) [\delta g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)], \end{aligned}$$

where  $Z^P = A^P F(I^P)$ . Combining the above results, the social welfare is

$$\begin{aligned} U &= Y + \frac{D_s^P - D_d^P}{R_D} - I^P + v(D_d^P) + \beta [p + (1-p)q] A^P F(I^P) \\ &+ \beta p [g(W) - (D_s^P - D_d^P)] + \beta(1-p) [\delta g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)]. \blacksquare \end{aligned}$$

## A.4 Proof of Proposition 2.

I first prove that welfare in the center unambiguously goes up.

**Step 1.** Let's denote the social welfare in country  $C$  by  $U^C = U^C(A^C, A^P; \cdot)$ , where the first argument is the marginal productivity of investment opportunities in country  $C$ , the second argument is the marginal productivity of investment opportunities in country  $P$ , the third argument is a dummy variable that indicates if the two countries are integrated. We are interested in computing the following difference

$$X^C = U^C(A^C, A^P; \text{integration}) - U^C(A^C, A^P; \text{autarky})$$

I can express the social welfare in country  $C$  as follows

$$U^C(A^C, A^P; \text{integration}) = \int_{A^C}^{A^P} \frac{dU^C(A^C, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A} + U^C(A^C, A^P; \text{integration})$$

Observe that if the two countries have the same level of  $A$  then there is no gains from integration. Formally,  $U^C(A^C, A^P; \text{integration}) = U^C(A^C, A^P; \text{autarky})$ . Thus, the variable of interest  $X^C$  can be expressed as follows

$$X^C = \int_{A^C}^{A^P} \frac{dU^C(A^C, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A} \tag{A.7}$$

**Step 2.** I now show that  $dU^C(A^C, \tilde{A}; \text{integration})/d\tilde{A} > 0$ . Thus, from (A.7) I will get that  $X^C > 0$ . In words, country  $P$  unambiguously benefits from integration when  $A^P > A^C$ . The social welfare function in country  $C$  is

$$\begin{aligned} U^C(A^C, \tilde{A}; \text{integration}) &= Y - I^C + \frac{D_s^C - D_d^C}{R_D} + v(D_d^C) + \beta [p + (1-p)q] A^C F(I^C) \\ &+ \beta \{ p [g(W) - (D_s^C - D_d^C)] + (1-p) [g(W - D_s^C) + D_s^C - (D_s^C - D_d^C)] \}. \end{aligned}$$

Rearranging last equation I get

$$\begin{aligned} U^C(A^C, \tilde{A}; \text{integration}) &= Y - I^C + (D_s^C - D_d^C) \left( \frac{1}{R_D} - \beta \right) + v(D_d^C) \\ &+ \beta \{ [p + (1-p)q] A^C F(I^C) + p g(W) + (1-p) [g(W - D_s^C) + D_s^C] \} \end{aligned}$$

Now, I take the full derivative of the above expression with respect to  $\tilde{A}$

$$\begin{aligned} \frac{dU^C(A^C, \tilde{A}; \text{integration})}{d\tilde{A}} &= -\frac{dI^C}{d\tilde{A}} + \left( \frac{dD_s^C}{d\tilde{A}} - \frac{dD_d^C}{d\tilde{A}} \right) \left( \frac{1}{R_D} - \beta \right) - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tilde{A}} + v'(D_d^C) \frac{dD_d^C}{d\tilde{A}} \\ &\quad + \beta \left\{ [p + (1-p)q] A^C F'(I^C) \frac{dI^C}{d\tilde{A}} + (1-p) [1 - g'(W - D_s^C)] \frac{dD_s^C}{d\tilde{A}} \right\} \end{aligned}$$

Rearranging I get

$$\begin{aligned} \frac{dU^C(A^C, \tilde{A}; \text{integration})}{d\tilde{A}} &= \left\{ \beta [p + (1-p)q] A^C F'(I^C) - 1 \right\} \frac{dI^C}{d\tilde{A}} \\ &\quad - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tilde{A}} \\ &\quad + \left[ -\frac{1}{R_D} + \beta + v'(D_d^C) \right] \frac{dD_d^C}{d\tilde{A}} \\ &\quad + \left\{ \frac{1}{R_D} - \beta + \beta(1-p) [1 - g'(W - D_s^C)] \right\} \frac{dD_s^C}{d\tilde{A}} \quad (\text{A.8}) \end{aligned}$$

If  $\tilde{A} > A^C$  which is the case of interest then  $D_s^C < D_s^P$ . It is also true that  $dQ^C/d\tilde{A} > 0$ ,  $dR_D/d\tilde{A} > 0$ ,  $dI^C/d\tilde{A} < 0$ ,  $dD_d^C/d\tilde{A} > 0$  and  $dD_s^C/d\tilde{A} < 0$ .<sup>29</sup> Before I simplify the above formula it useful to interpret all the terms to understand the effects of the marginal increase in  $\tilde{A}$ . Consider the **first line** of (A.8). An increase in  $\tilde{A}$  leads to decrease in investment in country  $C$ . This has two effects. First, the expected revenue of the bankers projects goes down which is represented by the first term in curly brackets. Second, the entrepreneurs in country  $C$  have now more endowment in period  $t = 0$  to consume which is represented by the second term. When the collateral constraint binds the net effect of these two effects is positive. This is because in equilibrium the marginal product of investment is smaller than the marginal financing cost of investment. This is because a unit of risky projects has additional benefit of increasing the amount of collateral for the bankers. See the first line of (A.9). Consider the **second line**. Because  $D_s^C < D_s^P$  country  $C$  is net lender of resources to country  $P$  in period  $t = 0$ . An increase in  $\tilde{A}$  leads to an increase in  $R_D$  which means that the entrepreneurs in country  $C$  have to lend less to banks in country  $H$  to get 1 unit return in the future. This is a benefit. Consider the **third line**. An increase in  $\tilde{A}$  increases demand for riskless securities in country  $C$ . This has a cost  $-1/R_D$  because the entrepreneurs give part of their endowment to buy the securities. It has two benefits: (i) the entrepreneurs get a unit of consumption at period  $t = 2$  but discount this at rate  $\beta$ ; (ii) the entrepreneurs benefit from using more riskless securities in their transactions which is captured by  $v'(D_d^C)$ . Observe that in equilibrium these two benefits exactly equal to the cost. This follows from the entrepreneurs optimality condition (5). Thus, the third line equals zero. Consider the **fourth line**. An increase in  $\tilde{A}$  leads to a decrease in riskless securities issuance  $D_s^C$ . A unit decrease in  $D_s^C$  have several effects on the welfare in country  $C$ . First, it decreases the amount of resources that the bankers in country  $C$  use to invest by  $1/R_D$ . Second, it decreases the amount of consumption goods that has to be paid out by the bankers in period  $t = 2$  which adds  $\beta$  to the welfare. Third, it decreases the reallocation of resources from the outside investors to the bankers in the bad state which has

<sup>29</sup>I don't show this formally here but it can be simply obtained by differentiating the integrated market equilibrium conditions with respect to  $A^P$ .

the following effect on welfare  $-\beta(1-p)[1-g'(W-D_s^C)]$ . That's, the output of projects that are run by the outside investors increases by  $g'(W-D_s^C)$  in the bad state while the bankers get 1 unit less of consumption goods. The bankers optimality condition with respect to riskless securities (16) can be used to simplify the fourth line. See the third line of (A.9).

$$\begin{aligned} \frac{dU^C(A^C, \tilde{A}; \text{integration})}{d\tilde{A}} &= -\beta\theta^C A^C F'(I^C) \frac{dI^C}{d\tilde{A}} \\ &\quad - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tilde{A}} \\ &\quad + \beta \frac{\theta^C}{Q^C} \frac{dD_s^C}{d\tilde{A}} \end{aligned} \tag{A.9}$$

Next, I use that  $\theta^C[D_s^C - Q^C A^C F(I^C)] = 0$  to combine the first and third line of the equation above to get

$$\frac{dU^C(A^C, \tilde{A}; \text{integration})}{d\tilde{A}} = \beta \frac{\theta^C A^C F(I^C)}{Q^C} \frac{dQ^C}{d\tilde{A}} - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tilde{A}}$$

The first term in the above formula is positive while the second one is negative which makes the overall expression positive. This completes the proof that the center benefits from the integration.

I consider the periphery next. The proof of this result uses the same idea as the proof of the previous result.

**Step 1.** Let's denote the social welfare in country  $P$  by  $U^P = U^P(A^P, A^C; \cdot)$ , where the first argument is the marginal productivity of investment opportunities in country  $P$ , the second argument is the marginal productivity of investment opportunities country  $C$ , the third argument is a dummy variable that indicates if the two countries are integrated. We are interested in computing the following difference

$$\begin{aligned} X^P &= U^P(A^P, A^C; \text{integration}) - U^P(A^P, A^C; \text{autarky}) \\ &= - \int_{A^C}^{A^P} \frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A}. \end{aligned} \tag{A.10}$$

**Step 2.** Repeating calculations in Step 2 of the previous proof I get

$$\frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} = \beta \frac{\theta^P A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tilde{A}} - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tilde{A}}$$

Because  $dQ^P/d\tilde{A} > 0$  the first term of this expression is positive. Because for  $\tilde{A} < A^P$  it is true that  $D_s^P > D_d^P$  and  $dR_D/d\tilde{A} > 0$  the second term is negative (taking into account the sign in front of this term). If I plug the above expression into (A.10) and take into account the negative sign in front of the integral the effects of the two terms in the above formula reverses. However, because the two terms have the opposite effects the net effect can be either negative or positive.

**Step 3.** Consider a case in which the marginal productivity of investment opportunities in the two countries are as follows  $(A^P, A^C) = (A + \epsilon, A)$  where  $A$  is some positive number and  $\epsilon$  is small and positive number. In this case I can write

$$X^P \approx -\frac{dU^P(A, A; \text{integration})}{d\bar{A}} \cdot \epsilon = -\beta \frac{\theta^P A^P F(I^P)}{Q^P} \frac{dQ^P}{d\bar{A}} \Big|_{(A^P, A^C)=(W, W)} \cdot \epsilon < 0$$

By continuity there exists  $\bar{A} > A^C$  such that for all  $A^P \in (A^C, \bar{A})$  it is true that  $X^P < 0$ . ■

## A.5 Proof of lemma 5.

$$\begin{aligned} \max_{\tau^P} Y - I^P + \frac{D_s^P - D_d^P}{R_D} + v(D_d^P) + \beta[p + (1-p)q]A^P F(I^P) \\ + \beta \{p [g(W) - (D_s^P - D_d^P)] + (1-p) [g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)]\}, \end{aligned}$$

subject to the following system of equilibrium conditions

$$\left. \begin{aligned} \frac{R_B}{R_D}(1 - \tau^P) - \left(p + \frac{(1-p)q}{Q^P}\right) - \frac{\theta^P}{Q^P} &= 0, \\ [p + (1-p)q] - R_B P_0^P + \theta^P &= 0, \\ g'(W - D_s^P) &= \frac{q}{Q^P}, \\ D_s^P \leq Q^P A^P F(I^P), \theta^P \geq 0, \\ \theta^P (D_s^P - Q^P A^P F(I^P)) &= 0, \\ R_D &= \frac{1}{\beta + v'(D_d^P)}, \\ P_0^P &= \frac{1}{A^P F'(I^P)}, \end{aligned} \right\} \text{periphery eq-um}$$

$$\left. \begin{aligned} \frac{R_B}{R_D}(1 - \tau^C) - \left(p + \frac{(1-p)q}{Q^C}\right) - \frac{\theta^C}{Q^C} &= 0, \\ [p + (1-p)q] - R_B P_0^C + \theta^C &= 0, \\ g'(W - D_s^C) &= \frac{q}{Q^C}, \\ D_s^C \leq Q^C A^C F(I^C), \theta^C \geq 0, \\ \theta^C (D_s^C - Q^C A^C F(I^C)) &= 0, \\ R_D &= \frac{1}{\beta + v'(D_d^C)}, \\ P_0^C &= \frac{1}{A^C F'(I^C)}, \end{aligned} \right\} \text{center eq-um}$$

$$D_d^P + D_d^C = D_s^P + D_s^C. \quad \left. \right\} \text{safe debt mkt clearing}$$

This system of fourteen equations and four constraints uniquely defines a mapping from  $\tau^P$  to fourteen variables  $P_0^P(\tau^P), P_0^C(\tau^P), I^P(\tau^P), I^C(\tau^P), Q^P(\tau^P), Q^C(\tau^P), D_s^P(\tau^P), D_s^C(\tau^P), D_d^P(\tau^P), D_d^C(\tau^P), R_D, \theta^P(\tau^P), \theta^C(\tau^P)$ . The uniqueness comes from the analysis similar to the one presented in section

3. The mapping is differentiable for any  $\tau^P$  except  $\tau^P$  for which the collateral constraints change from being binding to not being binding. Given an implicit mapping of  $\tau^P$  to all the equilibrium variables I can write the first order necessary condition by differentiating the welfare function with respect to  $\tau^P$

$$\begin{aligned} \frac{dU^P}{d\tau^P} = & -\frac{dI^P}{d\tau^P} + \left( \frac{dD_s^P}{d\tau^P} - \frac{dD_d^P}{d\tau^P} \right) \left( \frac{1}{R_D} - \beta \right) - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} + v'(D_d^P) \frac{dD_d^P}{d\tau^P} \\ & + \beta \left\{ [p + (1-p)q] A^P F'(I^P) \frac{dI^P}{d\tau^P} + (1-p) [1 - g'(W - D_s^P)] \frac{dD_s^P}{d\tau^P} \right\} = 0 \end{aligned}$$

Rearranging I get

$$\begin{aligned} \frac{dU^P}{d\tau^P} = & \left\{ \beta [p + (1-p)q] A^P F'(I^P) - 1 \right\} \frac{dI^P}{d\tau^P} \\ & - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} \\ & + \left[ -\frac{1}{R_D} + \beta + v'(D_d^P) \right] \frac{dD_d^P}{d\tau^P} \\ & + \left\{ \frac{1}{R_D} - \beta + \beta(1-p) [1 - g'(W - D_s^P)] \right\} \frac{dD_s^P}{d\tau^P} = 0 \end{aligned}$$

After plugging in the bankers and the entrepreneurs optimality conditions the regulator first condition can be written as follows

$$\begin{aligned} \frac{dU^P}{d\tau^P} = & -\beta \theta^P \left( A^P F'(I^P) \frac{dI^P}{d\tau^P} - \frac{1}{Q^P} \frac{dD_s^P}{d\tau^P} \right) - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} + \frac{R_B}{R_D} \tau^P \frac{dD_s^P}{d\tau^P} \\ = & \beta \theta^P \frac{A^P F'(I^P)}{Q^P} \frac{dQ^P}{d\tau^P} - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} + \frac{R_B}{R_D} \tau^P \frac{dD_s^P}{d\tau^P} \\ = & \beta \frac{A^P F'(I^P)}{Q^P} \frac{dQ^P}{d\tau^P} \left( \theta^P - \tau^P \frac{R^B Q^P}{R_D \tilde{c}_g^P} \right) - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} = 0, \end{aligned}$$

where the second line uses the observation that the derivative of  $\theta^P [D_s^P - Q^P A^P F(I^P)] = 0$  with respect to  $\tau^P$  equals

$$\theta^P \left[ \frac{dD_s^P}{d\tau^P} - \frac{dQ^P}{d\tau^P} A^P F(I^P) - Q^P A^P F'(I^P) \frac{dI^P}{d\tau^P} \right] = 0,$$

and the third line uses

$$\frac{1}{D_s^P} \frac{dD_s^P}{d\tau^P} = -\frac{1}{\tilde{c}_g^P Q^P} \frac{dQ^P}{d\tau^P}. \blacksquare$$

## A.6 Proof of Proposition 3.

Consider the periphery. The social welfare function change after integration equals

$$\begin{aligned} X^P = & U^P(A^P, A^C; \text{integration}) - U^P(A^P, A^C; \text{autarky}) \\ = & U^P(A^P, A^C; \text{integration}) - U^P(A^P, A^P; \text{autarky}) \\ = & - \int_{A^C}^{A^P} \frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A} \end{aligned}$$

The derivative of the social welfare function with respect to the marginal productivity of investment opportunities  $\tilde{A}$  in the center is

$$\frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} = \beta \frac{F(I^P)}{Q^P} \frac{dQ^P}{d\tilde{A}} \left( \theta^P - \tau^P \frac{R_B Q^P}{R_D \tilde{\epsilon}_g^P} \right) - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tilde{A}}$$

Using optimality condition of the regulator in country  $P$  from lemma 5 I obtain

$$\frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} = \frac{D_s^P - D_d^P}{R_D^2} \left[ \frac{dR_D}{d\tau^P} \frac{dQ^P/d\tilde{A}}{dQ^P/d\tau^P} - \frac{dR_D}{d\tilde{A}} \right]$$

where  $D_s^P - D_d^P > 0$ ,  $dR_D/d\tau^P < 0$ ,  $dQ^P/d\tilde{A} > 0$ ,  $dQ^P/d\tau^P > 0$ ,  $dR_D/d\tilde{A} > 0$ . This implies that  $dU^P(A^P, \tilde{A}; \text{integration})/d\tilde{A} < 0$ . Thus,  $X^P > 0$ .

Hence,  $X^C > 0$ . ■

## A.7 Proof of Proposition 4.

From lemma 5 the optimal level of taxes in country  $P$  satisfies

$$\beta \frac{A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tau^P} \left( \theta^P - \tau^P \frac{R_B Q^P}{R_D \tilde{\epsilon}_g^P} \right) - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} = 0. \quad (\text{A.11})$$

Similar equation holds for country  $C$

$$\beta \frac{A^C F(I^C)}{Q^C} \frac{dQ^C}{d\tau^C} \left( \theta^C - \tau^C \frac{R_B Q^C}{R_D \tilde{\epsilon}_g^C} \right) - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tau^C} = 0. \quad (\text{A.12})$$

Let's denote a solution to these equations, a Nash equilibrium, as  $(\hat{\tau}^C, \hat{\tau}^P)$ .

Next I consider the effect of the marginal change in  $\tau^P$  on the social welfare function in country  $C$  evaluated at a Nash equilibrium  $(\hat{\tau}^C, \hat{\tau}^P)$ . Repeating the algebra from lemma 5 I obtain

$$\frac{dU^C}{d\tau^P} = \beta \frac{F(I^C)}{Q^C} \frac{dQ^C}{d\tau^P} \left( \theta^C - \tau^C \frac{R_B Q^C}{R_D \tilde{\epsilon}_g^C} \right) - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tau^P} \quad (\text{A.13})$$

This formula is key to understanding the coordination failure result. A marginal increase in the taxes in the periphery has three effect: (i) it makes the welfare losses from the externality bigger (first term in the brackets); (ii) it decreases country  $C$  tax-induced bank funding costs; (iii) it decreases interest rate which makes entrepreneurs gain from investing in peripheral safe debt smaller.

Taking into account the optimality condition (A.12) I can rewrite the previous equation as follows

$$\frac{dU^C}{d\tau^P} \Big|_{(\hat{\tau}^C, \hat{\tau}^P)} = \frac{D_s^C - D_d^C}{R_D^2} \left[ \frac{dR_D}{d\tau^C} \frac{dQ^C/d\tau^P}{dQ^C/d\tau^P} - \frac{dR_D}{d\tau^P} \right] < 0 \quad (\text{A.14})$$

This expression is negative because (i) by the assumption of the proposition the center is a net buyer of safe debt  $D_s^C - D_d^C < 0$ , (ii) an increase in the tax level in the periphery decreases

the level of safe debt in the world making it more expensive which implies  $dR_D/d\tau^P < 0$ , (iii) analogously  $dR_D/d\tau^C < 0$ , (iv) an increase in taxes  $\tau^P$  increases the issuance of safe debt in the center (because the return on safe debt falls) which implies more severe fire-sale price decline (relative to fundamental value of the risky projects  $q$ )  $dQ_c^C/d\tau^P < 0$ , however, at the same time the fire-sale price in the periphery rises  $dQ_c^F/d\tau^P > 0$ . Negative sign in (A.14) implies that there is a gain for agents in country  $C$  from a marginal decrease in taxes in country  $P$ .

I can analogously compute the marginal effect of change in  $\tau^C$  on  $U^P$ .

$$\frac{dU^P}{d\tau^C} \Big|_{(\bar{\tau}^C, \bar{\tau}^P)} = \frac{D_s^P - D_d^P}{R_D^2} \left[ \frac{dR_D}{d\tau^C} \frac{dQ_c^C}{d\tau^C} - \frac{dR_D}{d\tau^C} \right] > 0$$

This expression is positive because  $D_s^P - D_d^P > 0$ ,  $dR_D/d\tau^C < 0$ ,  $dR_D/d\tau^P < 0$ ,  $dQ_c^P/d\tau^C < 0$  and  $dQ_c^F/d\tau^C > 0$ . Positive sign of this expression implies that there is gain for agents in country  $P$  from a marginal increase in taxes in country  $C$ .

Thus, the following perturbation  $d(\tau^C, \tau^P) = (-\Delta^C, \Delta^P)$ , where  $\Delta^C$  and  $\Delta^P$  are small and positive numbers, increases the social welfare functions in both countries. Hence, if the policy makers could coordinate on their decisions they could achieve higher welfare than in a Nash equilibrium by decreasing taxes in country  $P$  and increasing taxes in country  $C$ . ■

## A.8 Proof of proposition 5.

I start by defining the problem of the regulator in the periphery

$$\begin{aligned} \max_{\tau^P, \tau^C} Y - I^P + \frac{D_s^P - D_d^P}{R_D^P} + v(D_d^P) + \beta[p + (1-p)q]A^P F(I^P) \\ + \beta \{ p [g(W) - (D_s^P - D_d^P)] + (1-p) [g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)] \}, \end{aligned}$$

subject to the following system of equilibrium conditions

$$\frac{R_B}{R_D^P} (1 - \tau^P) - \left( p + \frac{(1-p)q}{Q^P} \right) - \frac{\theta^P}{Q^P} = 0, \quad (\text{A.15})$$

$$[p + (1-p)q]A^P F'(I^P) - R_B + \theta^P A^P F'(I^P) = 0, \quad (\text{A.16})$$

$$g'(W - D_s^P) = \frac{q}{Q^P},$$

$$D_s^P \leq Q^P A^P F(I^P), \theta^P \geq 0,$$

$$\theta^P (D_s^P - Q^P A^P F(I^P)) = 0,$$

$$R_D^P = \frac{1}{\beta + v'(D_d^P)}, \quad (\text{A.17})$$

$$\frac{R_B}{R_D^C} (1 - \tau^C) - \left( p + \frac{(1-p)q}{Q^C} \right) - \frac{\theta^C}{Q^C} = 0,$$

$$[p + (1-p)q]A^C F'(I^C) - R_B + \theta^C A^C F'(I^C) = 0,$$

$$g'(W - D_s^C) = \frac{q}{Q^C},$$

$$D_s^C \leq Q^C A^C F(I^C), \theta^C \geq 0,$$

$$\begin{aligned}
\theta^C(D_s^C - Q^C A^C F(I^C)) &= 0, \\
R_D^C &= \frac{1}{\beta + v'(D_d^C)}, \\
D_d^P + D_d^C &= D_s^P + D_s^C, \\
R_D^C &= (1 - \tau_f^P) R_D^P.
\end{aligned}$$

Instead of solving this problem I propose to solve less constrained problem and then show that the solution satisfies omitted constraints. The less constrained problem looks as follows

$$\begin{aligned}
\max_{D_d^P, D_s^P, I^P} Y - I^P + \frac{D_s^P - D_d^P}{R_D^P} + v(D_d^P) + \beta[p + (1-p)q]A^P F(I^P) \\
+ \beta \{p [g(W) - (D_s^P - D_d^P)] + (1-p) [g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)]\},
\end{aligned}$$

subject to the following subset of the equilibrium conditions

$$g'(W - D_s^P) = \frac{q}{Q^P},$$

$$\begin{aligned}
D_s^P &\leq Q^P A^P F(I^P), \\
\frac{R_B}{R_D^C} (1 - \tau^C) - \left( p + \frac{(1-p)q}{Q^C} \right) - \frac{\theta^C}{Q^C} &= 0, \\
[p + (1-p)q] A^C F'(I^C) - R_B + \theta^C A^C F'(I^C) &= 0, \\
g'(W - D_s^C) &= \frac{q}{Q^C}, \\
D_s^C &\leq Q^C A^C F(I^C), \theta^C \geq 0, \\
\theta^C (D_s^C - Q^C A^C F(I^C)) &= 0, \\
R_D^C &= \frac{1}{\beta + v'(D_d^C)}, \\
D_d^P + D_d^C &= D_s^P + D_s^C.
\end{aligned}$$

Observe that the regulator can directly affect the first two conditions. All the remaining conditions are affected through changes in  $D_s^P - D_d^P$  (because of the safe debt market clearing condition). These remaining conditions determine the equilibrium in the center conditional on  $D_s^P - D_d^P$ . Because only one variable from the center the peripheral welfare function and the first to constraints the only thing we need to know about the remaining conditions is how  $R_D^C$  depends on  $D_s^P - D_d^P$ . Hence, the problem can be written as follows

$$\begin{aligned}
\max_{D_d^P, D_s^P, I^P} Y - I^P + \frac{D_s^P - D_d^P}{R_D^P(D_s^P - D_d^P)} + v(D_d^P) + \beta[p + (1-p)q]A^P F(I^P) \\
+ \beta \{p [g(W) - (D_s^P - D_d^P)] + (1-p) [g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)]\},
\end{aligned}$$

subject to

$$\begin{aligned}
g'(W - D_s^P) &= \frac{q}{Q^P}, \\
D_s^P &\leq Q^P A^P F(I^P).
\end{aligned}$$

The optimal choice of  $I^P$  leads to

$$[p + (1 - p)q] A^P F'(I^P) - R_B + \theta^P A^P F'(I^P) = 0 \quad (\text{A.18})$$

The optimal choice of  $D_s^P$  leads to

$$\frac{R_B}{R_D^P} - \left( p + \frac{(1 - p)q}{Q^P} \right) - \frac{\theta^P}{Q^P} = -\theta^P D_s^P \frac{g''(W - D_s^P)}{q} + \frac{D_s^P - D_d^P}{R_D^P} \frac{R_B}{R_D^P} \frac{dR_D^C(D_s^P - D_d^P)}{dD_s^P}. \quad (\text{A.19})$$

The optimal choice of  $D_d^P$  leads to

$$\frac{1}{R_D^C} = \beta + v'(D_d^P) - \frac{D_s^P - D_d^P}{R_D^P} \frac{R_B}{R_D^P} \frac{dR_D^C(D_s^P - D_d^P)}{dD_d^P}. \quad (\text{A.20})$$

Note that

$$\frac{dR_D^C(D_s^P - D_d^P)}{dD_s^P} + \frac{dR_D^C(D_s^P - D_d^P)}{dD_d^P} = 0.$$

Finally, the complementarity slackness conditions should be satisfied

$$\theta^P [D_s^P - Q^P A^P F(I^P)] = 0.$$

I now show that the optimality conditions of the less constrained problem satisfy the condition omitted from the more constrained problem. Pick  $\tau_f^P$  such that

$$\tau_f^P = \frac{-\frac{R_D^P}{R_D^C} \frac{D_s^P - D_d^P}{R_D^C} \frac{dR_D^C(D_s^P - D_d^P)}{dD_d^P}}{1 - \frac{R_D^P}{R_D^C} \frac{D_s^P - D_d^P}{R_D^C} \frac{dR_D^C(D_s^P - D_d^P)}{dD_d^P}}. \quad (\text{A.21})$$

This  $\tau_f^P$  together with (A.20) implies (A.17). Next, (A.21) together with (A.19) and the following choice of  $\tau^P$

$$\tau^P = \theta^P \frac{\tilde{\epsilon}_g}{Q^P} \frac{R_D^P}{R_B}, \quad (\text{A.22})$$

imply (A.15). Next, (A.18) implies (A.16). Thus, I showed that the less constrained problem optimum is feasible under the more constrained problem optimum. ■