AUCTIONS, ACTIONS, AND THE FAILURE OF INFORMATION AGGREGATION

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We study a market in which k identical and indivisible objects are allocated using a uniform-price auction where $n > k$ bidders each demand one object. Before the auction, each bidder receives an informative but imperfect signal about the state of the world. The good that is auctioned is a common-value object for the bidders, and a bidder's valuation for the object is determined *jointly* by the state of the world and an action that he chooses after winning the object but before he observes the state. We show that there are equilibria in which the auction price is completely uninformative about the state of the world and aggregates no information even in an arbitrarily large auction. In the equilibrium that we construct, because prices do not aggregate information, agents have strict incentives to acquire costly information before they participate in the market. Also, market statistics other than price, such as the amount of rationing and bid distributions contain extra information about the state. Our findings sharply contrast with past work which shows that in large auctions where there is no ex-post action, the auction price aggregates information.

Keywords: Auctions, Large markets, Information Aggregation
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"We must look at the price system as a mechanism for communicating information if we want to understand its real function....The most significant fact about this system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action....by a kind of symbol, only the most essential information is passed on..." Hayek (1945).

1. Introduction

One important reason to trust markets arises from the belief that market prices accurately summarize the vast array of information held by market participants. Whether this belief is justified, that is, whether prices efficiently aggregate information dispersed among agents that are active in an economy is a central economic question addressed by past research. In certain auction markets, prices do in fact effectively aggregate dispersed information. Specifically, consider a market in which a large number of identical common-value objects are sold through a uniform-price auction. In such an auction, if the bidders each have an independent signal about an unknown state of the world and if this unknown state determines the value of the object, then the equilibrium price converges to the true value of the object as the number of objects and the number of bidders grow arbitrarily large. Therefore, the auction price reveals information about the unknown state of the world. Wilson (1977), Milgrom (1979), and Pesendorfer and Swinkels (1997) have shown that this remarkable result holds under quite general assumptions.

In many situations, however, the common value of an object is not determined solely by the unknown parameters of the environment, i.e., the unknown state of the world. Rather, the object's value is also a function of how the object is utilized; in turn, the optimal way to utilize the object can depend on the unknown state of the world. For example, suppose that a large tract of land is to be divided and sold to farmers in smaller parcels through a uniform-price auction. Each farmer who successfully acquires a parcel of land in the auction needs to decide which crop to grow (e.g., wheat or rice). However, there is uncertainty about future crop prices as well as which crop grows best on that land. Alternatively, consider a uniform-price auction in which bandwidth is sold to telecommunication companies. Each winner must decide whether to use conventional technology or adopt an unconventional new one. However, there is uncertainty about future demand drivers (such as customer tastes) which will determine which technology is more profitable. In both of these examples, the winner of an object in the auction (a piece of land in the first and bandwidth in the second) must choose an action which will itself affect the value that the winner derives from the object. Moreover, this action must be taken after the auction is finalized but before some payoff-relevant uncertainty is resolved.[1]

[1]Numerous other auctions share the characteristics of the two that we highlight here. Examples include

In the examples discussed above, if the auction price provides additional information that reduces uncertainty, i.e., if the auction price aggregates information, then the winners would make better decisions when choosing their action (which crop to grow or which technology to adopt). However, none of the past work on information aggregation in auctions explores how the information revealed by the auction price is used after the auction is completed. In contrast; in this paper we explicitly model how the information about the state of the world is used after a common-value auction is completed; in our model, the auction's winners must decide on an action in order to put the objects acquired into productive use and the optimal choice of action depends on the true state of the world. We show that such large common-value auctions have equilibria in which the equilibrium price reveals no information about the state of the world. Our result suggests that if information is useful for efficient decision making, then the equilibrium price may not aggregate all the information relevant for the decision. This finding stands in stark contrast to earlier studies which show that prices aggregate information if there is no immediate use for this information.

More specifically, we study a model in which k identical and indivisible objects are allocated using a uniform-price auction in which $n > k$ bidders each demand one unit of the good. Before the auction, each bidder receives an informative but imperfect signal about the state of the world. In the auction, bidders choose their bids as a function of their signal, the k highest bidders are allocated one unit of the object, and all bidders who win an object pay a uniform price equal to the $k+1$st highest bid. The good that is auctioned is a common-value object for the bidders and a bidder's valuation for the object is determined *jointly* by the state of the world and an action that he chooses after winning the object but before he observes the state. In a large market, if the market clearing price were to aggregate all information, then actions would be chosen efficiently and competition would necessarily drive the price of the object to its *efficient-use value*.

We explore a number of properties of markets as the numbers of bidders and the objects grow proportionately; however, our primary focus is on the informativeness of prices. An outsider who could observe the signals of an arbitrarily large number of bidders would learn the state of the world perfectly. Motivated by such an outsider's perspective, we say that prices fully aggregate information if an outsider can figure out the state of the world almost perfectly just by observing the equilibrium price of a large market.

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an auction for off-shore oil leases where the winner needs to undertake costly sunk investments in order to transform the oil reserves into productive use. However, exactly which sort of investment decision is wisest may depend on parameters unknown to the market participants. Another example is an auction for iron ore where a winner must decide on which end product he will manufacture from the ore (e.g., flat steel versus steel rods). However, the ore's value to the winner will depend crucially on future end product prices and, therefore, the product mix that he chooses to manufacture.

We present two main results. *In our first main result*, we construct a particular sequence of symmetric equilibria in which, as the market grows arbitrarily large, the limit price conveys no information about the true state of the world and remains strictly below the efficient use-value of the object. Moreover, we show that such a sequence of equilibria can be constructed for a generic set of parameter values. In the equilibria we construct, a strictly positive fraction of agents chooses the wrong action because the price conveys no new information. Therefore, inefficiency persists even in a large market whose outcome would have been efficient if one could observe all of the bidders' signals. Also, because the equilibrium price does not convey new information, agents have strict incentives to acquire costly information both before they participate in the auction and after the objects have been allocated.

A prominent property of the equilibrium which we construct is that equilibrium bids are nondecreasing in the signal that an agent receives. In order to explore the robustness of our first result, we also study arbitrary symmetric equilibria in which the bidding function is monotonic in the signal that agents receive. *In our second result*, we characterize equilibrium behavior in any symmetric equilibrium in which the bidding function is monotonic and we use this characterization to show that no sequence of such equilibria can fully aggregate information. In any symmetric equilibria where the bidding function is monotonic, the price fails to aggregate information and remains below the efficient-use value of the object.

In a nutshell, our results suggest that the auction price may not be a very good aggregator of information if the information content of this price is needed to make decisions that affect the value of the objects. In our model, market statistics other than price, such as the amount of rationing and bid distributions, are informative. Therefore, whether these statistics are observed after an auction is finalized can affect how much information is aggregated by prices.[2] Moreover, dynamic models in which traders engage in multiple rounds of activities may augment the accumulation of useful information.

Relation to the literature. This paper is closely related to earlier work which studies information aggregation in large auctions. Wilson (1977) studied second-price auctions with common value for one object for sale, and Milgrom (1979) extended the analysis to any arbitrary number of objects. Both of these papers show that as the number of bidders gets arbitrarily large, price converges to the true value of the object, but only provided that there are bidders with arbitrarily precise signals about the state of the world. Pesendorfer and Swinkels (1997) further generalize the previous analysis to the case where there are no arbitrarily precise signals. They show that prices converge to the true value of a common-value object in all symmetric equilibria if and only if both the number of identical objects and the number of bid-

[2] This is still an open and interesting question. The transparency of the market changes the incentives of the bidders at the first place, it may cause them to conceal their own information.

ders who are not allocated an object grow without bound. Pesendorfer and Swinkels (2000) generalize the analysis in Pesendorfer and Swinkels (1997) to a mixed private, common-value environment. Finally, Kremer (2002) shows that the information aggregation properties of auctions are more general than the particular mechanisms studied before; he does this by providing a unified approach that uses the statistical properties of certain order statistics.[3] The model that we present in this paper is closest to Pesendorfer and Swinkels (1997). The main difference from theirs is that in our model the object's value is jointly determined by the unknown state of the world and the action that the owner of the object later takes.[4]

Our work also relates to the literature on costly information acquisition in rational-expectations models, such as Grossman and Stiglitz (1976, 1980) and Grossman (1981). These papers explain the conceptual difficulties in interpreting prices as both allocation devices and information aggregators. Specifically, they argue that if consumers and producers need to undertake a costly activity in order to acquire information, then equilibrium prices cannot reveal the state of the world perfectly. Their reasoning is as follows: if prices were to reveal the state perfectly, then no agent would have an incentive to pay for information in the first place; but if no agent acquires information, then the prices cannot reveal the state as there is no information to aggregate. However, as was the case for auction markets, these papers do not explicitly consider how the information revealed by the market price could be used by the market participants once they have completed their trade in the market. In our model, since prices do not aggregate information, agents have a strict incentive to acquire information. This finding contrasts with the findings of Grossman and Stiglitz (1980), who argue that agents have no incentive to acquire information precisely because prices are so efficient in aggregating information.

Finally, our model is related to work by Bond and Eraslan (2010) which shows that trade is possible between two agents with the same preferences if the value of the object traded is jointly determined by an unknown state of the world and an ex-post action that the eventual owner of the object will undertake. In their model, trade is precluded by a no-trade theorem without any ex-post action. It is the ex-post action and the consequent value of information that lead to the possibility of trade. Our model shares the feature that the eventual owner of

[3]Also, see Hong and Shum (2004) for a calculation of the convergence rate of prices to the true value; Jackson and Kremer (2007), which shows that the result of Pesendorfer and Swinkels (1997) does not generalize once there are individualized prices, i.e., when one considers discriminatory price auctions; and Kremer and Skrzypacz (2005) for related results on the properties of order statistics.

[4]There is extensive work on the role of prices in various other market contexts. For example, see Reny and Perry (2006) and Cripps and Swinkels (2006) for work on the information aggregation properties of prices in large double auctions; Vives (2011) and Rostek and Weretka (2010) for the information aggregation properties of markets in which the objects are divisible; Rubinstein and Wolinsky (1985, 1990), Osborne and Rubinstein (2010), Lauermann (2007), Lauermann and Wolinsky (2011, 2012), Golosov et al. (2011), Ostrovsky (2009) for work on the properties of the equilibrium price in search markets.

an object undertakes an action once trading is complete. However, whereas they consider a bilateral bargaining framework and focus on the possibility of trade, we analyze an auction framework with a large number of strategic bidders and focus on information aggregation.

2. Model

We consider a sealed-bid, uniform-price auction. In this auction, there are n bidders with unit demand and k identical objects. We denote the ratio of objects to bidders (i.e., market tightness) in this auction by $\kappa := \frac{k}{n} < 1$. The set of states of the world is $\Omega := \{L, R\}$ and we denote a generic element of this set by ω. The state of the world is drawn according to a common prior $\pi \in [0, 1]$, where π denotes the prior probability that the state is R, and $1 - \pi$ denotes the prior probability that the state is L. Each bidder i observes a private signal s_i that belongs to the set of signals $S = [0, 1]$, and submits a bid $b_i \in [0, \infty)$. Each of the k highest bidders receives an object and is called a winner; all other bidders are called losers.[5] Each winner pays a price p which is equal to the $(k + 1)^{st}$ highest bid.

The payoff of a bidder who does not win an object is equal to zero. We assume that a bidder who wins an object must choose an action from a finite set of actions denoted by A. This action, together with the state of the world, determines the winner's valuation for the good. Although all our of arguments go through with an arbitrary, finite number of actions, to keep exposition simple, *we assume that* $A = \{l, r\}$. A winning bidder's payoff is jointly determined by the auction price p, the action that he chooses $a \in A$, and the state of the world $\omega \in \Omega$. In particular, we assume that a winning bidder's payoff is equal to $v(a, \omega) - p$, where the function $v(a, \omega)$ gives the winner's valuation for the object. In what follows, we assume, without loss of generality, that $v(r, R) \geq v(l, L)$ and we make the following main assumption:

Assumption 1 *The valuation function satisfies the following two inequalities:*

$$v(l, L) > v(r, L), \tag{1}$$

$$v(l, L) > v(l, R). \tag{2}$$

Note that if the valuation function does not satisfy inequality (1), then r is a weakly dominant action. Also, if the valuation function does not satisfy inequality (2), then a bidder's valuation for the good is higher in state R than in state L regardless of the action he chooses. In what follows, we normalize the valuation function such that

$$v(l, R) = v(r, L) = 0. \tag{3}$$

[5] To rank bids that are tied, nature picks a ranking of bidders at random with each ranking equally likely.

This normalization is without loss of generality if a valuation function satisfies Assumption 1. Under this normalization, Assumption 1 requires that $v(l, L) > 0$ and $v(r, R) > 0$, or in words, that the bidder's valuation for the good be positive if his action matches the state of the world, and his valuation for the good be zero if his action does not match the state of the world.

REMARK 1 *Assumption 1 is the substantive assumption which allows us to argue that information is not aggregated in our model. The main implication of Assumption 1 which we use in many of our arguments is the fact that a bidder's expected valuation for the good is a nonmonotonic function of the probability that he assigns to state R. In contrast, if either inequality (1) or (2) is not satisfied, then a bidder's expected valuation for the good is a monotonic function of the probability that he assigns to state R. In either of these two cases, Pesendorfer and Swinkels (1997)'s findings apply and therefore information is aggregated in every symmetric equilibrium of a large market. See section 5.1 for a more detailed discussion of this nonmonotonicity.*

2.1. Signals. The set of signals is $S := [0, 1]$, and the bidders' signals are independently and identically distributed conditional on the state of the world. Each bidder's signal distribution has a cumulative distribution function $F(.|w)$ with a *continuous density function* $f(.|w)$ for each $w \in \Omega$. If a bidder believes that the probability of state R is p, then we say that the bidder's likelihood ratio is $\rho := p/(1-p)$. For a bidder who receives signal $s \in [0, 1]$, we denote his likelihood ratio, slightly abusing notation, to be $\rho(s)$, defined as follows:

$$\rho(s) = \frac{\pi}{1-\pi}\frac{f(s|R)}{f(s|L)} = \rho_0\frac{f(s|R)}{f(s|L)},$$

where $\rho_0 := \pi/(1-\pi)$ denotes the prior likelihood ratio derived from the common prior π.

ASSUMPTION 2 (MLRP) *The likelihood ratio $\rho(s)$ is a strictly increasing function of s.*

Note that an implication of the MLRP assumption is that $f(0|L) \neq f(0|R)$.

ASSUMPTION 3 (Limited individual information) *There exists a number $\eta > 0$ such that $\eta < f(s|w) < \frac{1}{\eta}$ for every $s \in S, w \in \Omega$.*

This assumption requires that signals convey only a bounded amount of information. Hence, there is no bidder who possesses arbitrarily precise information based solely on the bidder's signal.

In what follows, we refer to the m^{th} highest value among n signals by Y_n^m. We define the unique signals $s_R^\kappa \in S$ and $s_L^\kappa \in S$ such that $F(s_R^\kappa|R) = 1-\kappa$ and $F(s_L^\kappa|L) = 1-\kappa$. Recall that $\kappa < 1$ is the market tightness, i.e., the ratio of objects to bidders. Intuitively, in a large market there are as many bidders with signals above s_R^κ as there are objects in state R. Therefore, if we were to allocate the objects to the bidders with higher signals first, then, in state R, the bidders who receive an object would be exactly those bidders whose signals exceed s_R^κ.

2.2. Strategies, equilibrium, and values. Each bidder submits a bid after observing his signal. A bidding strategy for player i is a measure H_i on $[0,1]\times[0,\infty)$ with marginal distribution $F(s) = \pi F(s|R) + (1-\pi)F(s|L)$ on its first coordinate (see Milgrom and Weber (1985)). The set of all bidding strategies is Σ. A strategy is pure if there is a function $b : [0,1] \to [0,\infty)$ such that $H(\{s, b(s)\}_{s\in[0,1]}) = 1$.[6]

Each winner chooses an action from the set of actions A. Hence, the action strategy is a mapping from a bidder's signal, his bid, and the winning price to an action, $a_i : S \times [0,\infty)\times[0,\infty) \to A$. Since the bidders' actions do not affect other bidders' payoffs, confining attention to pure strategy actions is without loss of generality.

In the rest of the paper, we will restrict our attention to pure symmetric Nash equilibria, which are equilibria where each bidder uses the same pure bidding strategy, i.e., $b_i = b_j$ for every two bidders i and j. The term $\Pr_b$ denotes the probability distribution induced by the pure and symmetric bidding strategy profile where each bidder uses the bidding strategy b over states of the world, signal and bid distributions, allocations, and prices.

Below we define a bidder's value as a function of his beliefs but we work with the likelihood ratio instead of working directly with beliefs for analytic convenience.[7] Below we introduce the value function u which gives a bidder's expected valuation for an object as a function of the likelihood ratio ρ. In particular, let $u : [0,\infty] \to \mathbb{R}$ be the function defined by

$$u(\rho) = \max_{a\in\{l,r\}} \left\{ \frac{1}{\rho+1} v(a,L), \frac{\rho}{\rho+1} v(a,R) \right\}.$$

This function gives the bidder's expected value for the object as a function of his beliefs about the state of the world, expressed as the relative likelihood ratio about the state of the world. Note that $u(0) = v(l,L)$ and $\lim_{\rho\to\infty} u(\rho) = v(r,R)$. Let $\rho^* \in (0,\infty)$ be the unique

[6] Of course, if b represents H, then so will any function that agrees with b for almost every $s \in [0,1]$.

[7] The whole analysis could be redone working directly with beliefs as there is a one-to-one mapping between likelihood ratios and beliefs.

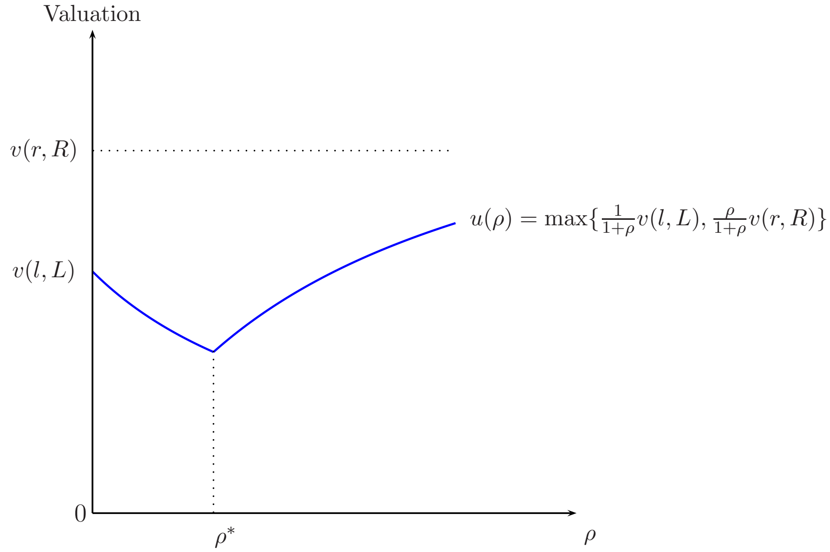

Figure 1: Continuation value as a function of the likelihood ratio of the bidder who wins a unit in the auction, before he makes the action choice. Assumption 1 implies that $u(\rho)$ is a nonmonotonic function which is minimized at ρ^* as depicted here.

solution to the equation:

$$\frac{1}{\rho+1}v(l,L) = \frac{\rho}{\rho+1}v(r,R).$$

This cutoff is the likelihood ratio that makes a bidder indifferent between action l and r. Such a cutoff always exists because of Assumption 1. *In what follows, we extensively use the fact that the value function $u(\cdot)$ is strictly decreasing in the interval $[0,\rho^*]$ and strictly increasing in the interval $[\rho^*,\infty)$.* See figure 1 for a depiction of the value function.

3. Large Markets and the Failure of Information Aggregation

In this section we present our main result as Theorem 1. In Theorem 1, we constuct a sequence of equilibria for auctions $\{\Gamma_n\}_{n=1}^{\infty}$ where the n^{th} auction Γ_n has n bidders and $\lfloor \kappa n \rfloor$ objects for sale.[8] In the remainder of the paper, we will proceed as if κn is an integer for expositional simplicity. We assume that the other parameters of the auctions, i.e., (v, F, π, κ), are constant along the sequence and satisfy all the assumptions that we have already made. For the sequence of equilibria we construct, equilibrium price reveals no information about the state of the world at the limit where there is an arbitrarily large number of bidders. Although the limit equilibrium price reveals no information, bidders *do* learn some information about the state of the world through rationing. However, the amount of information that they learn is limited, and incorrect ex-post actions are played frequently.

[8] The term $\lfloor \kappa n \rfloor$ refers to the highest integer not bigger than κn.

3.1. Information aggregation. Here we formally define information aggregation and its failure. Our object of study is a sequence of bidding functions $\boldsymbol{b} = \{b_n\}_{n=m}^{\infty}$. We say that the sequence $\boldsymbol{b}$ is an equilibrium sequence if b_n is part of a symmetric Nash equilibrium of Γ_n for each n.

Suppose that the number of bidders n is large. In this case, the law of large numbers implies that observing the signals $(s_1, ..., s_n)$ conveys precise information about the state of the world $\omega \in \{L, R\}$. The bidding function b_n determines a price p^* for the auction Γ_n given any realization of signals $(s_1, ..., s_n)$. We say that information is aggregated in the auction if this price p^* also conveys precise information about the state of the world. More precisely, (i) if the likelihood ratio $\frac{\Pr_{b_n}(p^*|R)}{\Pr_{b_n}(p^*|L)}$ is close to zero (i.e., if it is arbitrarily more probable that we observe such a price when $\omega = L$), then an outsider who observes price p^* learns that the state is L. Alternatively, (ii) if the likelihood ratio $\frac{\Pr_{b_n}(p^*|R)}{\Pr_{b_n}(p^*|L)}$ is arbitrarily large, then an outsider who observes price p^* learns that the state is R. If the probability that we observe a price that satisfies either (i) or (ii) is arbitrarily close to one, then we say that the equilibrium sequence $\boldsymbol{b}$ fully aggregates information. Conversely, if the likelihood ratio $\frac{\Pr_{b_n}(p^*|R)}{\Pr_{b_n}(p^*|L)}$ is close to one, i.e., if we are equally likely to observe price p^* in either of the two states, then an outsider who observes price p^* learns arbitrarily little information about the state of the world. If the probability that we observe such a price is arbitrarily close to one, then we say that the equilibrium sequence $\boldsymbol{b}$ aggregates no information. The precise definitions are as follows:

Definition 1 *An equilibrium sequence* **b** ***aggregates no information*** *if, for any $\epsilon > 0$,*

$$\lim_{n\to\infty} \Pr_{b_n}\left(p_n \in \left\{p \in [0,\infty) : \frac{\Pr_{b_n}(p|R)}{\Pr_{b_n}(p|L)} \in (1-\epsilon, 1+\epsilon)\right\}\right) = 1.$$

An equilibrium sequence **b** ***fully aggregates information*** *if, for any $\epsilon > 0$,*

$$\lim_{n\to\infty} \Pr_{b_n}\left(p_n \in \left\{p \in [0,\infty) : \frac{\Pr_{b_n}(p|R)}{\Pr_{b_n}(p|L)} \in [0,\epsilon) \cup (1/\epsilon, \infty))\right\}\right) = 1.$$

Remark 2 *Our definition of information aggregation differs from the definition provided by Pesendorfer and Swinkels (1997). In their model, the state of the world is defined as the value of the object and each bidder receives a signal about that value. Therefore, they say that information is aggregated if the equilibrium prices converge to the true value of the object (i.e., if the price converges to the state of the world) as the market grows large. In their setup, each state represents a distinct value for the object, so when information is aggregated in their model with their definition, then it is also aggregated under our definition. Therefore,* *if information aggregation fails using our definition, then it will also fail under the definition of Pesendorfer and Swinkels (1997).*

3.2. Failure of information aggregation. Our main theorem shows that if, in addition to Assumptions 1-3, two conditions are satisfied, then there exists an equilibrium sequence **b** which aggregates no information. The first condition that we require for the theorem is as follows:

Condition 1 $\rho(0) > \rho^*$.

If this condition is satisfied, then all the bidders would choose action r if they acted solely on the information contained in their private signal.

Recall that $s_R^\kappa \in S$ is the signal such that $F(s_R^\kappa|R) = 1 - \kappa$. The second condition we require for the theorem is as follows:

Condition 2 $u(\rho(s_R^\kappa)) < v(l, L)$.

Under this condition, if a bidder who received signal s_R^κ chooses an action based solely on this signal, then this bidder's expected valuation is lower than $v(l, L)$. Therefore, a sufficiently strong additional signal in favor of state L can increase the expected valuation of such a bidder. See figure 2 for a depiction of a situation where both Conditions 1 and 2 are satisfied.

Our main theorem is as follows:

Theorem 1 *Suppose that Assumptions 1-3 hold. If Condition 1 and Condition 2 also hold, then there exists an equilibrium sequence* $\boldsymbol{b}$ *which reveals no information.*

We prove this theorem by constructing an equilibrium sequence which aggregates no information. In this construction, each bidding function b_n in the equilibrium sequence $\boldsymbol{b}$ is a nondecreasing function of s. In this construction, Condition 1 allows us to construct an equilibrium sequence in which each bidding function b_n is nondecreasing in s. Condition 2, on the other hand, allows us to ensure that the equilibrium sequence that we construct aggregates *no information* about the state of the world.

In the particular equilibrium that we construct, information is not aggregated by the price because of the existence of pooling (see figure 3). Pooling by bidders with a range of different signals at a certain *pooling bid* makes the equilibrium price less sensitive to the information of the bidders and thus leads to *limited learning*. We now provide some intuition for Theorem 1 by *i*) arguing that if the bidding function is nondecreasing, then there must be pooling; and *ii*) arguing that pooling can be sustained in equilibrium.

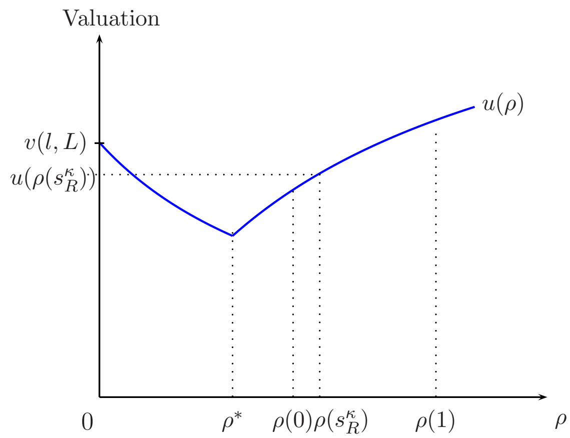

Figure 2: This figure shows the initial range of beliefs, expressed as likelihood ratios, on the belief-value graph. Note that the bidders who assign higher probability to state R, i.e., the bidders with higher signals, are the bidders with higher value. Moreover $u(\rho(s_R^\kappa)) < v(l, L)$.

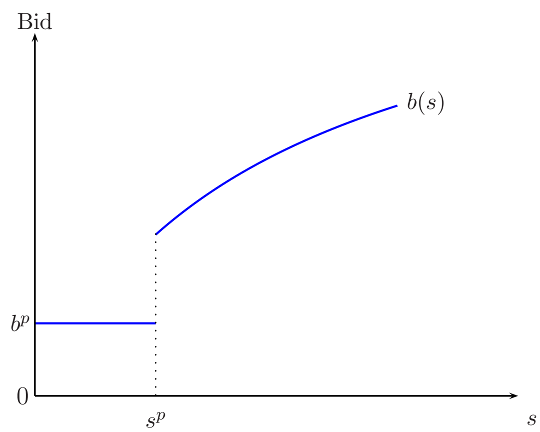

Figure 3: A typical equilibrium bidding function. Buyers with signals below a cutoff s^p bid a pooling bid b^p, and those with signals above s^p bid according to the bid function in Pesendorfer and Swinkels (1997), i.e., $b(s) = \rho(s_1 = s, Y_{n-1}^k = s)$ for $s > s^p$.

Assume that the bidding function is strictly increasing, i.e., assume that there is no pooling bid, and consider a bidder who receives the lowest signal. If the auction is sufficiently large, when the auction price is equal to this bidder's bid, it must be the case that this bidder is almost certain that the state is L. This is also true for a bidder who receives a signal $\epsilon > 0$ that is arbitrarily close to zero. However, then the bidder who receives signal zero would be willing to submit a bid that is greater than the bid of a bidder who receives signal ϵ, because the bidder with signal zero is more convinced that the state is L than the bidder with signal ϵ. This, however, contradicts the assumption that the bidding function is strictly increasing. Note that this argument crucially hinges on the *nonmonotonicity* of the value function (see figure 2 and also section 5.1).

In our construction, the pooling bid is sustained because agents have an incentive to learn the state in order to use this information while choosing their action. Specifically, when the price is equal to the pooling bid, objects are allocated using *rationing* among the bidders who choose the pooling bid.[9] A bidder who chooses the pooling bid and wins an object through rationing at a price equal to the pooling bid obtains more information about the state of the world, compared to the case in which he instead chooses a higher bid, avoids rationing, and wins an object. This is because winning an object when rationing is applied is more likely in state L than in state R. In other words, rationing is a lottery whose odds depend on the state of the world. Moreover, a sufficiently strong signal in favor of state L is valuable for some agent because the value function is *nonmonotonic*. If a bidder who chooses the pooling bid increases his bid, then he acquires an object more frequently because he avoids rationing when the price is equal to the pooling bid. However, in this case he forgoes the extra piece of information that comes from winning under rationing. Because this extra piece of information is sufficiently valuable for bidders who choose the pooling bid, these bidders refrain from increasing their bid even though they make strictly positive profits at the pooling bid.

3.3. Sketch of the construction. In this section, we sketch the ideas behind constructing the equilibrium sequence **b** whose existence Theorem 1 claims. Specifically, we construct an equilibrium in which no information is aggregated in a hypothetical market with a continuum of bidders with mass one and a continuum of objects with mass κ. Focusing on a hypothetical market with a continuum of bidders allows us to capture the main properties of the equilibrium sequence **b** for a market with a finite but large number of bidders while allowing us to avoid the more technical details involved in describing such equilibria for finite markets. In what follows we repeatedly use the fact that the value function $u(\cdot)$ is strictly decreasing in the interval $[0, \rho^*]$ and strictly increasing in the interval $[\rho^*, \infty)$.

We construct an equilibrium in which the equilibrium bidding function b is constant on the interval $[0, s^p)$ for some cutoff signal $s^p > s_R^\kappa$, which we calculate further below (i.e., $b(s) = b^p$ for all $s \in [0, s^p)$), and the bidding function is strictly increasing on the interval $(s^p, 1]$. We call the bid b^p (i.e., the bid submitted by all bidders with signals in the interval $[0, s^p)$) the *pooling bid* or *pooling price*. See figure 3 for a depiction of the bidding function b. In this equilibrium, the following properties hold true:

(i) The auction price is equal to the pooling price in either state of the world, and hence conveys no additional information about the state of the world. The auction price is always equal to the pooling price because s^p exceeds s_R^κ. The fact that s^p exceeds s_R^κ implies that

[9] See Stiglitz and Weiss (1981), Bester (1985, 1987), and Lauermann and Wolinsky (2012) for other models where there is rationing in equilibrium.

for any price $p' > b^p$, the mass of bidders who submit a bid greater than or equal to p' is strictly less than the mass of objects available, i.e., the measure of the set $\{s : b(s) \geq p'\}$ is strictly less than κ in both states.

(ii) Bidders with signals that exceed s^p, i.e., those bidders whose bid exceeds the pooling price, are always allocated an object and always choose action r. These bidders choose action r because they obtain no new information from the auction price and because choosing r is optimal based solely on their private signal.

(iii) Bidders with signals less than s^p who are allocated an object, i.e., the bidders who bid the pooling price, take action l. Although the price conveys no information about the state, the fact that a bidder wins an object by bidding the pooling price is a strong signal favoring state L which induces that bidder to choose action l. Winning an object by bidding the pooling price is a strong signal favoring state L because the mass of bidders bidding the pooling price exceeds the mass of objects to be allocated to bidders who bid the pooling price. Moreover, a bidder is more likely to be allocated a good in state L than in state R. We discuss this issue in more detail below.

We now discuss how to calculate the cutoff signal s^p. The cutoff signal s^p is the signal which leaves a bidder indifferent between bidding the pooling bid and bidding slightly above the pooling bid. As a first step in calculating s^p, we calculate a bidder's payoff if he bids slightly above the pooling bid, and if he bids the pooling bid and wins an object.

Payoff from bidding slightly above the pooling bid. If a bidder bids above the pooling bid, then she wins an object with certainty. The posterior belief of a bidder who wins an object by bidding above the pooling bid is equal to her initial beliefs. This is because the auction price is always equal to the pooling bid in this equilibrium and conveys no information. Consequently, the expected value of the object to a bidder with signal s if she bids above the pooling bid is $u(\rho(s))$.

Payoff from bidding the pooling bid. We now calculate the value of the object for a bidder who receives the cutoff signal s^p if he bids the pooling bid and wins a unit, when $s^p \geq s_R^\kappa$. In such an event, this bidder has an *extra* piece of information, which comes from the fact that he wins a unit while bidding the pooling bid. In particular, a fraction $1 - F(s^p|\omega)$ of bidders bid strictly above the pooling bid and each wins an object with certainty regardless of the state. The fraction of objects that remains to be delivered to bidders who choose the pooling bid is $\kappa - (1 - F(s^p|\omega))$. Since the number of objects remaining to be delivered is less than the number of bidders, there is *rationing* among the bidders at the pooling bid. Consequently, the belief of type s^p (represented as the likelihood ratio) if he bids the pooling bid and wins the object is as follows:

$$\rho^p(s^p) := \rho(s^p)\frac{\kappa-(1-F(s^p|R))}{F(s^p|R)}/\frac{\kappa-(1-F(s^p|L))}{F(s^p|L)} = \rho(s^p)\frac{\kappa-(1-F(s^p|R))}{\kappa-(1-F(s^p|L))}\frac{F(s^p|L)}{F(s^p|R)},$$

where the ratio $\Delta(s^p) := \frac{\kappa-(1-F(s^p|R))}{F(s^p|R)}/\frac{\kappa-(1-F(s^p|L))}{F(s^p|L)}$ reflects the extra information that a bidder learns from winning an object at the pooling bid. If a bidder with signal s^p bids the pooling bid and wins the object, then the expected value of the object to him is equal to $u\left(\rho^p(s^p)\right)$.

Remark 3 *It is straightforward to verify that $\Delta(s^p) < 1$, that is, winning an object at the pooling bid is more likely in state L than in state R; winning an object at the pooling bid is therefore an additional signal in favor of state L. In the context of the auction models of Pesendorfer and Swinkels (1997) or Milgrom and Weber (1982), the fact that $\Delta(s^p) < 1$ is commonly referred to as the* ***loser's curse****.[10] Intuitively, the loser's curse holds because if the state is L, then the MLRP assumption implies that fewer bidders choose a bid which exceeds the pooling bid, and therefore more goods are left over to be allocated to the bidders who choose the pooling bid.*

As we stated above, the cutoff signal s^p is the signal which leaves a bidder indifferent between bidding the pooling bid and bidding slightly above the pooling bid. In other words, the cutoff signal is defined implicitly by the following equation:

$$u\left(\rho(s^p)\right) = u\left(\rho^p(s^p)\right).$$

We now argue that this cutoff signal is unique. Specifically, we show that there is a unique signal $s > s_R^\kappa$ such that $u\left(\rho(s)\right) = u\left(\rho^p(s)\right)$, and we denote this signal by s^p. Note that $\rho^p(s_R^\kappa) = 0$ and $\lim_{s\searrow s_R^\kappa}\rho^p(s) = \rho^p(s_R^\kappa)$. By *Condition 2*, we have $v(l,L) > u(\rho(s_R^\kappa))$, and hence, $u(\rho^p(s_R^\kappa)) = u(0) = v(l,L) > u(\rho(s_R^\kappa)$. Let s^* denote the unique signal such that $\rho^p(s^*) = \rho^*$, and note that $s^* \in (s_R^\kappa, 1)$. Uniqueness of s^* follows because $\rho^p(s)$ is an increasing function, and $\rho^p(s_R^\kappa) = 0$ and $\rho^p(1) = \rho(1) > \rho^*$. The function $u(\rho(s))$ is strictly increasing in s, and $u(\rho(s)) > u(\rho^*) = u(\rho^p(s^*))$ for all s. The function $u(\rho(s))$ is strictly increasing because $\rho(s) > \rho^*$ (by Condition 1), because $\rho(s)$ is strictly increasing in s, and because $u(\rho)$ is strictly increasing for any $\rho \in (\rho^*, \rho]$. Also, the function $u(\rho^p(s))$ is strictly decreasing in s for all $s \in [s_R^\kappa, s^*]$, is strictly increasing in s for all $s \in [s^*, 1]$, and reaches its minimum at $u(\rho^p(s^*)) = u(\rho^*)$. Consequently, the two functions must cross at some point $s^p \in (s_R^\kappa, s^*)$.

[10]The loser's curse is defined in the setting with finitely many bidders; however, the idea extends naturally to the hypothetical setting with a continuum of bidders.

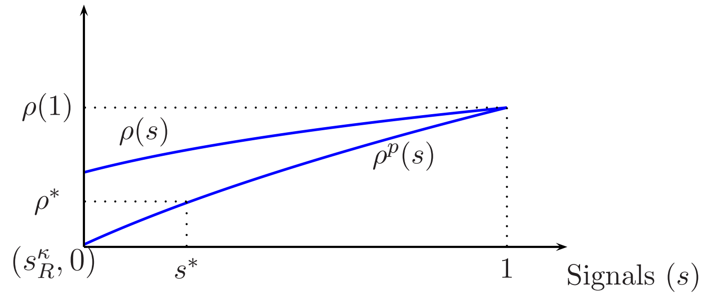

Figure 4: This figure depicts the functions ρ and ρ^p, in the range $[s_R^\kappa, 1]$. Notice that $\rho(s) \geq \rho^p(s)$; $\rho^p(s_R^\kappa) = 0$; and $\rho(1) = \rho^p(1)$.

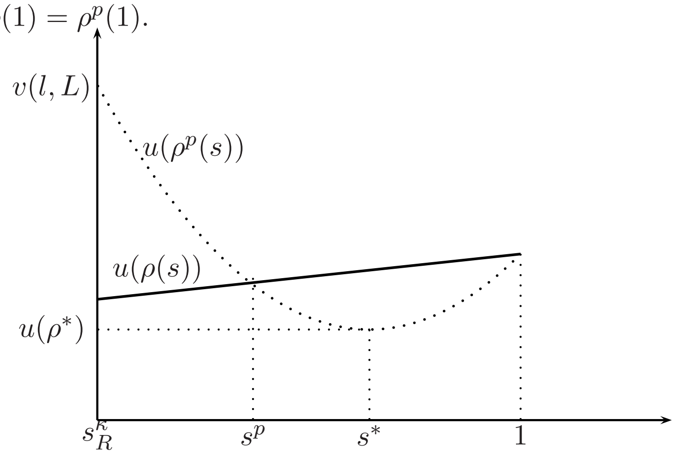

Figure 5: This figure depicts the value of the object to the cutoff type as a function of the choice of the cutoff type.

There is a unique such point, s^p, because $\rho^p(s) = \rho(s)\Delta(s) < \rho(s)$ for all $s \in [s_R^\kappa, 1)$. See figures 4 and 5 for depictions.

We now check that bidders will not want to deviate from the equilibrium we described. We first argue that bidders with signals lower than s^p cannot profitably deviate from the equilibrium by choosing a bid that exceeds the pooling bid. If a bidder with signal $s < s^p$ deviates and bids above the pooling bid, then she wins an object with certainty and pays the pooling bid $b^p = u(\rho(s^p))$. In this case, her posterior and prior likelihood ratios coincide and are equal to $\rho(s)$. However, *Condition 1* implies that $\rho^* < \rho(s) < \rho(s^p)$, and therefore we have $u(\rho(s)) < b^p = u(\rho(s^p))$, i.e., the auction price exceeds the expected valuation, conditional on winning, of the bidder with signal s. See figure 6 for a depiction of this argument for the case of $s = 0$.

We now argue that a bidder with signal $s > s^p$ cannot profitably deviate from equilibrium by choosing the pooling bid. If the bidder sticks to the equilibrium strategy, then she wins an object with certainty and her payoff is equal to $u(\rho(s)) - b^p$, a payoff which is strictly

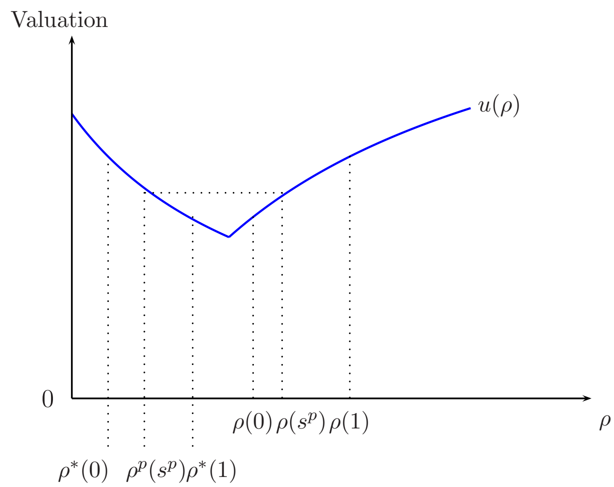

Figure 6: This figure depicts the posterior beliefs of bidders with signals 0, s^p, and 1 under two cases: (i) If they bid the pooling bid and win the object. In this case, their posterior likelihood ratios are $\rho^*(0) = \rho(0)\Delta(s^p)$, $\rho^p(s^p) = \rho(s^p)\Delta(s^p)$, and $\rho^*(1) = \rho(1)\Delta(s^p)$. ($ii$) if they bid above the pooling bid and win a unit at the pooling price. In this case, the bidders obtain no new information and therefore their posterior and prior likelihood ratios coincide, i.e., their posterior likelihood ratios are also equal to $\rho(0)$, $\rho(s^p)$, and $\rho(1)$. Bidders with signal 0 strictly prefer to bid the pooling bid, those with signal s^p are indifferent between bidding the pooling bid and above it, and those with signal 1 strictly prefer to bid above the pooling bid. Note that the pooling bid satisfies the equality $b^p = u(\rho(s^p)) = u(\rho^p(s^p))$.

positive. If she deviates instead and chooses the pooling bid, then, conditional on winning, her posterior is equal to $\rho^*(s) := \rho(s)\Delta(s^p)$. Note that $\rho^p(s^p) < \rho^*(s) < \rho(s)$. To see that this deviation is not profitable, consider two cases. First, if $\rho^*(s) \geq \rho^*$, then $u(\rho^*(s)) < u(\rho(s))$. This is because $\rho^*(s) = \rho(s)\Delta(s^p) < \rho(s)$ and because $u(\cdot)$ is increasing on $[\rho^*, \infty)$. Alternatively, if $\rho^*(s) \leq \rho^*$, then $u(\rho^*(s)) - b^p = u(\rho(s)\Delta(s^p)) - u(\rho^p(s^p)) < 0$. This is because $\rho^*(s) = \rho(s)\Delta(s^p) > \rho(s^p)$ and because $u(\cdot)$ is decreasing on $[0, \rho^*]$. See figure 6 for a depiction.

3.4. Properties of the equilibrium. There are a number of novel properties of the equilibrium that we constructed for Theorem 1. In particular, the properties listed below are satisfied as the number of bidders grow arbitrarily large. Note that none of these properties is present in a standard auction where there are no ex-post actions.

(i) The equilibrium price *aggregates no information* even in an arbitrarily large market.

(ii) Agents *learn from their own bids*. The posterior beliefs of the bidders who win an object depend on their bid. In particular, the posterior beliefs of the bidders who win an object by bidding above the pooling bid is equal to their prior belief. This is because the auction price reveals no new information. In contrast, the posterior of bidders who win an object by bidding the pooling bid is equal to their prior belief, $\rho(s)$, multiplied by the constant $\Delta(s^p) \in (0, 1)$. Hence, a bidder's bid affects the information that he has if he wins a unit.

(iii) A strictly positive fraction of bidders take the *wrong action in equilibrium* because prices convey no new information and because bidders' posterior beliefs are heterogeneous. Therefore, inefficiency persists even in a large market in which the outcome would have been efficient if one could use all of the signals observed by the bidders. In particular, *irrespective of the state of the world*, all bidders who win an object at the pooling bid choose action l and all other bidders who win an object choose action r. Consequently, the proportion of bidders choosing the wrong action is equal to $1 - F(s^p|L)$ and $\kappa - (1 - F(s^p|R))$ when the state of the world is L and R, respectively. Note that the total expected surplus in the equilibrium that we construct is equal to $\pi v(r, R) (1 - F(s^p|R)) + (1 - \pi) v(l, L) (\kappa - (1 - F(s^p|L)))$. Because $\rho(0) > \rho^*$ (by Condition 1), the equilibrium surplus is strictly decreasing in s^p.

(iv) *The expected profit of each bidder, except the bidder who receives signal s^p, is strictly positive in equilibrium.* Even though the equilibrium price is equal to the pooling bid, the bidders who submit the pooling bid also make positive profits.[11]

[11] In particular, the following profit function is obtained:

$$\Pi(s) := \begin{cases} \frac{\kappa - (1 - F(s^p|L))}{F(s^p|L)(1+\rho(s))} (v(l, L) - b^p(1 + \rho(s)\Delta(s^p))) & \text{if } s \leq s^p, \\ \frac{\rho(s)}{1+\rho(s)} v(r, R) - b^p & \text{if } s \geq s^p. \end{cases}$$

(v) *The value of information, i.e., the value of receiving a signal, is strictly positive for the bidders.* This is because winning an object at the pooling bid is only partially informative and the equilibrium price is uninformative, while, on the other hand, signals provide partial information about the state.

4. Information Aggregation Failures in Monotone Equilibria

In the previous section, we described equilibria in which no information is aggregated by the price. A prominent property of the equilibrium we described is that equilibrium bids are nondecreasing in the signal that a bidder receives. In this section, in order to demonstrate the robustness of Theorem 1, we characterize all symmetric equilibria in which the bid function is a *nondecreasing* function of signals (Lemma 1). We then use our characterization to show that information cannot be fully aggregated in equilibria in which the bidding function is nondecreasing (Theorem 2). Moreover, we show that equilibria in which the bidding function is nondecreasing exist under a mildly restrictive condition (Theorem 2). Consequently, our results in this section show that *i*) the failure of information aggregation is inherent in equilibria in which the bidding function is nondecreasing; and moreover, *ii*) such equilibria exist for a wide range of parameter values.

Recall that our object of study is a sequence of equilibrium bidding functions $\boldsymbol{b} = \{b_n\}_{n=m}^{\infty}$. We say that a bidding function is nondecreasing (nonincreasing) if $b(s)$ is a nondecreasing (nonincreasing) function of s; and we say that a sequence $\boldsymbol{b}$ is nondecreasing (nonincreasing) if b_n is a nondecreasing (nonincreasing) bidding function for each n. We begin by characterizing nondecreasing equilibrium bidding functions.[12]

Lemma 1 (Characterization) *Suppose that Assumptions 1-3 hold. Every equilibrium bidding function b that is nondecreasing satisfies the following conditions:*

(i) There is a cutoff signal $s^p \in [0,1]$ and a pooling bid b^p such that $b(s) = b^p$ for every $s < s^p$, and $b(s) > b^p$ for every $s > s^p$.

(ii) The bidding function $b(s)$ is strictly increasing in the range $(s^p, 1]$.

(iii) Bidders with signals above s^p choose action r and bidders with signals below s^p choose action l when they win an object.

The characterization lemma essentially states that any nondecreasing equilibrium resembles the equilibrium that we constructed in the previous section for Theorem 1. More specifically, Lemma 1 shows that in any nondecreasing equilibrium, there is at most one interval,

Note that $\Pi(s^p) = 0$ by construction, $\Pi(s) > 0$ for all $s < s^p$ and for all $s > s^p$.

[12] A straightforward modification of the lemma delivers a characterization of all nonincreasing equilibrium bidding functions as well.

which includes zero, over which the bidding function is constant and equal to the pooling bid; outside of this interval, the bidding function is strictly increasing. Moreover, the bidders who win an object choose l if they have submitted the pooling bid, and choose r if they have submitted a bid above the pooling bid.

As we noted in the previous section, if the bidding function is nondecreasing, then there must be pooling. We now provide an intuitive sketch of the remaining arguments for Lemma 1. **Bidders who bid a pooling bid choose action l if they win an object.** On the way to a contradiction, suppose that there is a bidder who bids the pooling bid and chooses action r if he wins an object. Notice that he wins an object only when the auction price is not more than the pooling bid. Moreover, when the price is equal to the pooling bid, there is rationing with strictly positive probability. When the price is equal to the pooling bid, losing a unit is a signal more favorable to state R because of the *loser's curse*. However, if this bidder deviates from such a strategy by increasings his bid slighly, he ensures that he wins an object whenever the auction price is equal to the pooling price. Such a deviation is profitable, because such a bidder chose action r when he won an object before the deviation (by the hypothesis), and after the deviation, he wins an object in those instances when he had been losing by bidding the pooling bid.

There is only one pooling bid. If the bidders who are bidding a pooling bid choose action l when they win an object at the price equal to the pooling bid, then they also choose action l if the price is lower than the pooling bid, and they make strictly positive profits when the price is lower than their bid. Therefore, the bidder with the lowest signal (i.e., signal zero) also would have chosen action l if she were to bid the highest pooling bid, and won an object. However, the bidder with signal zero then has the highest valuation for the object among all bidders whose bids are less than the highest pooling bid. This implies that the bid chosen by a bidder who receives signal zero must be at least as large as all the other pooling bids. Therefore, our assumption that the bidders use a nondecreasing bidding function implies that there is at most one pooling bid.

Bidders who submit bids above the pooling bid choose action r if they win an object. Suppose s^p is the highest signal for which $b(s^p)$ equals the pooling bid. Assume that a bidder who receives signal $s' > s^p$, where s' is arbitrarily close to s^p, plays l if he wins an object and the auction price is equal to the pooling bid. We now argue that this assumption leads to a contradiction. A bidder who receives signal s' prefers submitting a bid that exceeds the pooling bid to submitting the pooling bid because $s' > s^p$. Suppose that the bidder who receives signal s^p deviates and submits a bid that exceeds the pooling bid by an arbitrarily small amount and wins an object at the pooling price. In this event, the posterior of a bidder with signal s^p puts more weight on state l than the posterior of a bidder with signal s'. Therefore, if the bidder who receives signal s' prefers to submit a bid that exceeds the pooling bid, then so does a bidder who receives signal s^p. However, this contradicts the fact that $b(s^p)$ equals the pooling bid.

In the theorem below, we use the characterization given by Lemma 1 to show that monotone equilibria cannot fully aggregate information. Moreover, we establish that a monotone equilibrium sequence exists if Condition 1 holds, i.e., if $\rho(0) \geq \rho^*$.

THEOREM 2 *Suppose that Assumptions 1-3 hold. If* $\boldsymbol{b}$ *is a nondecreasing equilibrium sequence, then* $\boldsymbol{b}$ *does not fully aggregate information. Moreover, if Condition 1 is satisfied, then a nondecreasing equilibrium sequence* $\mathbf{b}$ *exists.*

The conclusion of the theorem follows because the auction price is equal to the pooling bid with strictly positive probability even when the state is R. Moreover, this probability is bounded strictly away from zero even as the number of bidders grows arbitrarily large. Therefore, an outside observer is uncertain about the state when she observes that the auction price is equal to the pooling bid. Moreover, this event occurs with strictly positive probability. To see why, suppose that the price is equal to the pooling bid with probability close to zero in state R. However, if this were the case, then nobody would be willing to choose action r when the price is equal to the pooling bid. But, this contradicts Lemma 1 which shows that all the bidders who submit a bid above the pooling bid play r when they win an object at the price of the pooling bid.

5. Discussion

5.1. Pooling bid, loser's curse and nonmonotonicity of the value function A key feature of our equilibrium construction that makes price uninformative about the state of the world is the existence of a pooling bid. In other words, in the equilibrium that we construct, there is an atom in the equilibrium bid distribution at b^p. In sharp contrast, the existence of such a pooling bid is not possible in the symmetric equilibria of the auction models of Pesendorfer and Swinkels (1997) or Milgrom and Weber (1982), where there is no ex-post action. The existence of a pooling bid in our model and the impossibility of pooling in auctions without ex-post actions are both consequences of the ***loser's curse***, i.e., the fact that the probability of winning an object at the pooling bid in state L is strictly higher than the probability of winning an object at the pooling bid in state R. Equivalently, a bidder is more convinced that the state is R when he does not win an object than when he does, provided that the price is the pooling bid and he bid the pooling bid.

Intuitively, not winning an object at the pooling bid, when the auction price is equal to the pooling bid, is a strong signal in favor of state R. Therefore, whenever the auction price is equal to the pooling price, a bidder would rather increase his bid slightly and ensure that he wins an object in Pesendorfer and Swinkels (1997)'s model, because any news in favor of state R is good news.

In our framework, however, the value function is nonmonotonic in the belief of the bidder that the state is R and this nonmonotonicity is a consequence of Assumption 1. In particular, the bidders who bid the pooling bid take action l when they win a unit. Hence, for such a bidder a signal in favor of state R is bad news, unless this signal is overwhelmingly strong. Therefore, the loser's curse argument does not preclude pooling. On the contrary, deviation from the pooling bid to a slightly higher bid makes such bidders win a unit more frequently, albeit with a different belief about the state, which makes them worse off.

To see why information could be aggregated if Assumption 1 is not satisfied and the value function is monotonic, suppose that $v(r, R) > v(l, L) = v(r, L) = v(l, R) = 0$, that is, action l is weakly dominated by action r. In this case, v satisfies neither inequality (1) nor (2) and our model coincides with Pesendorfer and Swinkels (1997)'s model with two states of the world $\Omega = \{L, R\}$ where the value of the object is equal to zero in state L and equal to $v(r, R)$ in state R. In the unique symmetric equilibrium of the auction with n bidders and k objects, the bidding function $b_n(s) = v(r, R) \Pr(\omega = R | s_1 = s, Y^k_{n-1} = s)$ for every $s \in (0, 1)$. This function is strictly increasing in s because the signal distribution satisfies MLRP. Notice that as n gets larger, bidders who receive signals s^κ_R and s^κ_L determine the equilibrium prices in states R and L respectively. A key observation that delivers information aggregation is that $\lim_{n\to\infty} b_n(s^\kappa_R) = v(r, R)$ and $\lim_{n\to\infty} b_n(s^\kappa_L) = 0$, i.e., prices converge to $v(r, R)$ and 0 in states R and L respectively. Intuitively, the bids of s^κ_R and s^κ_L are distinct from each other, and hence prices reveal which of these types won the last unit, and hence set the price (see also Figure 7).

In contrast, in the equilibrium that we construct in Theorem 1 there is a pooling bid that is chosen by bidders who receive signals in $[0, s^p]$. Moreover, $s^p > s^\kappa_R$, and therefore the price is equal to the pooling bid in both states and hence is uninformative. The key is that the bidder with signal s^κ_R and s^κ_L bid the same price, and hence price cannot distinguish the identity of the type who set the price.

6. Conclusion

In this paper, we have explored the role of market prices in aggregating information about the correct use of objects. In our set-up, multiple homogeneous goods are allocated among multiple bidders via a Vickrey-type auction. Our main finding is that, when prices contain information about the ex-post actions that the owners of the object will take, then prices may not reveal all the information available in the market. In the extreme case, prices reveal

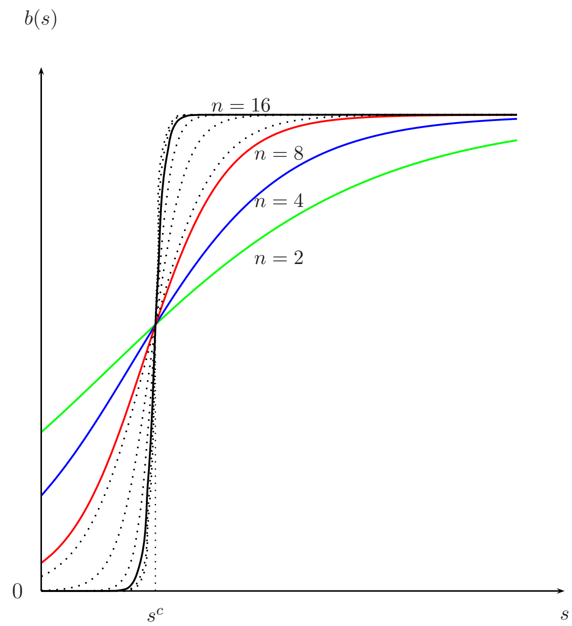

Figure 7: This figure depicts the equilibrium bidding functions in Pesendorfer and Swinkels (1997). These functions converge to a step function that has a jump at a signal s^c that satisfies $s^{\kappa}_L < s^c < s^{\kappa}_R$.

no information about the state of the world, and a nonnegligible fraction of the objects are thus used incorrectly.

There are trivial mechanisms that could aggregate information. In our model, there is no room for allocative inefficiency. Therefore, a direct mechanism that elicits the signals of bidders, allocates the objects randomly, discloses the signal profile would achieve full efficiency. However, we study Vickrey auctions for four main reasons. First, such auctions are frequently used in practice. Treasury bill auctions are prominent examples. Second, such auctions resemble competitive markets in which agents are price takers, since in a Vickrey auction, a bidder cannot change the price he pays for the object by altering his bid. Third, just as in competitive markets, there is a uniform price. Fourth, there is a large body of academic work that studies Vickrey auctions.

We interpret our results as suggesting that it is too much to expect prices alone to reveal the state of the world perfectly. Also, our results highlight that markets have several statistics other than price, such as the amount of rationing, volume of trade, and bid distributions, that are relevant for aggregating information.

A. Organization of the Appendix

We start by proving Theorem 2 instead of Theorem 1, because the construction we use for the former is used for the latter theorem. Later we prove Lemma 1, and then we present the proofs of some lemmata that we use in the proofs of our theorems and lemma 1.

B. Proof of Theorem 2

The first part of the theorem is proven in the main text. Here we prove the second part.

B.1. Method used for the construction The construction has two general steps. In the first step, we show that in a large market with size z, there exists a cutoff signal, s_z^p such that in a monotonic bidding profile b_z where all types below s_z^p bid a pooling bid, the following two properties are satisfied. i) the value of the object to bidders with signals $s < s_z^p$, who win a unit by bidding the pooling bid, is not less than the value of the object to such bidders if they win a unit by bidding above the pooling bid, and when the price is the pooling bid. ii) The value of the object to bidders with signals $s > s_z^p$ when they bid above the pooling bid and the price is the pooling bid is not less than if such bidders bid the pooling bid and win a unit. In this step, we also determine the value of the pooling bid.

The second step of the construction shows that under condition 1, the bidding profiles constructed in step 1 constitute an equilibrium of the auction game when z is sufficiently large. We do this by showing that no type has a profitable deviation from the bidding profile constructed in step 1.

B.2. Step 1: Cutoff type For any $s \in (0, 1)$, $s' \in S$ and $z \in \mathbb{Z}$, let $\rho_z^-(s', s)$ and $\rho_z^+(s', s)$ be

$$\begin{aligned} \rho_z^-(s', s) \; &: = \frac{P\left(Y_{z-1}^{\kappa z} \leq s, s_1 = s', 1 \text{ wins the lottery}|R\right)}{P\left(Y_{z-1}^{\kappa z} \leq s, s_1 = s', 1 \text{ wins the lottery}|L\right)}, \\ \rho_z^+(s', s) \; &: = \frac{P\left(Y_{z-1}^{\kappa z} \leq s, s_1 = s'|R\right)}{P\left(Y_{z-1}^{\kappa z} \leq s, s_1 = s'|L\right)}, \end{aligned}$$

The event that "1 wins the lottery" corresponds to the event that 1 wins a prize (or equivalently one unit of the object) in the following auxiliary lottery whose odds depend on the signal distribution across the bidders. The lottery has q prizes allocated equally likely to o people, where the number of prizes $q = max\left\{0, \kappa z - |j \in \{2, ..., z\} : s_j > s|\right\}$ and the number of people is $o = 1 + |j \in \{2, ..., z\} : s_j \leq s|$.[13] Intuitively, $\rho_z^-(s', s)$ is the posterior likelihood ratio of state R and L for type s', when he bids the pooling bid and wins a unit, where the bidders who bid the pooling bid are those with signals less than s. The second function, $\rho_z^+(s', s)$ is the posterior likelihood ratio of states R and L when a bidder with a signal s' wins a unit by bidding above the pooling bid, at a price equal to the pooling bid.

Remark 4 *Observe that, both $\rho_z^-(s, s')$ and $\rho_z^+(s, s')$ are continuous in both arguments. This is because the cdf $F(s|\omega)$ admits a positive density function f, and f is assumed to be a continuous density function.*

[13] The index sets exclude the number 1 since it is reserved for the bidder who is doing these calculations for her best response.

We also make the following definitions:

$\rho_z^-(s) := \rho_z^-(s,s)$

$\rho_z^+(s) := \rho_z^+(s,s)$

Note that we have used the same notation for both functions although they take different number of arguments. Intuitively, $\rho_z^-(s)$ is the posterior likelihood of a cutoff type s when she bids a pooling bid that only types in the range $[0,s]$ bid, and when she wins an object with such a bid.

REMARK 5 *From Pesendorfer and Swinkels (Lemma 7, page 1272) and from $f(0|L) \neq f(0|R)$ we know that i) $\rho_z^-(s) < \rho_z^+(s)$ for any $s \in (0,1)$, and ii) $\rho_z^-(s',s) < \rho_z^+(s',s)$ for any $s, s' \in (0,1)$. This is called the loser's curse. Moreover, $\rho_z^-(s,s')$ and $\rho_z^+(s,s')$ are both strictly increasing in s and s', since $f(0|L) \neq f(0|R)$, as we later show in Lemma 6.*

Since $\rho_z^-(s)$ and $\rho_z^+(s)$ are both increasing functions, $u(\rho_z^-(s))$ and $u(\rho_z^+(s))$ are both at most single-troughed functions. Now we make two observations about values of the functions $\rho_z^-(s)$ and $\rho_z^+(s)$ when s is close to zero and when s is close to one.

LEMMA 2

1. *$\exists \varepsilon > 0$ and a Z_1 such that $\rho_z^-(s) < \rho^*$ and $\rho_z^+(s) < \rho^*$ for every $s \leq \varepsilon$ and every $z > Z_1$.*
2. *$\exists \varepsilon > 0$ and a Z_2 such that $\rho_z^-(s) > \rho^*$ and $\rho_z^+(s) > \rho^*$ for every $s \geq 1-\varepsilon$ and every $z > Z_2$.*

PROOF:

1. $\exists \varepsilon > 0$ such that $\lim_{z\to\infty} \rho_z^+(\varepsilon) = 0$, because of MLRP. Since $\rho_z^-(s) < \rho_z^+(s)$ for $s \in (0,1)$, $\lim_{z\to\infty} \rho_z^-(\varepsilon) = 0$, and since $u(\rho)$ is strictly decreasing in the range $[0, \rho^*]$, we have that $u(\rho_z^-(s)) > u(\rho_z^+(s))$ for $s \leq \varepsilon$ when $z > Z_1$ for some integer Z_1.

2. For any $s > s_R^\kappa$, $\lim_{z\to\infty} \rho_z^-(1-\varepsilon) = \rho_0 \frac{\kappa-(1-F(s|R))}{\kappa-(1-F(s|L))}$ as we show in lemma 5. This, together with condition 1 imply that $\exists \varepsilon > 0$ such that $\lim_{z\to\infty} \rho_z^-(1-\varepsilon) > \rho^*$. Since $\rho_z^-(s) < \rho_z^+(s)$, and $\rho_z^-(s)$ and ρ_z^+ are strictly increasing , we have that $\rho_z^-(s) > \rho^*$ and $\rho_z^+(s) > \rho^*$ for every $s \geq 1-\varepsilon$ when z is sufficiently large. Since $u(\rho)$ is strictly increasing in the range $[\rho^*, 1]$, we have that $u(\rho_z^-(s)) < u(\rho_z^+(s))$ for $s \geq 1-\varepsilon$. □

LEMMA 3

1. *For every $z > \max\{Z_1, Z_2\}$, there is a unique $s_z^p \in (\varepsilon, 1-\varepsilon)$ that satisfies the equality $u(\rho_z^-(s)) = u(\rho_z^+(s))$.*
2. *When such an s_z^p exists, $\rho_z^+(s) > \rho^*$ for $s > s_z^p$ and $\rho_z^-(s) < \rho^*$ for $s < s_z^p$. Moreover $\rho_z^+(s_z^p) \geq \rho^*$.*

PROOF: 1. Let s_z^1 and s_z^2 be the unique signals that solve the equalities $\rho_z^-(s) = \rho^*$ and $\rho_z^+(s) = \rho^*$ respectively. Note that $s_z^1, s_z^2 \in (0,1)$ and are well-defined when $z > \max\{Z_1, Z_2\}$ from lemma 2, and because ρ_z^- and ρ_z^+ are both continuous and strictly increasing. Moreover, because $\rho_z^-(s) < \rho_z^+(s)$, $z_1 > z_2$. In the range $[0, z_2)$, $u(\rho_z^-(s)) > u(\rho_z^+(s))$, and in the range $(z_1, 1]$, $u(\rho_z^+(s)) > u(\rho_z^-(s))$.

In the range (z_2, z_1), $u(\rho_z^-(s))$ is strictly decreasing and $u(\rho_z^+(s))$ is strictly increasing. Therefore, $u(\rho_z^-(s)) - u(\rho_z^+(s))$ is strictly negative in $[0, z_2)$, strictly increasing in $[z_2, z_1]$, and strictly positive in $(z_1, 1]$. Therefore, by the intermediate value theorem, there is a unique signal s_z^p, in the range $[z_2, z_1]$ that satisfies the equality $u\left(\rho_z^-(s)\right) = u\left(\rho_z^+(s)\right)$.

2. As we argued above, such a signal is in the range $[z_2, z_1]$, and hence $\rho_z^+(s) > \rho^*$ for $s > s_z^p$ and $\rho_z^-(s) < \rho^*$ for $s < s_z^p$. Also $\rho_z^+(s_z^p) \geq \rho^*$ because $s_z^p > s_z^2$. □

B.3. Setting the pooling bid and its properties We now determine the bidding function, b_z, when $z > \max\{Z_1, Z_2\}$. The bidding function is constant and equal to $b_z^p := u(\rho_z^-(s_z^p)) = u(\rho_z^+(s_z^p))$ for bidders with signals in the interval $[0, s_z^p)$ and is strictly increasing and equal to $u(\rho(s_1 = s, Y_{z-1}^{\kappa z} = s))$ in the region $(s_z^p, 1)$. Notice that, the part of the bidding function that is strictly increasing coincides with Pesendorfer and Swinkel's equilibrium bidding function, in the case where bidders are taking action r. Moreover, $u(\rho(s_1 = s, Y_{z-1}^{\kappa z} = s)) > u(\rho(s_1 = s_z^p, Y_{z-1}^{\kappa z} \leq s_z^p)) = u(\rho_z^+(s_z^p)) = b_z^p$ for every $s > s_z^p$, because $\rho_z^+(s_z^p) > \rho^*$ (from lemma 3) and together with MLRP.

We state the following remark that summarizes some of the findings up to now, before we proceed:

REMARK 6 *The posterior likelihood ratio of types lower than s_z^p, conditional on winning at price b_z^p is less than ρ^*, and types higher than s_z^p, conditional on the price being b_z^p has a posterior likelihood ratio that is more than ρ^*. In particular,*

$$\rho(s_1 = s, Y_{z-1}^{\kappa z} \leq s_z^p, 1 \text{ wins with } b_z^p) \leq \rho^* \text{ for } s \leq s_z^p, \text{ and}$$
$$\rho(s_1 = s, Y_{z-1}^{\kappa z} \leq s_z^p) \geq \rho^* \text{ for } s \geq s_z^p.$$

B.4. Step 2: Checking deviations In this step, we will show that the bid function we constructed in step 1 (i.e., b_z) is an equilibrium when z is large (i.e., when $z > Z_3$ for some integer Z_3) by showing that no type has a profitable deviation from the proposed bidding strategy profile. In the following we assume that z is large enough that s_z^p exists, i.e., $z > \max\{Z_1, Z_2\}$.

B.4.1. *Bidders with signals above s_z^p* Pick a type $s > s_z^p$. We will first show that for any type $s' \in (s_z^p, s)$ the following inequality holds:

$$(4) \qquad u(\rho(s_1 = s, Y_{z-1}^{\kappa z} = s')) > b_z(s') = u(\rho(s_1 = s', Y_{z-1}^{\kappa z} = s'))$$

This inequality follows because $\rho(s_1 = s, Y_{z-1}^{\kappa z} = s') > \rho(s_1 = s', Y_{z-1}^{\kappa z} = s')$ by MLRP and because $\rho(s_1 = s', Y_{z-1}^{\kappa z} = s') > \rho(s_1 = s', Y_{z-1}^{\kappa z} \leq s') = \rho_z^+(s') \geq \rho^*$, again by MLRP and Lemma 3. Therefore, type s has no profitable deviation to bid $b_z(s')$ for any $s' \in (s_z^p, s)]$. A similar calculation shows that such a type has no profitable deviation to bid above $b(s)$. Next, we will argue that such a type doesn't find it a profitable deviation to bid b_z^p. To show this, we will prove two inequalities:

$$(5) \qquad u(\rho_z^+(s, s_z^p)) > b_z^p$$

$$(6) \qquad u(\rho_z^+(s, s_z^p)) \geq u(\rho_z^-(s, s_z^p))$$

To see that these inequalities suffice to prove that bidding b_z^p is not a profitable deviation, notice that, first, such a type makes a strictly positive profit when the price is above b_z^p and below $b_z(s)$. The first inequality above says that, when bidding above b_z^p, type s has a positive payoff when the price is equal to b_z^p. The second inequality says that, the payoff to a bidder when he bids above b_z^p and the price is b_z^p is not less than when he bids b_z^p and wins a unit. Moreover, the probability of winning a unit by bidding above b_z^p is strictly larger than winning by bidding b_z^p. Hence, bidding b_z^p cannot not be a profitable deviation. Now, we will prove the two inequalities above.

The first inequality follows, because $\rho_z^+(s, s_z^p) > \rho_z^+(s_z^p) \geq \rho^*$ by MLRP and Lemma 3.

Now we will show the second inequality. There are two cases to consider. Either $\rho_z^-(s, s_z^p) \geq \rho^*$ or $\rho_z^-(s, s_z^p) < \rho^*$. In the former case, $\rho_z^+(s, s_z^p) \geq \rho_z^-(s, s_z^p)$ together with the facts that both are at least ρ^* and u is increasing when $\rho \geq \rho^*$ deliver the desired inequality. In the latter case, $\rho_z^-(s, s_z^p) > \rho_z^-(s_z^p)$. Since both are less than ρ^*, and since $u(\rho)$ is decreasing in that range, $u(\rho_z^-(s, s_z^p)) < u(\rho_z^-(s_z^p)) = b_z^p < u(\rho_z^+(s, s_z^p))$.

B.4.2. *Bidders with signals below s_z^p* In this part of the proof, we will need z sufficiently large and we will need our restriction on the priors that $\rho(0) > \rho^*$. Remember that s_R^κ is the signal such that $F(s_R^\kappa|R) = 1 - \kappa$. We will show in the next lemma that such types have a negative payoff if they bid above b_z^p and if the price is equal to b_z^p.

Lemma 4 *$\exists Z_5 \in \mathbb{Z}$ such that $u(\rho_z^+(s, s_z^p)) \leq b_z^p$ for every $s < s_z^p$, every $z > Z_5$.*

Proof: If $\rho_z^+(s, s_z^p) \geq \rho^*$, then $\rho_z^+(s_z^p) = b_z^p > \rho_z^+(s, s_z^p)$ because of MLRP and because $u(\rho)$ is increasing when $\rho \geq \rho^*$. The rest of the proof shows either directly that $u(\rho_z^+(s, s_z^p)) \leq b_z^p$

or indirectly by showing that $\rho_z^+(s, s_z^p) \geq \rho^*$ for every $s < s_z^p$ when z is sufficiently large. In the latter case, since $\rho_z^+(0, s_z^p) \leq \rho_z^+(s, s_z^p)$, proving that $\rho_z^+(0, s_z^p) \geq \rho^*$ suffices.

We'll make our argument under the assumption that the limit of the sequence $\{s_z^p, b_z^p\}_{z=1}^{\infty} = (s^p, b^p)$ exists and then we will verify this in lemma 5. The next three claims are the steps of the proof. □

Claim 1 $s^p \geq s_R^\kappa$.

Proof: On the way to a contradiction, suppose that $s^p < s_R^\kappa$. Then, $\lim_{z\to\infty} \rho_z^+(s_z^p) = 0$, because $\lim_z P\left(Y_{z-1}^{\kappa z} \leq s_z^p | L\right) = 1$ and $\lim_z P\left(Y_{z-1}^{\kappa z} \leq s_z^p | R\right) = 0$ if $s^p < s_R^\kappa$. This contradicts the assertion in Remark 6 that $\rho(s_1 = s, Y_{z-1}^{\kappa z} \leq s_z^p) \geq \rho^*$ for $s > s_z^p$. □

Claim 2 *If* $s^p > s_R^\kappa$*, then* $\exists Z_3 \in \mathbb{Z}$ *such that* $\rho_z^+(0, s_z^p) \geq \rho^*$*, for every* $z > Z_3$.

Proof: Since $F(s^p|\omega) > 1 - \kappa$ for $\omega \in \Omega$, $\lim_{z\to\infty} Pr(Y_{z-1}^{\kappa z} \leq s_z^p) = 1$ for $\omega \in \Omega$. Hence, $\lim_{z\to\infty} \rho(s_1 = 0, Y_{z-1}^{\kappa z} \leq s_z^p) = \rho(s_1 = 0)$. Since we assumed $\rho(0) > \rho^*$ in condition 1, $\exists Z_3 \in \mathbb{Z}$ such that $\rho_z^+(0, s_z^p) \geq \rho^*$, for every $z > Z_3$. □

Claim 3 *If* $s^p = s_R^\kappa$, $u(\rho_z^+(s, s_z^p)) \leq b_z^p$.

Proof: This is the case when prices may indeed reveal some information. We'll start by arguing that pooling bids, b_z^p converges to $u(0)$.

The crucial observation in this case is that $\lim_{z\to\infty} \Pr(\omega = L | p_z = b_z^p, 1 \text{ wins with } b_z^p) = 1$. The reason for the above limit calculation is the following. Fix an $\epsilon > 0$. Then, $\lim_{z\to\infty} \Pr(Y_{z-1}^{(\kappa-\epsilon)z} > s_z^p | \omega = R) = 0$. Therefore, $\lim_{z\to\infty} \Pr(p_z = b_z^p, 1 \text{ wins with } b_z^p | \omega = R) \leq \frac{\epsilon}{1-\kappa+\epsilon}$. Since this is true for every $\epsilon > 0$, it has to be that

$$\lim_{z\to\infty} \Pr(p_z = b_z^p, 1 \text{ wins with } b_z^p | \omega = R) = 0. \tag{7}$$

On the other side $\lim_{z\to\infty} \Pr(p_z = b_z^p | \omega = L) = 1$ because $1 - F(s_R^\kappa | \omega = L) < \kappa$. Moreover, there is an $\epsilon > 0$ such that $Pr(|(\text{signals above } s_R^\kappa)| \leq (\kappa - \epsilon)z | \omega = L) = 1$. Therefore,

$$\lim_{z\to\infty} \Pr(p_z = b_z^p, 1 \text{ wins with } b_z^p | \omega = L) > 0. \tag{8}$$

Combining equation 7 and inequality 8 delivers that

$$\lim_{z\to\infty} \Pr(\omega = L | p_z = b_z^p, 1 \text{ wins with } b_z^p) = 1. \tag{9}$$

Now we'll finish the argument that pooling bids converge to $u(0)$. Since each signal has bounded information, $\lim_{z\to\infty} b_z^p = u\left(\rho_z^-\left(s_z^p, s_z^p\right)\right) = u(0)$, which follows from equality 9.

Note that if $\rho^+(s, s_z^p) \geq \rho^*$, then the proof is complete. So we assume that $\rho^+(s, s_z^p) < \rho^*$. Note that, there is an $\epsilon \in (0, \rho^*)$ such that $\liminf_{z\to\infty} \rho^+(0, s_z^p) > \epsilon$. This is because, $\rho^+(s, s_z^p) \geq \rho^*$ for every $s > s_z^p$ (by lemma 3) and because of the limited individual information assumption on the signal distributions. Therefore, $u(\rho^+(0, s_z^p)) < u(\epsilon) < u(0)$. Since we have shown that $\lim_z b_z^p = u(0)$, when z is sufficiently large, $u(\rho^+(0, s_z^p)) < b_z^p$, proving the claim for this case. □

We have proven the lemma. Now we will argue that bidders with signals less than s_z^p don't have profitable deviations to bid strictly above s_z^p when the hypothesis of lemma 4 is satisfied. As shown in lemma 4, such types lose money if they bid above b_z^p and if the price is equal to b_z^p. If a type $s < s_z^p$ bids strictly above $b(s')$ for some $s' > s_z^p$, we will now argue that he loses money when the price is equal to $b(s')$. If $\rho(s_1 = s, Y_{z-1}^{\kappa z} = s') \geq \rho^*$, then $u(\rho(s_1 = s, Y_{z-1}^{\kappa z} = s')) < b(s') = \rho(s_1 = s', Y_{z-1}^{\kappa z} = s')$. If $\rho(s_1 = s, Y_{z-1}^{\kappa z} = s') < \rho^*$, then $b_z(s') > b_z^p \geq u(\rho_z^+(s, s_z^p)) = u(\rho(s_1 = s, Y_{z-1}^{\kappa z} \leq s_z^p)) > u(\rho(s_1 = s, Y_{z-1}^{\kappa z} = s'))$, where the second inequality follows from lemma 4, and the third inequality follows from MLRP.

C. Proof of Theorem 1

Proof: We will prove this theorem by using the same construction that we used to prove Theorem 2. Notice that the assumption that $\rho(0) \geq \rho^*$ facilitates that the hypothesis of Theorem 2 is satisfied, and hence the constructed bidding strategies constitute an equilibrium when z is sufficiently large.

We will now show that if $u(0) > u(\rho(s_R^\kappa))$, then the limit of the cutoff types as z goes to infinity, which we denote by s^p is strictly larger than s_R^κ. The implication of this inequality is that equilibrium prices become the pooling bid in both states of the world with probabilities approaching one, and hence prices reveal no information as the market gets arbitrarily large.

We have already proven in the proof of Theorem 2 that any limit point of the cutoffs has to be at least s_R^κ. Thus it remains to show that s_R^κ is not a limit point of the cutoff types constructed in the sequence of bidding functions.

On the way to a contradiction, suppose our claim is not true, i.e., s_R^κ is the limit point of the cutoff types. Then as we argued in the proof of Theorem 2, the pooling bid, b_z^p goes to $u(0)$ as z goes to ∞. On the other side, $\rho_z^+(s_z^p) \geq \rho^*$ by Lemma 3. Moreover, $\rho_z^+(s_z^p) < \rho(s_z^p)$. Hence, $b_z^p = u(\rho_z^+(s_z^p)) < u(\rho(s_z^p))$. However, $\lim b_z^p = u(0)$ and $\lim u(\rho(s_z^p)) = u(s_R^\kappa) < u(0)$ which contradicts that $b_z^p < u(\rho(s_z^p))$ for every z. □

D. Proof of Lemma 1

Proof: There are two cases to consider, either b is strictly increasing or there is an atom in the bid distribution.

Case 1: If b is strictly increasing, then the first part of the lemma is true by picking $s^p = 0$. The second part of the lemma for this case claims that b is a la Pesendorfer and Swinkels (1997) (abbreviated as PS in the following). This is a slight modification of the arguments in PS, the second part of 'proof of proposition 1' in page 1272.

Case 2: Suppose that the bid function has an atom at some bid b^p. Then the monotonicity of the bidding function implies that $b(s) = b^p$ for an interval of signals, $S(b^p) = (s', s^p)$ with $s' < s^p$ and $b(s) = b^p$ for every $s \in S(b^p)$ and $b(s) > b^p$ for every $s > s^p$. In steps 1, 2 and 3 below we will show that there can be at most one atom in the bid distribution and that $s' = 0$.

Step 1: The first step is to show that $\rho(s_1 = s^p, p = b^p, 1 \text{ wins with } b^p) < \rho^*$. On the way to a contradiction, suppose that it's not true. Then due to winner's and loser's curse (see PS, page 1272), types in $S(b^p)$ would deviate and bid slightly above b^p. This follows from the monotonicity of the bidding function b.

Step 2: We will now argue that

$$\rho(s_1 = s, p, 1 \text{ wins with } b(s)) < \rho^*$$

for every $s < s^p$ and $p \leq b(s)$. We first claim that the following is true for every $p' < b^p$ which is in the range of b.

$$\rho(s_1 = s, p') < \rho(s_1 = s, p = b^p, 1 \text{ wins with } b^p).$$

This is a non-trivial claim and the proof is in Lemma 7. Moreover,

$$\rho(s_1 = s, p, 1 \text{ wins with } p) \leq \rho(s_1 = s, p).$$

This inequality is a standard argument from lemma 7 of PS, at page 1272. Combining the two inequalities in this step with the result in step 1 delivers the claim.

Step 3: We will now argue that all types below s^p bid b^p.

On the way to a contradiction, assume that a positive measure of types bid strictly below b^p and let $s'' < s'$ be such a type. By lemma 7, the probability that type s'' puts on state L were she to bid b^p and the price is any price between her bid and b^p is weakly higher than that of types who are bidding b^p. Formally, for any $p' \leq b^p$ that is in the range of b, the following holds:

$$\Pr(\omega = L | s_1 = s'', p', 1 \text{ wins by bidding } b^p) \geq \Pr(\omega = L | s_1 = s', p', 1 \text{ wins by bidding } b^p).$$

Since bidding slightly below b^p is a feasible strategy, we have that,

$$u(\rho(s_1 = s', b^p, 1 \text{ wins by bidding } b^p)) \geq b^p.$$

Therefore, b^p is weakly less than the value of the object to types who bid b^p conditional on the price being b^p and they winning the object. Since this value is strictly less than the value when the price is strictly lower than b^p, these types make strictly positive profits when the price is strictly less than b^p. And finally, the bid of s'' cannot be an atom because the value of the object conditional on losing when the price is her bid is strictly larger than the value if the price was strictly above her bid but not higher than b^p, which contradicts her bid being an atom (This is a completely symmetric argument as lemma 7). More precisely, for any $p' \in (b(s''), b^p]$ such that $b(s) = p'$ for some $s \in (s'', s^p)$,

$$\Pr(\omega = L | s_1 = s'', p = b(s''), 1 \text{ loses by bidding } b(s'')) \geq \Pr(\omega = L | s_1 = s, p', 1 \text{ wins by bidding } p')$$

Therefore,

$$u(\rho(s_1 = s'', p = b(s''), 1 \text{ loses by bidding } b(s''))) \geq u(\rho(s_1 = s, p', 1 \text{ wins by bidding } p')) \geq p' > b(s'').$$

Therefore, type s'' would have an incentive to bid strictly above $b(s'')$, yielding a contradiction to $b(s'')$ being an atom. Since $b(s'')$ is not an atom, s'' has a strict incentive to bid b^p, yielding the contradiction.

Step 4: Now we consider bids above b^p and will show that $\rho(s_1 = s^p, p = b^p) > \rho^*$

Since we have shown that there can be at most one atom, b does not have a constant part above s^p. Therefore, it should be that $\rho(s_1 = s^p, p = b^p) > \rho^*$. This follows from monotonicity of b and the winner's curse. The reason is that otherwise signals lower and arbitrarily close to s^p would have a provitable deviation to bid above b^p (see PS, page 1272 again). Moreover $\rho(s_1 = s, p = b(s')) > \rho^*$ for $s, s' > s^p$ from MLRP.

We now conclude that b has to be a la PS for types above s^p, i.e., for $s > s^p$, $b(s) = u(\rho(s_1 = s, Y^k_{n-1} = s))$. This follows from PS, because the value of the object is strictly increasing in the probability that the bidder assigns to state R for types above s^p. $\square$

E. Miscellaneous Results

Lemma 5 *For any* $s > s_R^\kappa$, $\lim_z \rho_z^-(s) = \rho(s)\frac{\kappa-(1-F(s|R)))}{F(s|R)}\frac{F(s|L)}{\kappa-(1-F(s|L)))}$. *There is a unique signal* s^p *which is a limit point of the cutoffs* $\{s_z^p\}_{z\geq 0}$. *Moreover either* $s^p = s_R^\kappa$ *or* s^p *is the unique signal with the property that satisfies for* $\bar{\rho}(s) := \rho(s)\frac{\kappa-(1-F(s|R)))}{F(s|R)}\frac{F(s|L)}{\kappa-(1-F(s|L)))}$, $u(\bar{\rho}(s)) = u(\rho(s))$.

Proof: Let s^p be a limit point of the sequence, and renumber the new seuence so that its limit is s^p. We have already shown that $s^p \geq s_R^\kappa$ in claim 1 of proof of theorem 2. So now assume that $s^p > s_R^\kappa$. Let *objects taken* denote the random variable which is equal to the minimum of κz and the number of bidders with signals higher than s_z^p. We first note that, $\rho_z^-(s_z^p)$ can be more conveniently expressed by the following equality:

$$\rho_z^-(s_z^p) = \rho(s_z^p)\frac{E\left[\frac{\kappa z-(\text{objects taken})}{z-(\text{objects taken})}|R\right]}{E\left[\frac{\kappa z-(\text{objects taken})}{z-(\text{objects taken})}|L\right]}$$

Our first observation is that

$$\frac{\text{objects taken}}{z}|\omega \rightarrow_{\text{in probability}}^{z\rightarrow\infty} 1 - F(s^p|\omega)$$

Therefore as $z \rightarrow \infty$,

$$E\left[\frac{\rho z - (\text{objects taken})}{z - (\text{objects taken})}|\omega\right] \rightarrow \frac{\kappa - (1 - F(s^p|\omega))}{F(s^p|\omega)}$$

and hence,

$$\frac{E\left[\frac{\rho z-(\text{objects taken})}{z-(\text{objects taken})}|R\right]}{E\left[\frac{\rho z-(\text{objects taken})}{z-(\text{objects taken})}|L\right]} \rightarrow \frac{\frac{\kappa-(1-F(s^p|R))}{F(s^p|R)}}{\frac{\kappa-(1-F(s^p|L))}{F(s^p|L)}}$$

Therefore as $z \rightarrow \infty$,

$$\rho_z^-(s_z^p) \rightarrow \rho(s^p)\frac{\frac{\kappa-(1-F(s^p|R))}{F(s^p|R)}}{\frac{\kappa-(1-F(s^p|L))}{F(s^p|L)}}$$

Since all of the above apply to any arbitrary $s > s_R^\kappa$, the first claim of the lemma is proven. Since $s^p > s_R^\kappa$, as $z \rightarrow \infty$, $\rho_z^+(s_z^p) \rightarrow \rho(s^p)$.

Since each s_z^p has the feature that $u(\rho_z^-(s_z^p)) = u(\rho_z^+(s_z^p))$, and since for each $s > s_R^\kappa$,

$$\rho_z^-(s) \to \rho(s)\frac{\frac{\kappa-(1-F(s|R))}{F(s|R)}}{\frac{\kappa-(1-F(s|L))}{F(s|L)}}$$

we have that for $s \in (s_R^\kappa, 1)$,

$$\Delta(s) := u\left(\rho(s)\left(\frac{\kappa-(1-F(s|R))}{F(s|R)}\frac{F(s|L)}{\kappa-(1-F(s|L))}\right)\right) - u\left(\rho(s)\right) \overset{(\geq)}{\leq} 0 \text{ for } s \overset{(\leq)}{\geq} s^p.$$

The term $\Delta(s)$ is strictly decreasing in the interval $(s_R^\kappa, 1)$ and is strictly negative when s is close to 1, therefore, there should be at most one signal s^p that can be a limit point in the range $(s_R^\kappa, 1)$.

Now suppose that s_R^κ is a limit point. We will show that no signal $s > s_R^\kappa$ can be a limit point. If s_R^κ is a limit point of the sequence, then it should be that $\lim_z u(\rho_z^-(s_z^p)) = u(0)$, and $\limsup_z u(\rho_z^+(s_z^p)) \leq u(s_R^\kappa)$. But then, for every $s > s_R^\kappa$, $\Delta(s) < 0$.

Hence we have shown that if s_R^κ is a limit point, then it is the unique limit point, and if it is not, and if an $s > s_R^\kappa$ is a limit point, then it is unique. This completes the argument that the sequence has a unique limit point. □

Lemma 6 *If $f(0|L) \neq f(0|R)$, then $\rho_z^-(s) > \rho_z^+(s)$ for any $s \in (0,1)$. Moreover both of these functions are strictly increasing in s.*

Proof: The first claim in this lemma is identical to the argument in Lemma 7 in page 1272 of Pesendorfer and Swinkels (1997), and is called loser's curse. The claim that $\rho_z^+(s)$ and $\rho_z^-(s)$ are strictly increasing is standard and follows from the MLRP assumption. The proof can be found in the technical appendix of Milgrom and Weber (1982). □

Lemma 7 *In an increasing equilibrium bidding function b, if there is an atom at bid b^p, then for any $p < b^p$ the following holds:*

$$\Pr(\omega = L|s_1 = s, p) > \Pr(\omega = L|s_1 = s, p = b^p, 1 \textit{ wins with } b^p).$$

Proof: Let the interval of types who are bidding at the atom bid be (s', s''). Then $\Pr(\omega = L|s_1 = s, p) > f(\omega = L|s_1 = s)\Pr(\omega = L|Y_{n-1}^k = s')$. The term $\Pr(\omega = L|s_1 = s, p = b^p, 1$

wins with b^p) is calculated using the following steps:

$$\begin{aligned}
1-F_t(s',s''|\omega) &:= \frac{F(s''|\omega)-F(s'|\omega)}{F(s''|\omega)}\\
C_j^{n-1-i}(\omega) &:= \binom{n-1-i}{j}(1-F_t(s',s''|\omega))^j\,(F_t(s',s''|\omega))^{n-1-i-j}\\
D^i(\omega) &:= \binom{n-1}{i}(1-F(s''|\omega))^i(F(s''|\omega))^{n-1-i}\sum_{n-1-i\geq j\geq k-i} C_j^{n-1-i}(\omega)\frac{k-i}{j+1}\\
\Pr(s_1=s,p=b^p,1 \text{ wins with } b^p|\omega) &= f(s|\omega)\sum_{0\leq i\leq k-1} D^i(\omega)\\
\Pr(\omega=L|s_1=s,p=b^p,1 \text{ wins with } b^p) &= \frac{f(s|L)\sum_{0\leq i\leq k-1}D^i(L)}{f(s|R)\sum_{0\leq i\leq k-1}D^i(R)}
\end{aligned}$$

Explanation: The probability that 1 wins with b^p, the price is b^p conditional on ω can be calculated as the sum of the probabilities of winning in each of the following events, $w^{i,j}$ where $i\leq k-1$ bidders bid above s'', and $k-i\leq j\leq n-1-i$ bidders bid the pooling bid. The probability of winning conditional on event $w^{i,j}$ is $\frac{k-i}{j+1}$, since there are $k-i$ objects remaining for the $j+1$ bidders bidding the pooling bid. The above expressions calculate the probability of each event $w^{i,j}$ in each state and calculate the total winning probability in each state. Similarly the term $f(\omega=L|s_1=s)\Pr(\omega=L|Y^k_{n-1}=s')$ is calculated using the following steps:

$$\begin{aligned}
\Pr(Y^k_{n-1}=s'|\omega) &= \tbinom{n-1}{1}f(s'|\omega)\textstyle\sum_{0\leq i\leq k-1}\tbinom{n-2}{i}(1-F(s''|\omega))^i(F(s''|\omega))^{n-2-i}C^{n-2-i}_{k-i-1}(\omega)\\
f(\omega=L|s_1=s)\Pr(\omega=L|Y^k_{n-1}=s') &= \tfrac{f(s|L)\Pr(Y^k_{n-1}=s'|L)}{f(s|R)\Pr(Y^k_{n-1}=s'|R)}.
\end{aligned}$$

We will now show the following:

$$\frac{f(s|L)\Pr(Y^k_{n-1}=s'|L)}{f(s|R)\Pr(Y^k_{n-1}=s'|R)} > \frac{f(s|L)\sum_{0\leq i\leq k-1}D^i(L)}{f(s|R)\sum_{0\leq i\leq k-1}D^i(R)},$$

or equivalently the following,

$$\frac{\binom{n-1}{1}f(s'|L)\sum_{0\leq i\leq k-1}\binom{n-2}{i}(1-F(s''|L))^i(F(s''|L))^{n-2-i}C^{n-2-i}_{k-i-1}(L)}{\binom{n-1}{1}f(s'|R)\sum_{0\leq i\leq k-1}\binom{n-2}{i}(1-F(s''|R))^i(F(s''|R))^{n-2-i}C^{n-2-i}_{k-i-1}(R)} > \frac{\sum_{0\leq i\leq k-1}D^i(L)}{\sum_{0\leq i\leq k-1}D^i(R)}.$$

Let,

$$E^i(\omega) := \binom{n-1}{1}f(s'|\omega)\binom{n-2}{i}(1-F(s''|\omega))^i(F(s''|\omega))^{n-2-i}C^{n-2-i}_{k-i-1}(\omega).$$

We first obtain the following identity by direct algebra:

$$\frac{D^i(\omega)}{E^i(\omega)} = \frac{(1-F_t(s',s''|\omega))F(s''|\omega)}{f(s'|\omega)} \sum_{k-i\le j\le n-i-1} \frac{(k-i)!(n-k-1)!}{(j+1)!(n-j-i-1)!} \left(\frac{1-F_t(s',s''|\omega)}{F_t(s',s''|\omega)}\right)^{j+i-k}.$$

A simplification of the above identity via a change of variables by letting $u := j-k+i$ delivers the following:

$$D^i(\omega) = E^i(\omega)\frac{(1-F_t(s',s''|\omega))F(s''|\omega)}{f(s'|\omega)} \sum_{0\le u\le n-k-1} \frac{(k-i)!(n-k-1)!}{(k-i+u+1)!(n-k-u-1)!} \left(\frac{1-F_t(s',s''|\omega)}{F_t(s',s''|\omega)}\right)^{u}.$$

The following are consequences of MLRP:

$$\frac{(1-F_t(s',s''|L))F(s''|L)}{f(s'|L)} < \frac{(1-F_t(s',s''|R))F(s''|R)}{f(s'|R)},$$

and, for any positive integer u,

$$\left(\frac{1-F_t(s',s''|L)}{F_t(s',s''|L)}\right)^{u} < \left(\frac{1-F_t(s',s''|R)}{F_t(s',s''|R)}\right)^{u}.$$

For any fixed $u \in \{0,...,n-k-1\}$, the term $\frac{(k-i)!(n-k-1)!}{(k-i+u+1)!(n-k-u-1)!}$ is strictly increasing in i.

Our final observation is that $\frac{E^i(L)}{E^i(R)}$ is strictly decreasing in i. This observation also follows from the MLRP assumption and some algebra.

These four observations yield the desired result. □

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