Productivity, welfare and reallocation: theory and firm evidence*

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Abstract

We prove that the change in welfare of a representative consumer is summarized by the current and expected future values of the standard Solow productivity residual. The equivalence holds if the representative household maximizes utility while taking prices parametrically. This result justifies TFP as the right summary measure of welfare (even in situations where it does not properly measure technology) and makes it possible to calculate the contributions of disaggregated units (industries or firms) to aggregate welfare using readily available TFP data. Based on this finding, we compute firm and industry contributions to welfare for a set of European OECD countries (Belgium, France, Great Britain, Italy, Spain), using industry-level (EU-KLEMS) and firm-level (Amadeus) data. After adding further assumptions about technology and market structure (firms minimize costs and face common factor prices), we show that welfare change can be decomposed into three components that reflect respectively technical change, aggregate distortions and allocative efficiency. Then, using theoretically appropriate firm-level data, we assess the importance of each of these components as sources of welfare improvement in the same set of European countries.

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1 Introduction

How much of growth comes from innovation and technical advances, and how much from changes in allocative efficiency? This question arises in a variety of contexts, in fields as diverse as growth and development, trade, and industrial organization. Yet, despite the importance of the question, there is no consensus regarding the answer. A large number of papers have proposed a bewildering variety of methods to measure the importance of allocative efficiency, leading to a wide range of numerical estimates. Much of the confusion is due to the lack of an organizing conceptual framework for studying this issue. We propose such a framework, and then provide a quantitative answer using one particular set of data.

In starting such a project, one immediately faces the question: What do we mean by allocative efficiency? Indeed, what do we mean by growth? We take the view that growth is an improvement in social well-being. While growth is commonly described in terms of GDP per worker or consumption per capita, these statistics are usually viewed as indicators of some deeper target. Their virtue, a considerable one, is that they can be generated from aggregate data, which are usually readily available. We ask if we can produce a more complete description of economic welfare and its change, while also restricting ourselves to aggregate data. Given an empirical method for characterizing aggregate welfare, allocative efficiency is naturally defined as the increment to welfare achieved by reallocating productive resources to more efficient uses, holding constant the aggregate quantities of resources used in production.

We undertake three tasks. First, we begin from a utility-maximization problem that is standard in the economics of growth and business cycles: We assume that a representative household with an infinite horizon values both consumption and leisure, and maximizes utility subject to a standard intertemporal budget constraint.[1] We show that this standard specification of the objective function implies, to a first order, that welfare is proportional to the present discounted value of total factor productivity (TFP) for the aggregate economy. This result is "TFP without firms"—it is derived purely from the standard model of a price-taking, competitive household. Thus, our result holds for *all* specifications of technology and market structure, including ones where TFP does not measure technology, as long as consumers are free to choose the quantities of goods they purchase at prices they take as being outside their control. Here we follow the intuition of Basu and Fernald (2002), and supply a general proof of their basic proposition that TFP is relevant for welfare.

Second, we use this result to show that we can calculate the welfare contributions of particular sectors of the economy—which can be as large as industries and as small as individual firms. We present industry and firm contributions to welfare for a set of European OECD countries (Belgium, France, Great Britain, Italy, Spain), using industry data from EU-KLEMS and the Amadeus firm-

[1] While the valuation of leisure is not common in a growth context, it is quantitatively very important. Reviewing a large number of social goods that are valued by consumers but not counted in GDP, Nordhaus and Tobin (1973) found the omission of leisure the most significant (with another imputation for the use of non-market time in home production the second most important). Our household maximization framework also corrects automatically for two other gaps that Nordhaus and Tobin find are significant: The need to subtract depreciation (moving to a NDP rather than a GDP framework), and the need to adjust for a growing population.

level data set. Among other things, we use these data to compare the distributions of firm-level productivities relative to the country means across the countries in our sample, and ask how much welfare would increase if, for example, Italian firms had the same relative productivity distribution as those in Great Britain. This analysis is akin to that of Hsieh and Klenow (2009), but it has a direct welfare interpretation and is more general because it does not require assumptions about the production technology.

Third, we show how to decompose welfare—aggregate TFP—into components due to technology, aggregate distortions, and allocative inefficiency. Any such decomposition *does* depend on assumptions about production technology, adjustment costs, and industrial organization. Different assumptions will lead to different decompositions, but within the same overarching social-welfare framework. Finally, we implement one specific decomposition, again based on Basu and Fernald (2002), using firm-level data from a number of European countries represented in the Amadeus data set. We find that welfare grows significantly faster than technology changes, but improvements in allocative efficiency usually account for a modest fraction of the gap between the two.

Our first result clarifies the nature of the important link between welfare on the one hand and aggregate productivity and national income measurement on the other.[2] Our main goal in this section is to provide a clear objective for any decomposition of productivity. To have an economic interpretation, any such decomposition should indicate how productivity contributes to the ultimate target, which is social welfare. Under the usual assumptions and to a first-order approximation, that target *is* a measure of productivity, aggregate TFP. But the method is more important than the specific result. A different specification of the consumer's problem may deliver a different result about the relationship between welfare and productivity. (In fact, we derive results in the paper showing that under certain conditions—for example, if there are distortionary taxes—the correct welfare measure may differ substantially from the usual Solow residual.) But it is still important for researchers interested in decomposing productivity or studying allocative efficiency to relate their empirical method to the solution to some well-specified maximization problem so that the implications of their decompositions for some ultimate welfare objective, which are usually left implicit in any such study, can be made explicit, and the necessary assumptions can be examined closely.

One benefit of starting from a well-defined objective function is that it enables the researcher to take consistent, model-based positions on a variety of issues that bedevil the measurement of productivity and allocative efficiency. For example, Baker and Rosnick (2007), reasoning that the ultimate object of growth is consumption, make the reasonable conjecture that one should deflate nominal productivity gains by a consumption price index to create a measure they call "usable productivity." We begin from the assumption that consumption (and leisure) at different dates are the only inputs to economic wellbeing, but nevertheless show that output should be calculated in the conventional way, rather than being deflated by consumer prices.[3] To take another example, there

[2] Earlier works also make a connection between the two. Some of the most important are Nordhaus and Tobin (1973), Weitzman (1976, 2003) and Hulten (1978). Our approach closely follows that of Basu and Fernald (2002).

[3] The other main adjustment by Baker and Rosnick, moving to a net measure of output as a starting point for

is no consensus in the literature about the proper treatment of scale economies. Most researchers examine allocative efficiency by asking whether firms with higher levels of Hicks-neutral technology produce more output. Others pose the same question in terms of labor productivity, which includes scale economies but does not subtract capital's contribution to output. Using our framework, it is easy to show that only the Solow TFP index gives the correct welfare accounting. Unlike a pure technical change measure, the Solow residual includes scale effects, which do contribute to welfare by producing more output for given inputs. Unlike labor productivity, the TFP residual subtracts the change in capital input valued at its opportunity cost to the consumer.

Our analytical results create several links between productivity and welfare. One important message is that welfare depends on the entire expected future path of TFP. Not surprisingly, the same size change in current TFP has very different effects on welfare if it is expected to be persistent than if it is expected to be transitory. This result suggests new ways of assessing the importance of reallocation. To our knowledge, the literature does not examine the time-series properties, especially the persistence, of measures of allocative efficiency. But our derivation shows that to understand the contribution that reallocation makes to growth, it is important to know the persistence as well as the mean. In principle, the allocative efficiency component of TFP might be either more or less persistent than total TFP, making reallocation either more or less important than its average share would suggest.

So far we have been vague about whether our results relate to TFP in levels or in growth rates. In fact, our results apply to both. We show that the *level* of welfare for a representative consumer is, up to a first-order approximation, proportional to the present discounted value of expected log levels of TFP. Welfare *change* for the consumer, on the other hand, is proportional to the change in log levels, i.e., to the present discounted value of TFP growth as we define it (equal to the standard Solow productivity residual if there are zero economic profits), plus an "expectation revision" term that depends on the difference in expectations of future log levels of TFP between time t-1 and time t. Under perfect foresight, the expectation revision term is identically zero, and the change in welfare is proportional to the present discounted value of current and future Solow residuals alone.

Starting from a well-posed optimization problem also forces us to confront two issues in national income and welfare measurement. First, our derivation shows that "consumption" should be defined as any good or service that consumers value, whether or not it is included in GDP. Similarly, "capital" should include all consumption that is foregone now in order to raise consumption possibilities for the future. These items include, for example, environmental quality and intangible capital. Of course, both are hard to measure and even harder to value, since there is usually no explicit market price for either good. But our derivation is quite clear on the principle that the environment, intangibles and other non-market goods should be included in our measure of "welfare TFP." We follow conventional practice in restricting the output measure for our TFP variable to market output (and the inputs to measured physical capital and labor), but in so doing we, and almost everyone else, are mismeasuring real GDP and TFP. Second, our starting point of a

productivity measurement, follows a long tradition of research on this topic, and is fully supported by our derivation.

representative-consumer framework implies that we automatically ignore issues of distribution that intuition says should matter for social welfare. We believe that distributional issues are very important. However, our objective of constructing a better welfare measure from aggregate data alone implies that we cannot incorporate measures of distribution into our framework. Thus, we maintain the representative-consumer framework, but without in any way minimizing the importance of issues that cannot be handled within that framework.

Having established that aggregate TFP is the natural measuring stick for aggregate welfare, we then ask the next natural question: Can one show what contribution a subset of the economy (which may be as small as a single firm) makes to the aggregate welfare index? The answer is yes, as shown by Domar (1961). Domar established that a correctly-weighted average of sectoral TFP residuals sums to Solow's familiar aggregate index. We use a variant of his result to present the welfare contributions made by large sectors of the economy using the EU-KLEMS dataset. We compare the sources of welfare differences across countries, asking what fraction of cross-country differences are due to differences in industrial structure as opposed to differences in the welfare contributions of the same sector across countries. We then do a similar exercise using our firm-level data over the period 1998-2004, and investigate the extent to which differences in the relative productivity distribution of firms across countries contributes to differences in welfare.

Finally, we decompose aggregate TFP into components. As we noted, while TFP is itself meaningful in welfare terms without any additional assumptions, we need to make assumptions about firm technology and behavior in order to decompose TFP in a meaningful way. We use a variant of the decomposition of Basu and Fernald (2002), which is derived by assuming that firms minimize costs and are price-takers in factor markets, but may have market power for the goods they sell and might produce with increasing returns to scale. As we also noted, different assumptions about technology would give different decompositions, without changing the essential features of the results. For example, Basu and Fernald (2002) assume that factors are freely mobile across firms, without adjustment costs, while Basu, Fernald and Shapiro (2001) extend the framework to include costly factor adjustment. Abel (2003) and Basu et al. (2001) show that adjustment costs are a special type of intangible capital, of a sort that needs to be accumulated in fixed proportions with physical capital. Thus, accounting for adjustment costs in the empirical results would require us to impute an addition to measured output, which is conceptually the same issue as accounting for non-market consumption goods or for more general forms of intangible capital accumulation.

Some of the components in the decomposition we use can be clearly identified as being due to reallocation, since they depend on marginal products of identical inputs not being equalized across firms. Other components depend on aggregate distortions, such as the average degree of market power and various tax rates. In order to estimate the reallocation terms, we need to estimate firm-level marginal products. We do so using firm-level data for a number of manufacturing industries across six European countries, as represented in the Amadeus data base.[4] We extend the existing

[4]Petrin, White and Reiter (2009) also use firm-level data to implement a variant of the Basu-Fernald (2002) decomposition. They use U.S. Census data for manufacturing industries. We compare our results to theirs in Section 6.

decomposition to study reallocation both within and between industries, since the two kinds of reallocation may have different policy implications.

We use the Amadeus data to estimate production functions for firms within a number of manufacturing industries across six countries. We experimented with a variety of estimation methods to ensure that our main results were robust. We found that there is usually a substantial gap between our estimates of technical change for each manufacturing industry and that industry's contribution to aggregate TFP growth (and hence welfare). However, for most countries, the majority of this gap is due to the aggregate distortions (especially when taxes are included in the decomposition). Reallocation strictly defined usually accounts for a small fraction of the gap.

The paper is organized as follows. We present the key equations linking productivity and welfare in Section 2, with the full derivation presented in an appendix. While our derivations link welfare to both TFP levels and growth rates, we choose to work mostly in growth rates, since there are well-known difficulties in comparing TFP levels across industries and countries. Then, in Section 3, we present our data, and decompose productivity growth for each country into a variety of components, at both the sectoral and firm level. We then switch to an econometric framework for decomposing the sources of welfare change in Sections 4 and 5, and present the results in Section 6. We discuss the relation of our work to the existing literature in Section 7. We conclude in Section 8 with some reflections, and suggestions for future research.

2 The Productivity Residual and Welfare

It is intuitive that technology growth matters for welfare purposes, since our intuition suggests that technological progress is responsible for the secular increase in the standard of living. However, should we care about the Solow residual in an economy with non-competitive output markets, non-constant returns to scale, and possibly other distortions? Here we build on the intuition of Basu and Fernald (2002) that a slightly modified form of the Solow residual is welfare relevant even in those circumstances and derive rigorously the relationship between a modified version of the productivity residual (in growth rates or log levels) and the intertemporal utility of the representative household. The fundamental result we obtain is that, to a first-order approximation, utility reflects the present discounted values of productivity residuals.

Our results are complementary to those in Solow's classic (1957) paper. Solow established that if there was an aggregate production function then his index measured its technical change. We now show that under a very different set of assumptions, which are disjoint from Solow's, the familiar TFP index is also the correct welfare measure. The results are parallel to one another. Solow did not need to assume anything about the consumer side of the economy to give a technical interpretation to his index, but he had to make assumptions about technology and firm behavior. We do not need to assume anything about the firm side (which includes technology, but also firm behavior and industrial organization) in order to give a welfare interpretation, but we do need to assume the existence of a representative consumer. Both results assume the existence of a potential function (Hulten, 1973), and show that TFP is the rate of change of that function. Which result is more useful depends on the application, and the tradeoff that one is willing to make between having a result that is very general on the consumer side but requires very precise assumptions on technology and firm behavior, and a result that is just the opposite.

2.1 Approximating around the steady state

More precisely, assume that the representative household maximizes intertemporal utility:

$$V_t = E_t \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} U(C_{1,t+s}, .., C_{Z,t+s}; \overline{L} - L_{t+s}) \tag{1}$$

where $C_{i,t}$ is the capita consumption of good i at time t, L_t are hours of work per capita, $\overline{L}$ is the time endowment, and N_t population. H is the number of households, assumed to be fixed and normalized to one from now on. X_t denotes Harrod neutral technological progress, assumed to be common across all sectors. Population grows at constant rate n and X_t at rate g. For a well defined state state in which hours of work are constant we assume that the utility function has the King Plosser and Rebelo form(1988):

$$U(C_{1,t+s}, .., C_{Z,t+s}; L - \overline{L}_s) = \frac{1}{1-\sigma} C(C_{1,t+s}, .., C_{Z,t+s})^{1-\sigma} \nu(\overline{L} - L_{t+s})$$

with $0 < \sigma < 1$ or $\sigma > 1$.[5] We assume that $C()$ has constant returns to scale. Define $c_{i,t+s} = \frac{C_{i,t+s}}{X_{t+s}}$. We can rewrite the utility function in a normalized form as follow:

$$v_t = \frac{V_t}{N_t X_t^{(1-\sigma)}} = E_t \sum_{s=0}^{\infty} \beta^s U(c_{1,t+s}, .., c_{Z,t+s}; \overline{L} - L_{t+s}) \tag{2}$$

where $\beta = \frac{(1+n)(1+g)^{1-\sigma}}{(1+\rho)}$ is assumed to be less than one. The budget constraint (with variables scaled by $N_t X_t$) is:

$$k_t + b_t = \frac{(1-\delta)}{(1+g)(1+n)} k_{t-1} + \frac{(1+r_t)}{(1+g)(1+n)} b_{t-1} + p_t^L L_t + p_t^K k_t + \pi_t - \sum_{i=1}^{Z} p_{i,t}^C c_{i,t} \tag{3}$$

New capital goods are the numeraire, $k_t = \frac{K_t}{X_t N_t}$ denotes capital per unit of effective labor, , $b_t = \frac{B_t}{P_t^I X_t N_t}$ are real bonds. $p_t^K = \frac{P_t^K}{P_t^I}$, $p_t^L = \frac{P_t^L}{P_t^I X_t}$, $p_{i,t}^C = \frac{P_{i,t}^C}{P_t^I}$ denote, respectively, the user cost of capital, the wage per hour of effective labor, and the price of consumption goods. $(1 + r_t)$ is the real interest rate (again in terms of new capital goods) and $\pi_t = \frac{\Pi_t}{P_t^I X_t N_t}$ denotes profits.

Log linearizing around the non stochastic steady state, intertemporal household utility can be

[5] If $\sigma = 1$, then the utility function must be $U(C_1, .., C_G; \overline{L} - L) = \log(C) - \nu(\overline{L} - L)$. See King, Plosser and Rebelo (1988).

written as:

$$v_t - v = E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[\sum_{i=1}^{Z} p_i^C c_i \widehat{c}_{i,t+s} + i \widehat{i}_{t+s} - \ p^L L \widehat{L}_{t+s} - \ p^K k \widehat{k}_{t+s} \right] \tag{4}$$

where v is the steady state value of utility, $\widehat{x} = \log x_t - \log x$ denote log deviation from the steady state.In obtaining this result we have used the FOC of the household maximization problem:

$$U_{c_{i,t}} - \lambda_t p_{i,t}^C = 0 \tag{5}$$

$$U_{L_t} + \lambda_t p_t^L = 0 \tag{6}$$

$$\lambda_t \left(p_t^K - 1 \right) + \beta \frac{(1-\delta)}{(1+g)(1+n)} E_t \lambda_{t+1} = 0 \tag{7}$$

$$-\lambda_t + \beta \frac{1}{(1+g)(1+n)} E_t (1+r_t) \lambda_{t+1} = 0 \tag{8}$$

and the log linear approximation of the budget constraint around the steady state:

$$k \ \widehat{k}_t + b\widehat{b}_t = \frac{(1-\delta)}{(1+g)(1+n)} k\widehat{k}_{t-1} + \frac{(1+r)}{(1+g)(1+n)} b\widehat{b}_{t-1} + p^L L \widehat{L}_t + p^K k \ \widehat{k}_t + p^L L \widehat{p}_t^L + p^K k \ \widehat{p}_t^K$$
$$+ \pi \widehat{\pi}_t - \sum_{i=1}^{Z} p_i^C c_i \widehat{c}_{i,t} - \sum_{i=1}^{Z} p_i^C c_i \widehat{p}_{i,t}$$

Equation (4) says that intertemporal utility (in log deviation from the steady state) equals the expected present discounted value of terms that represent the sum of the components of final demand (in log deviation from the steady state), weighted by their steady state contribution to demand, minus primary inputs (in log deviation from the steady state) times their respective steady state factor prices.

2.2 Connecting with the productivity residual

We are now close to relating utility to a modified version of the Solow residual. There are two options here. The first one is to obtain a first order approximation for the log level of utility in terms of the log level productivity residual. Simple manipulations allow us to rewrite log level utility as a function of expected future Solow residuals plus an initial (log) level productivity residual. The second one focuses instead on approximating the log change in utility over time.

To connect utility with the Solow residual, we will rely on the following (Divisia) definition of growth in normalized value added:

$$\Delta \log y_t = \sum_{i=1}^{Z} \frac{p_i^C c_i}{p^Y y} \Delta \log c_{i,t+s} + \frac{i}{p^Y y} \Delta \log i_t \tag{9}$$

Using the fact that nominal value added $P_tY_t = \sum_{i=1}^{Z} P_{i,t}^C C_{i,t} N_t + P_t^I I_t$, it is also true that non-normalized value added growth, $\Delta \log Y_t$, equals:

$$\Delta \log Y_t = \sum_{i=1}^{Z} \frac{P_i^C C_i N}{P^Y Y} \Delta \log(C_{i,t} N_t) + \frac{P^I I}{P^Y Y} \Delta \log I_t \tag{10}$$

where the growth rate of each demand component is aggregated using constant steady state shares.[6]

To establish a relationship with the (log) level of productivity, we will, instead, use the fact that, to a first order approximation, the level of value added (in terms of normalized variables):

$$\widehat{y}_t = \log y_t - \log y = \sum_{i=1}^{Z} \frac{P_i^C C_i N}{P^Y Y} \widehat{c}_{it} + \frac{P^I I}{P^Y Y} \widehat{i}_t = \sum_{i=1}^{Z} s_{c_i} \widehat{c}_{it} + s_i \widehat{i}_t \tag{11}$$

Starting from this latter case, using (11), intertemporal utility in (4) can be written as:

$$v_t - v = (\lambda p^Y y) E_t \sum_{s=0}^{\infty} \beta^s \left[\widehat{y}_t - \frac{p^L L}{p^Y y} \widehat{L}_{t+s} - \frac{p^K k}{p^Y y} \widehat{k}_{t+s} \right] \tag{12}$$

which, after some manipulations detailed in the appendix can be rewritten as:

$$\frac{v_t - v}{v} = \frac{(1-\sigma)(1-\beta)}{s_C} E_t \sum_{s=0}^{\infty} \beta^s \log pr_{t+s} + \Lambda(t) \tag{13}$$

where:

$$\log pr_t = \log Y_t - s_L \log N_t L_t - s_K \log K_t \tag{14}$$

is the log level of aggregate value added, $\log Y_t$, minus aggregate factor inputs, $\log N_t L_t$ and $\log K_t$ multiplied by their respective distributional shares, s_L and s_K. $s_C = \frac{\sum_{i=1}^{Z} p_i^C c_i}{p^y y}$ is the share of consumption goods in value added and $\Lambda(t)$ is a deterministic function of time:

$$\begin{aligned}\Lambda(t) \quad = \quad & -\frac{(1-\sigma)}{s_C} \left[\log y - s_L \log L - s_K \log k + \frac{\beta}{(1-\beta)} \left[g(1-s_K) + n(1-s_L-s_K) \right] \right] \\ & -\frac{(1-\sigma)}{s_C} \left[(1-s_K) \log X_t + (1-s_L-s_K) \log N_t \right]\end{aligned} \tag{15}$$

Utility, therefore, is an increasing function of the sequence of (log) level aggregate productivity residuals, appropriately discounted.[7]

To establish the relationship with the Solow residual (a growth rate concept) there are two options. One option is to use the fact that, for any variable x :

$$E_t \widehat{x}_{t+s} = E_t(\log x_{t+s} - \log x) = E_t \sum_{i=1}^{s} (\log x_{t+i} - \log x_{t+i-1}) + \log x_t - \log x$$

[6] Here we are departing slightly from convention, as value added is usually calculated with time varying shares.
[7] Note that the utility index v is positive for $0 < \sigma < 1$ and negative for $\sigma > 1$.

In the appendix we show that log level utility,(4), implies that per capita (log) intertemporal utility can also be written as:

$$\frac{v_t - v}{v} = \frac{(1-\sigma)}{s_C} E_t \sum_{s=1}^{\infty} \beta^s \Delta \log pr_{t+s} + \frac{(1-\sigma)}{s_C} \log pr_t + \Lambda(t) \tag{16}$$

where $\Delta \log pr_t$ denotes the "modified" Solow productivity residual:

$$\Delta \log pr_{t+s} = \Delta \log Y_{t+s} - s_L \Delta \log N_{t+s} L_{t+s} - s_K \Delta \log \ K_{t+s} \tag{17}$$

We use the word "modified," for two reasons. First, we do not assume that the distributional shares of capital and labor add to one, as they would if there were zero economic profits. Zero profits are guaranteed in the benchmark case with perfect competition and constant returns to scale, but can also arise with imperfect competition and increasing returns to scale, as long as there is free entry, as in the standard Chamberlinian model of imperfect competition. Second, the distributional shares are calculated at their steady state values and, hence, are not time varying. Rotemberg and Woodford (1991) argue that in a consistent first-order log-linearization of the production function the shares of capital and labor should be taken to be constant, and Solow's (1957) use of time-varying shares amounts to keeping some second-order terms while ignoring others. Now log level productivity has been written as a combination of expected future Solow residuals and one initial productivity term in levels. Assume one is willing to make the assumption that an economy at time t-1 was at the steady state, so that $\log y_{t-1} - s_L \log L_{t-1} - s_K \log k_{t-1} = \log y - s_L \log L - s_K \log k$ In this special case simple algebra shows that v_t depends upon the expected present discounted value of Solow residuals (from the present to infinity)

$$\frac{v_t - v}{v} = \frac{(1-\sigma)}{s_C} E_t \sum_{s=0}^{\infty} \beta^s \Delta \log pr_{t+s} + \Lambda_0 \tag{18}$$

where:

$$\Lambda_0 = \frac{(1-\sigma)}{(1-\beta) s_C} \left[g(1 - s_K) + n(1 - s_K - s_L) \right] \tag{19}$$

An alternative and more satisfactory way to illustrate the relationship between welfare and the Solow residual (with no level term) is to return to (4) and take its difference through time ($\Delta v_t = v_t - v_{t-1}$). Using only the definition of value added in growth terms, equation (9), the growth rate of per capita utility can be written as follows:

$$\begin{aligned} \frac{\Delta v_t}{v} &= \frac{(1-\sigma)(1-\beta)}{s_C} E_t \sum_{s=0}^{\infty} \beta^s \Delta \log pr_{t+s} + \Lambda_1 \\ &+ \frac{(1-\sigma)(1-\beta)}{s_C} \sum_{s=0}^{\infty} \beta^s \left[E_t \log pr_{t+s} - E_{t-1} \log pr_{t+s} \right] \end{aligned} \tag{20}$$

where $E_t \log pr_{t+s} - E_{t-1} \log pr_{t+s}$ represents the revision in expectations of the log level of the productivity residual, based on the new information received between t-1 and t. Moreover, the constant Λ_1 is:

$$\Lambda_1 = \frac{(1-\sigma)}{s_C} \left[g(1-s_K) + n(1-s_K-s_L) \right] \tag{21}$$

Note that the revision term in the second summation will reduce to a linear combination of the innovations in the stochastic shocks affecting the economy at time t. Moreover, if we assume that the modified Solow residual follows a simple stable first order autoregressive process, then the current Solow residual, $\Delta \log pr_t$, is a sufficient statistic for all the terms in the first summation. In this case, the growth in expected per capita utility is a linear function only of today's actual Solow residual and of innovations at time t in the stochastic processes driving the economy.

2.3 Extensions

We now show that our method of using TFP to measure welfare can be extended to cover multiple types of capital and labor, taxes, and government expenditure. The first extension modifies our baseline results in only a trivial way, but the others all require more substantial changes to the formulas above. These results show that the basic idea of using TFP to measure welfare holds in a variety of economic environments, but also demonstrate the advantage of deriving the welfare measure from an explicit dynamic model of the household. The model shows exactly what modifications to the basic framework are required in each case, and demonstrates that some of these modifications are quantitatively significant.

2.3.1 Multiple Types of Capital and Labor

The extension to the case of multiple types of labor and capital is immediate. For simplicity, we could assume that each individual is endowed with the ability to provide different types of labor services, $L_{h,t}$ and that the utility function can be written as:

$$U(C_{1,t+s},..,C_{Z,t+s},\overline{L},L_{1,t+s},...,L_{H_L,t+s}) = \frac{1}{1-\sigma} C(C_{1,t+s},..,C_{Z,t+s})^{1-\sigma} \nu \left[\overline{L} - L(L_{1,t+s},...,L_{H_L,t+s}) \right] \tag{22}$$

where $L(.)$ is an homogenous function of degree 1, H_L is the number of types of labor and $P_t^{L_h}$denotes the payment to a unit of $L_{h,t}$.[8] Similarly consumers can accumulate different types of capitals $K_{h,t}$ and rent them out at $P_t^{K_h}$. Proceeding exactly as before, the same equations will characterize the realtionship between utility and the Solow residual, with the only difference that the latter is defined now as:

$$\log pr_t = \log Y_t - \sum_{h=1}^{H_K} s_{L_h} \log N_t L_{h,t} - \sum_{h}^{H_L} s_{K_h} \log K_{h,t}$$

[8] We assume that the nature of the utility function is such that positive quantities of all types of labors are supplied.

2.3.2 Taxes

Our derivation of section 2.2 requires only reinterpretation to apply exactly to an environment with either distortionary and/or lump-sum taxes. The reason is that all prices in the budget constraint, equation (4), are from the point of view of the consumer. Thus, if there are taxes, the prices should all be interpreted as after-tax prices. Therefore our derivation implicitly allows for proportional taxes on capital and labor income as well as sales or value-added taxes levied on consumption and/or investment goods. The variable that we have been calling "profits," π, is really any transfer of income that the consumer takes as exogenous. Thus, it can be interpreted to include lump-sum taxes or rebates.

However, for the sake of exposition, we shall interpret all prices in equation (4) as being from the point of view of a firm, and thus before all taxes. To modify (4) to allow for taxes, we define some notation. Let τ^K be the tax rate on capital income, τ^L be the tax rate on labor income, τ_i^C be the *ad valorem* tax on consumption goods of type i, and τ^I be the corresponding tax on investment goods. We assume that the revenue so raised is distributed back to individuals using lump-sum transfers. (We consider government expenditures in the next sub-section.) Then it is apparent that we arrive at the following modified version of equation (4):

$$v_t - v = \lambda E_t \sum_{s=0}^{\infty} \beta^s \left[\sum_{i=1}^{Z} \left(1+\tau_i^C\right) p_i^C c_i \widehat{c}_{i,t+s} + \left(1+\tau^I\right) i\widehat{i}_{t+s} - \left(1-\tau^L\right) \; p^L L \widehat{L}_{t+s} - \left(1-\tau^K\right) \; p^K k \widehat{k}_{t+s} \right] \tag{23}$$

To make contact with the data, note that the national accounts define nominal expenditure using prices as perceived from the demand side. Thus, equation (11) can be written exactly as before and still be consistent with standard national accounts data:

$$\widehat{y}_{t+s} = \sum_{i=1}^{Z} s_{c_i} \widehat{c}_{it} + s_i \widehat{i}_t \tag{24}$$

On the other hand, the national accounts define factor prices as perceived by firms, before income taxes. Thus, the data-consistent definition of the welfare residual with taxes needs to be based on a new definition of $\log pr_t$. Rewrite equation (14) as:

$$\begin{aligned} \log pr_{t+s} &= \log Y_{t+s} - \frac{\left(1-\tau^L\right) p^L L N}{p^Y y} \log N_{t+s} L_{t+s} - \frac{\left(1-\tau^K\right) p^K k}{p^Y y} \log K_{t+s} \\ &= \log Y_{t+s} - \left(1-\tau^L\right) s_L \log N_{t+s} L_{t+s} - \left(1-\tau^K\right) s_K \log K_{t+s} \end{aligned} \tag{25}$$

This new definition of $\log pr_t$ then needs to be applied to equations such as (12) and (13) in section 2.2.

While it is easy to incorporate taxes into the analysis—as noted above, they are present implicitly in the basic expressions derived in section 2.2—the quantitative impact of modeling taxes explicitly can be large. Suppose that output is produced using an aggregate, constant-returns-to-scale production function of capital, labor and technology, as in Solow's classic (1957) paper. Then, without distortionary taxes, only changes in technology change welfare.

Now suppose the average marginal tax rate on both capital and labor income is 30 percent, and the share of consumption in output is 0.60. Suppose the government manages to raise aggregate capital and labor inputs by 1 percent permanently without a change in technology (perhaps via a small cut in tax rates). Then the flow increase in utility is equivalent to an increase in steady-state consumption of 0.5 percent. If the discount factor is 0.95 on an annual basis, the present value of this policy change is equivalent to a one-year increase in consumption of 10 percent of the steady-state level!

2.3.3 Government expenditure

With some minor modification, our framework can be extended to allow for the provision of public goods and services. We illustrate this under the assumption that government activity is financed with lump-sum taxes. Using the results from the previous subsection, it is straightforward to extend the argument to the case of distortionary taxes.

Assume that government spending takes the form of public consumption valued by consumers. We rewrite the instantaneous utility function as

$$U(C_{1,t+s},..,C_{Z,t+s},\overline{L},L_{1,t+s},...,L_{H_L,t+s}) = \frac{1}{1-\sigma}C(C_{1,t+s},..,C_{Z,t+s};G_{t+s})^{1-\sigma}\nu(\overline{L}-L_{t+s}) \quad (26)$$

where G denotes per-capita public consumption, and we continue to assume that $C(.)$ is homogenous of degree one in its arguments. The relevant welfare residual in equation (4) now becomes:

$$v_t - v = \lambda E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{U_g g \widehat{g}_{t+s}}{\lambda} + \sum_{i=1}^{Z} p_i^C c_i \widehat{c}_{i,t+s} + i\widehat{i}_{t+s} - \ p^L L \widehat{L}_{t+s} - \ p^K k \widehat{k}_{t+s} \right] \quad (27)$$

where $g_t = \frac{G_t}{X_t}$. The definition of GDP in deviation from steady state is now:

$$\widehat{y}_t = \sum_{i=1}^{Z} s_{c_i}\widehat{c}_{it} + s_i \widehat{i}_t + s_g \widehat{g}_t$$

where $s_g = \frac{P^G G}{P^Y Y}$ and P^G is the public consumption deflator. Let $s_g^* = \frac{U_g g \widehat{g}_{t+s}}{\lambda}$. Then we can write:

$$v_t - v = \lambda p^Y y E_t \sum_{s=0}^{\infty} \beta^s \left[\widehat{y}_{t+s} - s_L \widehat{L}_{t+s} - s_K \widehat{k}_{t+s} + \left(s_g^* - s_g\right)\widehat{g}_{t+s} \right]$$

Hence in the presence of public consumption the Solow residual needs to be adjusted up or down depending on whether public consumption is under- or over-provided (i.e., $s_g^* > s_g$ or $s_g^* < s_g$ respectively). If the government sets public consumption exactly at the utility-maximizing level, $s_g^* = s_g$ and no correction is necessary. In turn, in the standard neoclassical case in which public consumption is pure waste $s_g^* = 0$, the welfare residual is computed on the basis of private final demand – i.e., GDP minus government purchases.

What if government purchases also yield productive services to private agents? This could be the case if, for example, the government provides education or health services, or public infrastructure, which may be directly valued by consumers and may also raise private-sector productivity. In such case, the above expression remains valid, but it is important to note that the net contribution of public expenditure to welfare would not be fully by captured by $(s_g^* - s_g)\widehat{g}_{t+s}$. To this term we would need to add a measure of the productivity of public services, which in the expression is implicitly included in the productivity residual $\widehat{y}_{t+s} - s_L\widehat{L}_{t+s} - s_K\widehat{k}_{t+s}$.

3 Decomposing the Productivity Residual: Firm and Sector Level Contributions

The fundamental result from the previous section is that the growth in welfare is related to the expected present discounted value of the aggregate (modified) Solow productivity residual. In this section we will argue that this aggregate effect can be decomposed into the contribution of individual firms (or subset of firms). In order to do this we will look at aggregate value added, not from the expenditure side as we have done so far, but from the product side. More specifically, define aggregate value added as the following Tornqvist/Divisia index of firm-level value added:

$$\Delta \log Y_t = \sum_i w_i \Delta \log Y_{i,t} \tag{28}$$

The corresponding index for producer prices is:

$$\Delta \log P^Y{}_t = \sum_i w_i \Delta \log P^Y_{i,t} \tag{29}$$

Moreover, one can easily show that the following is true as an approximation:

$$s_K \Delta \log K_t = \sum_i w_i s_{K,i} \Delta \log K_{i,t}$$

and

$$s_L \Delta \log N_t L_t = \sum_i w_i s_{L,i} \Delta \log N_t L_{i,t}$$

As a result the aggregate Solow residual can be written as the weighted sum (with value-added weights) of the firm-level Solow residuals. More specifically:

$$\Delta \log pr_t = \sum_i w_i \Delta \log pr_{it}$$

where $\Delta \log pr_{it}$ is defined as:

$$\Delta \log pr_{it} = \Delta \log Y_{i,t} - s_{K,i} \Delta \log K_{i,t} - s_{L,i} \Delta \log N_t L_{i,t} \tag{30}$$

We can use this result to examine the sectoral sources of productivity growth, which is the key to welfare change, within a country. We can ask a variant of the same question for firms, as we explain in the results sub-section. Finally, we can compare cross-sectional summary statistics. For example, we can ask whether small or large firms contribute more to national welfare improvement.

4 Data and Measurement

Our main source of information is Amadeus, a comprehensive firm-level pan-European database developed by Bureau Van Dijk. For every firm it provides data on the industry where it operates (at the 4-digit NACE level), its location, the year of incorporation, the ownership structure and the number of employees, in addition to the complete balance sheets and the profit and loss accounts. The data set includes both publicly traded and non traded companies. We limit our analysis to a subset of countries: Belgium, France, Great Britain, Italy, and Spain. We focus on manufacturing companies with operating revenues greater than or equal to 2 million Euros and continuous observations within the period of analysis. (We restrict ourselves to the balanced panel because Amadeus does not supply census data; there is no way to distinguish between entry into the sample and actual entry into the economy.)

We also use industry-level yearly data from the EU-KLEMS project, which provides output, input and price data for industries at roughly the 2-digit level of aggregation across a large number of countries up to 2005. These countries are mostly, but not exclusively, European; the project also gives data for non-EU countries like Australia, Japan, Korea and the United States. The EU-KLEMS data are extensively documented by O'Mahony and Timmer (2009).

In addition to the non-parametric welfare-relevant index numbers presented in the next section, we will also estimate production functions using firm level data, allowing the coefficients to vary across 2-digit industries for the period 1998-2005.[9] Before 1998 the number of firms in the survey is significantly smaller in most countries. Between 1998 and 2000 many firms enter in the data set. The coverage provided by the dataset varies across these countries. In 2005 the aggregated sales of the firms represented in Amadeus represent between 20 percent and 45 percent of the manufacturing sector's total production value, as documented in EU-KLEMS.

Our gross output proxy is (firm level) revenues deflated by the sectoral value added deflator obtained from the EU-KLEMS data set, at the 2 digit level. All deflators used here will be at the 2 digit level and are obtained from EU-KLEMS. We are aware that using industry deflators in place of firm-level prices can cause problems (Klette and Grilliches (1996)), but firm-level price data for output are not available in Amadeus. Our proxy for labor input is manpower costs deflated by the labor services deflator. (For some countries, such as Italy, the number of employees figure is not reliable, since there is not a reporting requirement for the number of employees in the main section of the balance sheet.) Capital is the historical value of tangible fixed assets divided by the price index for investment. We have also experimented with the perpetual inventory method, obtaining

[9] The use of a finer sectoral disaggregation is questionable if one wants to have enough firms in each sector for estimation purposes.

similar results. A measure for materials, intermediates and other services used in production has been computed using the following formula: materials = Operating Revenues - (Operating Profits+Manpower costs+Depreciation). The figure obtained in this way is then deflated by the materials and services deflator. Given gross output and materials input, value added is constructed as a Tornqvist/Divisia index.

5 Sources of Welfare Differences

Welfare change depends on the expected present discounted value of expected TFP growth as shown by equations (18) and (20). It is therefore important to investigate the time-series property of TFP growth. We do so in Table 1, using annual data from EU-KLEMS up to 2005 for the entire private economy for Belgium, France, Great Britain, Italy, and Spain. We use the measure of TFP developed in EU-KLEMS, based on the assumption of zero profits and time varying distributional shares and present both country by country and pooled results. The persistence of TFP growth is a key statistic, since it shows how the entire summation of expected productivity residuals changes as a function of the new information about the TFP growth rate. For most countries the log level of the TFP index is well described by an AR(1) stationary process around a country-specific linear trend. Additional lags of log TFP are not significant and the residual is white noise, as suggested by the Lagrange Multiplier test for residual serial correlation. The only exception is Spain, where the coefficent of log TFP (t-3) is significantly different from zero at the 5% level and the LM test rejects the hypothesis of no serial correlation (up to the third order). Thus, for most countries the growth rate of TFP is well described by an ARMA(1,1) model. We henceforth focus on the current TFP growth rate, since for most countries the data do not reject the proposition that the current growth rate (or its innovation) gives all necessary information about the entire future path of TFP, and hence welfare.

We first ask which sectors contributed the most to welfare change in these countries over the period of our study. The results are in Table 2. We look at the contributions of five major industry groups: Manufacturing, Utilities, Construction, Wholesale and Retail Trade and FIRE. For each country, we present in line 1 the mean of the Tornqvist index of TFP growth for these industries, which represent the overwhelming majority of private output. Interestingly, average TFP growth over this period is less than 1 percent per annum, even for the leading economies, France and Britain. The sectoral decompositions are also interesting. The next five lines give average sectoral TFP growth rates (not growth contributions, which would multiply the growth rates by the respective sectoral weights, and give a mechanical advantage to large sectors). Manufacturing makes a positive contribution for each country. The contribution of FIRE (Finance, Insurance, Real Estate), on the other hand, is often negative, especially in Britain, which has become a financial hub for the world.[10] But the humble utility sectors are the largest source of productivity growth on average

[10] However, measures of both nominal and real financial sector output are often unreliable. See Wang, Basu and Fernald (forthcoming) for a model-based method for constructing financial sector output. Basu, Inklaar and Wang (forthcoming) apply this theory to construct nominal bank output measures, and Inklaar and Wang (2007) provide

(in every country other than Italy). Alesina et al (2005) suggest an explanation for this pattern based on deregulation of the utilities sectors in many European countries (with Italy a laggard in terms of the timing and pace of deregulation).

In Table 3, we look at the contributions of different groups of firms to welfare growth, now using our firm-level data from Amadeus for these countries. We now look at the average TFP growth rates of small and large firms, from 1998-2004. No very clear pattern emerges. Large firms have higher TFP growth rates in two countries (Belgium and Spain); small firms have higher growth rates in two others (Italy and Great Britain), and the contributions are basically identical in the remaining country, France.

We can further decompose productivity differences across countries by applying the following decomposition, based on Griliches and Regev (1995). We wish to ask whether the difference in the productivity growth rate of any pair of countries is due to differences in their sectoral compositions or to differences in the growth rates for each sector. Let i now index sectors (not firms) and C be one of the countries in our sample other than the UK.

$$\begin{aligned}\sum_i w_i^C \Delta \log pr_{it}^C - \sum_i w_i^{UK} \Delta \log pr_{it}^{UK} &= \sum_i \frac{(w_i^C + w_i^{UK})}{2} (\Delta \log pr_{it}^C - \Delta \log pr_{it}^{UK}) \\ &+ \sum_i \frac{(\Delta \log pr_{it}^C + \Delta \log pr_{it}^{UK})}{2} (w_i^C - w_i^{UK})\end{aligned}$$

In Table 4, we examine the results of the Griliches-Regev (1995) decomposition, investigating the sources of growth of each country's TFP relative to that of Britain, which is the TFP growth leader over our period. The first column describes the difference in productivity change between Great Britain and the other economies in our sample (and is of course negative in all cases). The second column gives the amount of the difference accounted for by cross country differences in TFP growth for each sector, while the third column gives the amount of the difference due to differences in industrial structure (the share of each industry in the aggregate for that country). In most cases, cross country differences in the growth rate of the same sector account for the great majority of the gap with the UK. The exception is France, which actually grows faster than Britain comparing the same sector in the two countries, but loses nearly two-tenths of a percentage point of TFP growth per year due to differences in industrial structure.

In Table 5, we do an exercise designed to show whether the productivity patterns in each country are related to cross-country differences in the shape of the distribution of productivity growth rates across firms. This is an exercise in the spirit of Hsieh and Klenow (2009). However, Hsieh and Klenow expended considerable effort (and had to make a number of strong assumptions) in order to isolate firm-level technology within each country-sector. Our results show that if the object is to investigate the reasons for differences in welfare change across countries, it is not necessary (and indeed not sufficient) to understand how technology differs across firms; we should concentrate on

a theory-consistent measure of real bank output in the United States.

differences in the Solow residual instead. We do the following exercise. For our full sample of firms within each country, we calculate TFP, and then divide firm-level TFP growth by TFP growth for the aggregate of the firms in that country. We then divide the range of productivity growth rates into 10 bins, and ask what percentage of firm value-added is produced by firms in each standardized productivity decile. (We experimented with dividing the range of growth rates more finely, into 20 bins, with qualitatively similar results.) Finally, we ask how much faster or slower aggregate TFP would have grown if the standardized distribution for the country had been replaced by the standardized distribution for Great Britain.

The results are in Table 5. For ease of viewing the results, we also plot the distributions for each country and the distribution for Britain in Figure 1. We find that replacing the distributions in Belgium and Spain with the British distribution would actually have caused those two countries to grow slightly more slowly. However, the same exercise for France and Italy shows that those two countries would each have had half a percentage point higher TFP growth per year over the full six years. This is a significant difference, especially for Italy where it approximately doubles the annual TFP growth for our aggregate of firms. Thus, there is some evidence that a portion of the TFP growth differences relative to Britain, which is the probably the least regulated and most "US-like" of the countries in our sample, is driven by differences in institutions that allow weak firms to linger or prevent strong firms from expanding. The evidence is particularly strong in the case of Italy, which has been a conspicuous laggard in its rate of productivity growth over the last decade.

6 Decomposing the Productivity Residual: The Role of Reallocation and Technology

The great benefit of an index-number approach, such as the one we take in the previous section, is that it provides interesting results without requiring formal econometrics. The cost is that we cannot then identify the components of productivity growth, such as technical change or scale economies. Having established that aggregate TFP is the natural measuring stick for aggregate welfare, we now proceed to decompose aggregate TFP into components. We choose to work in growth rates, since there are well-known difficulties in comparing TFP levels across industries and countries. As we noted, while TFP growth is itself meaningful in welfare terms without any additional assumptions, we need to make assumptions about firm technology and behavior in order to decompose it in a meaningful way. We use the decomposition of Basu and Fernald (2002), which is derived by assuming that firms minimize costs and are price-takers in factor markets, but may have market power for the goods they sell and might produce with increasing returns to scale. Some of the components in the decomposition we use can be clearly identified as being due to reallocation, since they depend on marginal products of identical inputs not being equalized across firms. Other components depend on aggregate distortions, such as the average degree of market power and various tax rates.

6.1 Summary of the Basu and Fernald decomposition

Following Basu and Fernald (2002), in this paragraph we decompose changes in aggregate productivity into changes in aggregate technologies and changes in three non-technological components reflecting imperfections and frictions in output and factor markets. Suppose that each firm i has the following production function:

$$Q_i = F^i(K_i, L_i, M_i, T_i^Q) \tag{31}$$

where Q_i is the gross output, K_i, L_i and M_i are inputs of capital, labor and materials, T_i^Q is a technology index and F^i is an homogenous function. Assume that firms are price takers in factor markets but have market power in the output markets. Call P_{Ji} the price for factor J faced by firm i and μ_i^Q the mark up that firm i imposed over marginal costs. For any input J, let F_J^i be the marginal product. Firm i's first order condition implies:

$$P_i F_J^i = \mu_i^Q P_{Ji} \tag{32}$$

Output growth, $d\log Q_i$, can be written as:

$$d\log Q_i = \mu_i^Q \left[s_{L,i}^Q d\log L_i + s_{K,i}^Q d\log K_i + s_{M,i}^Q d\log M_i \right] + \frac{F_{T^Q}^i T_i^Q}{F^i} d\log T_i^Q = \mu_i^Q d\log X_i^Q + \frac{F_{T^Q}^i T_i^Q}{F^i} d\log T_i^Q$$

where $s_{J,i}^Q$ is the revenue share of input J out of gross output, $d\log T_i^Q$ denotes technology growth and $d\log X_i^Q$ is revenue share weighted total input growth. Remember that our ultimate goal is decomposing the aggregate Solow residual. In the national account identity in closed economy, total expenditure equals the sum of firms' value added. Consider the standard Divisia index of firm level value added:

$$d\log Y_i = \frac{d\log Q_i - s_{M,i}^Q d\log M_i}{1 - s_{M,i}^Q} = d\log Q_i - \frac{s_{M,i}^Q}{1 - s_{M,i}^Q} \left(d\log M_i - d\log Q_i \right)$$

and define the change in aggregate primary inputs, $d\log X_i$, as the share-weighted sum of the growth rates of capital and labor:

$$d\log X_i = \frac{s_{K,i}^Q}{1 - s_{M,i}^Q} d\log K_i + \frac{s_{L,i}^Q}{1 - s_{M,i}^Q} d\log L_i = s_{K,i} d\log K_i + s_{L,i} d\log L_i$$

After some algebra, taking into account that the firms' value added productivity residual $d\log pr_i$ equals $d\log Y_i - d\log X_i$, we obtain:

$$d\log pr_i = (\mu_i - 1) d\log X_i + (\mu_i - 1) \frac{s_{M,i}^Q}{1 - s_{M,i}^Q} \left(d\log M_i - d\log Q_i \right) + d\log T_i$$

where:
$\mu_i = \mu_i^Q \frac{1-s_{M,i}^Q}{1-\mu_i^Q s_{M,i}^Q}$
$d\log T_i = \frac{F_{T^Q}^i T_i^Q}{F^i} \frac{d\log T_i^Q}{1-\mu_i^Q s_{M,i}^Q}$

Let us move now to aggregate quantities. Define aggregate inputs as the simple sums of firm-level quantities: $K = \sum_{i=1}^I K_i$ and $L = \sum_{i=1}^I L_i$.

Now define aggregate output growth as a Divisa index of firm level value added:

$$d\log Y = \sum_{i=1}^{I} w_i d\log Y_i$$

where w_i is firm i's share of nominal value added: $w_i = P_i^Y Y_i / P^Y Y$ and define aggregate primary input growth as:

$$d\log X = \frac{s_K^Q}{1-s_M^Q} d\log K + \frac{s_L^Q}{1-s_M^Q} d\log L = s_K d\log K + s_L d\log L$$

where s_J is the share of input J out of total value added. After some algebraic manipulation, $d\log X$ can be written in terms of the weighted average of firm level primary input growth: $d\log X = \sum_{i=1}^I w_i d\log X_i$. Aggregate productivity growth, $d\log pr$, is the difference between aggregate output growth $d\log Y$ and aggregate inputs growth $d\log X$. Basu and Fernald shows that after some manipulations, $d\log pr$ can be decomposed in the following way:[11]

$$d\log pr = (\overline{\mu} - 1) d\log X + (\overline{\mu} - 1) d\log M/Q + R_\mu + R_M + d\log T \tag{33}$$

where:
$\overline{\mu} = \sum_{i=1}^I w_i \mu_i$
$d\log M/Q = \sum_{i=1}^I w_i \frac{s_{M,i}^Q}{1-s_{M,i}^Q} (d\log M_i - d\log Q_i)$
$R_\mu = \sum_{i=1}^I w_i (\mu_i - \overline{\mu}) d\log X_i$
$R_M = \sum_{i=1}^I w_i (\mu_i - \overline{\mu}) \frac{s_{M,i}^Q}{1-s_{M,i}^Q} (d\log M_i - d\log Q_i)$
$d\log T = \sum_{i=1}^I w_i d\log T_i$

It is easy to provide an intuition for the welfare relevance of each term in which we have decomposed aggregate productivity. The first term, $(\overline{\mu} - 1) d\log X$, is a direct consequence of imperfect competition. Consumers would prefer to provide more labor and capital and consume the extra goods produced, since their utility value exceeds the disutility of producing them. Hence aggregate productivity and welfare increases with aggregate primary input growth, and this is true even if firms have the same markup. In this sense, $(\overline{\mu}-1) d\log X$ reflects an aggregate distortion and

[11] We are assuming here that the price paid by each firm for capital and labor is the same. If it is allowed to differ, Basu and Fernald (2002) show that two additional terms should be added to the right hand side of (33): $R_K \equiv \overline{\mu} \sum_{i=1}^I w_i s_{K,i} \left[\frac{P_{Ki} - P_K}{P_{Ki}}\right] d\log K_i$ and $R_L \equiv \overline{\mu} \sum_{i=1}^I w_i s_{L,i} \left[\frac{P_{Li} - P_L}{P_{Li}}\right] d\log L_i$. These input reallocation terms represent gains from directing primary inputs towards firms where they have higher social valuation.

should not be counted as part of "reallocation," which we use as shorthand for allocative efficiency.

The third term, R_μ, represents the increase in productivity and welfare coming from the fact that primary inputs are directed towards firms with higher-than-average markups, since higher prices and markups express higher social valuation.

The terms $(\overline{\mu}-1)d\log M/Q$ and R_M reflect the fact that a markup greater than one reduces the use of materials as well as primary inputs below the socially optimal level. This distortion is greater the greater is the markup. Note that if materials had to be used in fixed proportion to output, $d\log M_i - d\log Q_i$ would equal zero and so would both $(\overline{\mu}-1)d\log M/Q$ and R_M. (In other words, the distortions regarding primary inputs would summarize fully the distortions in input use due to markups that exceed one.) More specifically, $(\overline{\mu}-1)d\log M/Q$ reflects the distortion generated by an average markup above unity and R_M reflects reallocation across firms with different markups (relative to $\overline{\mu}$). Only the latter should be counted as part of reallocation. Finally the term $d\log T$ represents the contribution to productivity and welfare of changes in aggregate technology.

The Basu and Fernald decomposition can be extended by disaggregating R_μ into a within sectors and between sectors component. This is useful in assessing whether the gain from reallocation (if any) occur because resources are reallocated across industries or within industries across firms. Basu and Fernald used industry level data in their empirical exercise so they could at best evaluate the between component. (We say "at best" because if there are within-industry reallocation terms, then Basu and Fernald's estimation using industry-level data would not give a consistent estimate of even the average industry markup, $\overline{\mu}$. In general, one can estimate $\overline{\mu}$ correctly only by taking the average of firm-level markups, estimated using firm-level data.) If one uses firm-level data, one can discuss the relative importance of the within and between components. R_M can also be decomposed into a within and between component, but there is a residual term.

Let $P^{YJ}Y^J = \sum_{i\epsilon J} P_i^Y Y_i^J$ be the total value added produced in industry J, $w^J = P^{YJ}Y^J/P^YY$ the share of industry J out of aggregate output and $w_i^J = P_i^Y Y_i^J/P^{YJ}Y^J$ the share of value added of firm i in industry J. Denote with Q_i a firm gross output and with P_i^Q its price. Then $w_i^{QJ} = P_i^Q Q_i^J/P^{QJ}Q^J$, where $P^{QJ}Q^J = \sum_{i\epsilon J} P_{ii}^Q Q^J$, represents the firm share of industry gross output. Finally, the primary inputs growth in industry J is $d\log X^J = s_K^J d\log K^J + s_L^J d\log L^J$, where $s_K^J = \frac{\sum_{i\epsilon J} P_{K,i}K_i}{P^{YJ}Y^J}$ and $s_L^J = \frac{\sum_{i\epsilon J} P_{L,i}L_i}{P^{YJ}Y^J}$. Define R_μ^J and R_μ^Jas the industry equivalent of the reallocation terms R_μ and R_M when aggregating over industry J rather than the entire economy, i.e $R_\mu^J = \sum_{i\epsilon J} w_i^J \left(\mu_i - \overline{\mu}^J\right) d\log X_i$ and $R_M^J = \sum_{i\epsilon J} w_i^J \left(\mu_i - \overline{\mu}^J\right) \frac{s_{M,i}^Q}{1-s_{M,i}^Q} \left(d\log M_i - d\log Q_i\right)$, where $\overline{\mu}^J = \sum_{i\epsilon J} w_i^J \mu_i$. We can decompose the reallocation term for primary inputs, R_μ, into a within and a between component (denoted by superscripts W and B, respectively) as follows:

$$R_\mu = R_\mu^W + R_\mu^B$$

where $R_\mu^W = \sum_{J=1}^K w^J R_\mu^J$ and $R_\mu^B = \sum_{J=1}^K w^J \left(\overline{\mu}^J - \overline{\mu}\right) d\log X^J$.Note that the between component can be calculated on the basis of industry data only.

The decomposition for the reallocation term for materials, R_M, is instead:

$$R_M = R_M^W + R_M^B + R_{w-w^Q}$$

where $R_M^W = \sum_{J=1}^K w^J R_M^J$, $R_M^B = \sum_{J=1}^K w^J \left(\overline{\mu}^J - \overline{\mu}\right)(d\log Q^J - d\log Y^J)$ and $R_{w-w^Q} = \sum_{J=1}^K w^J \left(\overline{\mu}^J - \overline{\mu}\right)$ $(\ w_i^J - w_i^{QJ})d\log Q_i$. In the between component, $d\log Y^J = \sum_{i\epsilon J} w_i^J d\log Y_i$ is the divisia index of industry value added,. $d\log Q^J = \sum_{i\epsilon J} w_i^{QJ} d\log Q_i$ is the divisia index of gross output (using w_i^{QJ} as weights). The residual term, R_{w-w^Q}, reflects the difference between value added weights and gross output weights in aggregating firm level gross output within an industry.

7 Econometric Framework

The modified Solow productivity residual can be essentially calculated from the data and requires no estimation if the distributional shares are observable (or if we observe the labor share and assume approximately zero profits). However, in order to break down the productivity residual into components that reflect aggregate distortions, reallocation and technology growth we must obtain estimates of the markups and of technology growth. We will do that by assuming that the (gross) production function in sector j is Cobb Douglas:

$$\log Q_{it} = \varepsilon_L^j \log L_{it} + \varepsilon_K^j \log K_{it} + \varepsilon_M^j \log M_{it} + \eta_{jt} + \alpha_i + \omega_{it} \tag{34}$$

where i denotes firms $(i = 1, .., I_j)$, t time $(t = 1, .., T_j)$, and small case variables logs. η_{jt} is an industry specific common component of productivity, α_i a time invariant firm level component and ω_{it} an idiosyncratic component. In our application using the Amadeus data set, T_j is small and N_j large.

We will experiment with different estimation methods: OLS, LSDV, Olley and Pakes, Difference and System GMM (assuming that ω_{it} is either serially uncorrelated, or that it follows an AR(1) process). The advantages and disadvantages of each choice are well known, although there is no agreement on which estimator one should ultimately choose. One fundamental estimation problem is the endogeneity of the input variables, which are likely to be correlated both with α_i and ω_{it}. Correlation with ω_{it} may reflect both simultaneity of input choices or measurement errors. Given the shortness of the panel, elimination of α_i through a within transformation is not the appropriate strategy. Differencing of (34) and application of the difference GMM estimator (Arellano and Bond (1991)) is a possibility, but appropriately lagged values of the regressors may be poor instruments if inputs are very persistent. Application of the GMM System estimator (Blundell and Bond (1998) and Blundell and Bond *(*2000*)* is probably a better option. An alternative approach is the one proposed by Olley and Pakes (1996). This estimator addresses the simultaneity (and selection) problem by using firm investment as a proxy for unobserved productivity and requires the presence of only one unobserved state variable at the firm level and monotonicity of the investment function. We are not interested to take a stand in this paper on which one is the preferable estimation strategy. Fortunately for us, the results of the decomposition are insensitive to the choice of a particular estimator.

Having obtained estimates of the output elasticity for each factor we will recover the firm specific markup from the first order conditions for materials, equation (32). In the Cobb Douglas case, this can be expressed as:

$$\widehat{\mu}_i^Q = \frac{\widehat{\varepsilon}_M^j}{s_{M,i}^Q} \tag{35}$$

where $s_{M,i}^Q$ is the time average of the firm specific revenue share of materials for firm i. A hat denotes estimated values. We have chosen to focus on the FOC for materials because they are likely to be the most a flexible input. Whereas the labor share, $s_{L,i}^Q$, can be easily recovered from the data, the same is not true for the capital share, $s_{K,i}^Q$, unless one is willing to make assumptions about the user cost of capital, which is problematic in the presence of firm heterogeneity in the cost of finance. We have recovered the capital share from estimates of the markup described above and of the elasticity of output with respect to capital, using:

$$s_{K,i}^Q = \frac{\widehat{\varepsilon}_K^j}{\widehat{\mu}_i^Q} \tag{36}$$

Alternatively we have obtained $s_{K,i}^Q$ from:

$$s_{ki} = 1 - s_{L,i}^Q - s_{M,i}^Q - \frac{\Pi_i}{Y_i} = 1 - s_{L,i}^Q - s_{M,i}^Q - (1 - \frac{\widehat{\theta}^j}{\widehat{\mu}_i^Q}) \tag{37}$$

where $\widehat{\theta}^j = \widehat{\varepsilon}_K^j + \widehat{\varepsilon}_L^j + \widehat{\varepsilon}_M^j$ is the degree of returns to scale in sector j. The result are robust to this choice.

8 Results

We will discuss now the empirical results obtained when the production function is estimated on the firm level data contained in Amadeus for Belgium, France, Great Britain, Italy, Spain over the period 1998-2005. To avoid overburdening the reader, we report results for selected estimators (OLS, System GMM, and Olley and Pakes) for only one of our countries, Belgium.

The estimation results for the elasticity of output with respect to each factor, for constant returns to scale and for average markups are reported in the tables 6, 7, and 9. Estimates are pretty standard and vary somewhat across estimators. Recall that materials include services together with materials and intermediates. The degree of returns to scale is very close to one in most sectors using OLS and System GMM, while it is slightly smaller, but still close to one, with the Olley and Pakes estimator. The estimate of ε_K^j is greater for the OLS estimator and the smallest for the Olley and Pakes estimator. For five sectors it is negative using the GMM System estimator with serially uncorrelated errors, although not significantly so. The test of overidentifying restrictions and the test of second order serial correlation for the GMM System do not suggest major misspecification issues for most sectors, which leads us to focus on this version of the GMM estimator, instead of the one allowing for an AR(1) error component in the level equation. The average estimated markup, obtained using (35), exceeds one in all sectors, whatever the estimator used. Moreover it is strictly greater than one for 64% of firms, using OLS, 70% using System GMM, and 63% using Olley and Pakes.

We find markup estimates that are quite reasonable compared to existing estimates in the micro-econometric literature [12], albeit somewhat high relative to the macro literature. The numerical estimates in Tables 6 through 9, usually in the range of 1.10 to 1.25, seem quite small, but one needs to remember that these are markups on gross output. Converting to markups on value added using a representative materials share of 0.7, the markups are in the range of 1.43 to 3. Similarly, the implied profit rates are a bit on the high side. Using equation 37, the profit rate can be calculated as $(1 - \frac{\widehat{\theta}^j}{\widehat{\mu}_i^Q})$. Taking constant returns as our modal estimate, the markup range just discussed corresponds to profit rates in the range of nine to 20 percent, expressed as a percentage of gross output.

Our estimates of the markup and thus of the profit rate are probably upper bounds. We do not control for variations in firm-level input utilization (changes in the number of shifts or variations in labor effort), except through our use of time fixed effects. Thus, we remove variations in utilization due to common industry effects but not due to firm-specific demand variation over time. Basu (1996) suggests that variable utilization is likely to bias upward the output elasticity of materials in particular, which is the parameter that has the largest impact on our estimates of markups and profit rates. Unfortunately, we do not have the firm-level data on hours worked per employee that would be necessary to implement the utilization control derived from the optimizing model of Basu, Fernald and Kimball (2006). Thus, our estimates of the average distortions coming from markup pricing, as summarized by the first two terms in equation (33) are likely to be on the high side. But the fact that our estimated average markups are large does not create any particular bias in our estimates of the reallocation terms, which are our particular focus, since the reallocation terms involving markups depend on the gaps between firm-level and average markups.

In light of this discussion, it is interesting to look at the estimates of the various reallocation terms for our sample of six countries, which are presented in Table 10 and in 11. In Table 10 we report for each country in our sample, average productivity growth, $d \log pr$, the sum of aggregate distortions, $(\overline{\mu}-1)d\log X + (\overline{\mu}-1)d\log M/Q$, the sum of the reallocation terms for primary factors and materials, $R_\mu + R_M$, and technology growth, $d \log T$. The last column reports as residual the difference between productivity, on the one hand, and the sum of aggregate distortions, of the reallocation terms, and of technological progress, on the other, i.e. the difference between the left hand side and the right hand side of (33). This equation may not hold as an equality for three reasons: first we do not observe the true value of the markup, but only its estimated value; (ii) whereas the labor share is observed in the data, calculations of the capital share depends upon a

[12] For example, Dobbelaure and Mairesse (2008) find very similar markups using panel data for French firms.

zero profit assumptions or an estimate of the markup and of the degree of returns to scale; (iii) as Basu and Fernald (2002) show, if the price paid for capital and labor differs across firms, additional terms involving the difference of factor prices for each firm from the average, multiplied by each factor growth rate will appear on the right hand side of (33).[13]

First of all, we see from Table 10 robust average annual productivity growth for all countries in our sample of large firms. The case of Italy is particularly striking, since our sample of firms has an average productivity growth rate, $d\log pr$, of 2.8 percent, while the EU-KLEMS database shows that for all of Italian manufacturing average TFP declined at a rate of 1.2 percent per year over our sample period. Second, we see that technical change was also positive for all countries, and over 1 percent per year in all countries except Spain, where it averaged 0.5 percent. The strongest rates of technical change, in excess of 4 percent per year, were registered in France, which is usually found to be a high-productivity country in most cross-country studies, and in the United Kingdom, which had 2.2 percent average TFP growth in manufacturing over this time period.

Before discussing the results on reallocation, note that the residual is sizeable and we decide to allocate it to the aggregate distortion, reallocation, and technology growth component in proportion to their relative size. In Table 11 we report the proportion of aggregate productivity accounted for by each component, after this adjustment. The results suggest, first, that in most countries most of productivity growth is accounted by technology growth. More specifically, technological progress accounts for the totality of productivity growth in Great Britain and in France, for a large fraction in Italy (.66%) and for a sizeable, but smaller fraction in Belgium and Spain (43% and 21% respectively). Second, aggregate distortions are quite important in Spain Belgium, and Italy, where they account for 85%, 55%, and 33% of productivity growth respectively. They are, instead rather small in Great Britain and in France. The reallocation terms for primary factors or materials accounts for a small proportion of productivity growth in all countries.[14] It follows that, unless one is willing to treat the entire residual as part of reallocation term, factor reallocation does not appear to be an important component of productivity growth.[15] Here the nature of the sample may work against finding strong results, since most of the firms are quite large in all the years they are observed. Reallocation effects are most clearly apparent when firms that are small initially grow to a large size due to their superior productivity. There are probably fewer such firms in our sample than in the population, thus reducing the quantitative impact of reallocation. Petrin, White and Reiter (2009) come to the different conclusion that reallocation represents a large fraction of productivity growth, using manufacturing plant level data for the US. They calculate

[13] See footnote 11. Petrin, Reiter and White (2009) argue that changes in fixed costs create yet another gap between the two sides of equation *(33).* However, changes in fixed costs are equivalent to an additive technology shock, and to a first-order approximation both additive and multiplicative technology shocks are already incorporated into the estimate of $dlogT$. Thus, changes in fixed costs are not an additional gap between productivity growth and technological change.

[14] Because the reallocation term is so small, not much is learned from presenting its the decomposition in a within and between component.

[15] If we treat the residual as reflecting the difference in primary factor prices faced by firms and treat it entirely as part of the reallocation term, as in Basu and Fernald (2002), reallocation would account for approximately a third of productivity growth in Great Britain.

their reallocation term as the difference between a Divisia index of firm level productivity growth and a Divisia index of technology growth. Thus, they include aggregate distortions as part of reallocation, which should not be the case if one wants to estimate and index of allocative efficiency strictly defined. We also find that aggregate distortions can be substantial for some countries.[16]

Finally, although reallocation of factors towards uses where they have a higher social valuation has not been a large part of the improvement in productivity and welfare for the sample period we have analyzed, does not mean that a benevolent central planner could not achieve large welfare improvement from factor reallocation. This distinction between the historical decomposition we have presented and what could be potentially obtained should be kept in mind when drawing inferences from these results.

9 Conclusions

We show that the present value of aggregate TFP growth is a complete welfare measure for a representative consumer, up to a first-order approximation. This result rigorously justifies TFP, rather than technical change or labor productivity, as the central statistic of interest in any exploration of productivity, at all levels of aggregation. Importantly, the result holds even when TFP is not a correct measure of technical change, for example due to increasing returns, externalities, or imperfect competition. It also suggests that productivity decompositions should be oriented towards showing how particular features or frictions in an economy either promote or hinder aggregate TFP growth, which is the key to economic welfare. Our theoretical results point to a key role for the persistence of aggregate TFP growth, since welfare change is related to the entire expected time path of productivity growth in addition to the current growth rate. Finally, our derivation shows that in order to create a proper welfare measure, TFP has to be calculated using prices faced by households rather than prices facing firms. In modern, developed economies with high rates of income and indirect taxation, the gap between household and firm TFP can be considerable.

We use these central results to show that one can explore the sources of welfare change using both non-parametric index numbers and formal econometrics. The non-parametric approach has the great advantage of simplicity, and avoids the need to address issues of econometric identification. Many interesting cross-country comparisons can be performed using the index-number approach, including calculating summary statistics of allocative efficiency for each country based on firm-level data. However, if one wants to ask how much of aggregate TFP growth is due to technical change, as opposed to scale economies or allocative inefficiency, one does need to make additional assumptions and estimate production functions at the firm level. We show how one can decompose aggregate TFP growth in such a manner using firm-level data.

The results suggest that in the majority of OECD countries we have analyzed (Belgium, France, Great Britain, Italy, and Spain) most of productivity growth in manufacturing is accounted for by technology growth. This is particularly true for Great Britain and France. Moreover, aggregate

[16] Petrin, White and Reiter (2009) implement the decomposition proposed by Petrin and Levinsohn (2008) which is a variant of the Basu and Fernald (2002) decomposition.

distortions are quite important in many countries, such as Spain Belgium, and Italy. Finally, the reallocation terms for primary factors or materials account for a small proportion of productivity growth in all countries over the period 1995-2005. We will explore in future research whether this results extends to other countries or time periods, or to other data sets less biased towards larger firms.

A final comment is in order. In a deep sense, neither the non-parametric approach nor the production-function approach can answer the most interesting questions regarding the sources of welfare change. The reason is that neither approach allows us to answer the most interesting counterfactual questions, such as "How much lower would welfare be if there had been no technical change in sector X over an interval of time Y?" In order to answer such questions, one needs to estimate a full general-equilibrium model. However, realistic GE models that allow for dynamic imperfect competition and non-trivial firm-level heterogeneity are very complex objects to specify, let alone to estimate. Our results based on theory allow us to suggest an exercise that would be rigorous, without requiring a full GE model. One interesting question is the effects of various government policies on welfare. These policies might be trade policies, like joining NAFTA, or purely domestic, such as a change in the income tax rate. Suppose one can isolate exogenous measures of policy change. This is difficult but not impossible, as the literature on identifying exogenous monetary and fiscal policy shocks suggests. Then, knowing that the entire welfare-relevant effects of these policy changes are summarized by their effects on the time path of national TFP, one can simply regress TFP on current and lagged measures of policy changes (in a single time series or using a panel of countries), and then take the present discounted value of the impulse response. The results would tell us the effects of a particular policy change on national welfare, without requiring us to write down a GE model and specify all the channels through which that policy might work. A similar exercise can be conducted for the components of productivity growth due to technology or to reallocation. This is a topic that we leave for future work.

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A Appendix A: Derivations

A.1 Making the problem stationary

The representative household maximizes intertemporal utility:

$$V_t = \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} U(C_{1,t+s}, .., C_{Z,t+s}; \overline{L} - L_{t+s}) \tag{A.1}$$

where $C_{i,t+s}$ is the capita consumption of good i at time t+s, L_{t+s} are hours of work per capita, $\overline{L}$ is the time endowment, and N_{t+s} population. H is the number of households, assumed to be fixed and normalized to one from now on.Consider the laws of motion for N_t and for X_t, where the latter denotes Harrod neutral technological progress (so that total labor input in efficiency units is $(N_t X_t L_t)$:

$$N_t = N_0(1+n)^t \tag{A.2}$$

$$X_t = X_0(1+g)^t \tag{A.3}$$

and normalize $H = 1$.

We can rewrite the utility function as:

$$V_t = N_t \sum_{s=0}^{\infty} \frac{(1+n)^s}{(1+\rho)^s} U(C_{1,t+s}, .., C_{Z,t+s}; \overline{L} - L_{t+s}) \tag{A.4}$$

For a well defined state in which hours of work are constant we assume that the utility function has the King Plosser and Rebelo form(1988):

$$U(C_{1,t+s}, .., C_{Z,t+s}; L - \overline{L}_s) = \frac{1}{1-\sigma} C(C_{1,t+s}, .., C_{Z,t+s})^{1-\sigma} \nu(\overline{L} - L_{t+s})$$

We assume that $C()$ is homogenous of degree 1. Define $c_{i,t+s} = \frac{C_{i,t+s}}{X_{t+s}}$. We can rewrite the utility function in the following form:

$$U(C_{1,t+s}, .., C_{Z,t+s}; L - \overline{L}_{t+s}) = \frac{1}{1-\sigma} X_{t+s} C(c_{1,t+s}, .., c_{Z,t+s})^{1-\sigma} \nu(\overline{L} - L_{t+s})$$

or

$$U(C_{1,t+s}, .., C_{Z,t+s}; L - \overline{L}_s) = (1+g)^{s(1-\sigma)} X_t^{(1-\sigma)} \frac{1}{1-\sigma} C(c_{1,t+s}, .., c_{Z,t+s})^{1-\sigma} \nu(\overline{L} - L_{t+s})$$

Inserting this into V_t, we get:

$$V_t = N_t X_t^{(1-\sigma)} \sum_{s=0}^{\infty} \beta^s U(c_{1,t+s}, .., c_{Z,t+s}; \overline{L} - L_{t+s}) \tag{A.5}$$

where: $\beta = \frac{(1+n)(1+g)^{1-\sigma}}{(1+\rho)}$.

A.2 Budget constraint

Start from the usual budget constraint:

$$P_t^I K_t + B_t = (1-\delta)\, P_t^I K_{t-1} + (1+i_t)\, B_{t-1} + P_t^L L_t N_t + P_t^K K_t + \Pi_t - \sum_{i=1}^{Z} P_{i,t}^C C_{i,t} N_t \qquad \text{(A.6)}$$

Divide both sides by $P_t^I X_t N_t$ to get:

$$\begin{aligned} \frac{K_t}{X_t N_t} + \frac{B_t}{P_t^I X_t N_t} \quad = \quad & (1-\delta)\, \frac{K_{t-1}}{X_{t-1}N_{t-1}} \frac{X_{t-1}N_{t-1}}{X_t N_t} + (1+i_t)\, \frac{B_{t-1}}{P_{t-1}^I X_{t-1} N_{t-1}} \frac{P_{t-1}^I}{P_t^I} \frac{X_{t-1}N_{t-1}}{X_t N_t} \\ & + \frac{P_t^L}{P_t^I} \frac{L_t N_t}{X_t N_t} + \frac{P_t^K}{P_t^I} \frac{K_t}{X_t N_t} + \frac{\Pi_t}{P_t^I X_t N_t} - \sum_{i=1}^{Z} \frac{P_{i,t}^C}{P_t^I} \frac{C_{i,t} N_t}{X_t N_t} \end{aligned}$$

Define: $k_t = \frac{K_t}{X_t N_t}$, $b_t = \frac{B_t}{P_t^I X_t N_t}$, $p_t^K = \frac{P_t^K}{P_t^I}$, $p_t^L = \frac{P_t^L}{P_t^I X_t}$, $p_{i,t}^C = \frac{P_{i,t}^C}{P_t^I}$, $(1+r_t) = \frac{(1+i_t)}{(1+\pi_t)}$, $\pi_t = \frac{\Pi_t}{P_t^I X_t N_t}$. The budget constraint can be rewritten as:

$$k_t + b_t = \frac{(1-\delta)}{(1+g)\,(1+n)} k_{t-1} + \frac{(1+r_t)}{(1+g)\,(1+n)} b_{t-1} + p_t^L L_t + p_t^K k_t + \pi_t - \sum_{i=1}^{Z} p_{i,t}^C c_{i,t} \qquad \text{(A.7)}$$

A.3 Optimality conditions

The representative household solves the following maximization:

$$Max\ v_t = Max \frac{V_t}{N_t X_t^{(1-\sigma)}} = E_t \sum_{s=0}^{\infty} \beta^s \{ U(c_{1,t+s}, .., c_{Z,t+s}; \overline{L} - L_{t+s})$$

$$+\lambda_t \left(-k_t - b_t + \frac{(1-\delta)}{(1+g)\,(1+n)} k_{t-1} + \frac{(1+r_t)}{(1+g)\,(1+n)} b_{t-1} + p_t^L L_t + p_t^K k_t + \pi_t - \sum_{i=1}^{Z} p_{i,t}^C c_{i,t} \right) \}$$

where $v_t = \frac{V_t}{N_t X_t^{(1-\sigma)}}$ is normalized intertemporal utility. The FOCs are:

$$U_{c_{i,t}} - \lambda_t p_{i,t}^C = 0 \qquad \text{(A.8)}$$

$$U_{L_t} + \lambda_t p_t^L = 0 \qquad \text{(A.9)}$$

$$\lambda_t \left(p_t^K - 1\right) + \beta \frac{(1-\delta)}{(1+g)(1+n)} E_t \lambda_{t+1} = 0 \tag{A.10}$$

$$-\lambda_t + \beta_t \frac{1}{(1+g)(1+n)} E_t (1+r_t) \lambda_{t+1} = 0 \tag{A.11}$$

A.3.1 Approximation around SS

Define with $\widehat{x} = \log x_t - \log x$ the log deviation from the steady state of a variable (x is the steady state value of x_t). Loglinearize the normalized value function around the steady state:

$$\begin{aligned}
v_t - v \quad = \quad & E_t[\sum_{s=0}^{\infty} \beta^s (\sum_{i=1}^{Z} U_{c_i} c_i \widehat{c}_{i,t+s} + U_{L_t} L \widehat{L}_{i,t+s} \\
& + \lambda p^L L \widehat{L}_{i,t+s} - \lambda \sum_{i=1}^{Z} p_i^C c_i \widehat{c}_{i,t+s} + \lambda (p^K - 1) k \widehat{k}_{t+s} - \lambda b \widehat{b}_{t+s}) \\
& + \sum_{s=0}^{\infty} \beta^{s+1} \left(\lambda \frac{(1-\delta)}{(1+g)(1+n)} k \widehat{k}_{t+s} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t+s} \right) \\
& + \sum_{s=0}^{\infty} \beta^s \lambda \left(p^L L \widehat{p}_{t+s}^L + p^K k \ \widehat{p}_{t+s}^K - \sum_{i:0}^{Z} p_{i,t}^C c_i \widehat{p}_{i,t+s} + \pi \widehat{\pi}_{t+s} \right)]
\end{aligned}$$

Using the first order conditions the first three lines equal zero so that:

$$v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^L L \widehat{p}_{t+s}^L + p^K k \ \widehat{p}_{t+s}^K + \pi \widehat{\pi}_{t+s} - \sum_{i=1}^{Z} p_{i,t}^C c_i \widehat{p}_{i,t+s} \right] \tag{A.12}$$

Now log linearize the budget constraint:

$$\begin{aligned}
& k \ \widehat{k}_t + b \widehat{b}_t - \frac{(1-\delta)}{(1+g)(1+n)} k \widehat{k}_{t-1} - \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1} - p^L L \widehat{L}_t - p^K k \ \widehat{k}_t - p^L L \widehat{p}_t^L - p^K k \ \widehat{p}_t^K \\
& - \pi \widehat{\pi}_t + \sum_{i=1}^{Z} p_i^C c_i \widehat{c}_{i,t} + \sum_{i=1}^{Z} p_i^C c_i \widehat{p}_{i,t} = 0
\end{aligned}$$

Using this result and the fact that $b = 0$ in (A.12) gives us:

$$v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[\sum_{i=1}^{Z} p_i^C c_i \widehat{c}_{i,t+s} + k \widehat{k}_{t+s} - \frac{(1-\delta)}{(1+g)(1+n)} k \widehat{k}_{t+s-1} - p^L L \widehat{L}_{t+s} - p^K k \ \widehat{k}_{t+s} \right] \tag{A.13}$$

Notice that the law of motion of capital: $K_t = (1-\delta) K_{t-1} + I_t$, can be rewritten as: $\frac{K_t}{X_t N_t} = (1-\delta) \frac{K_{t-1}}{X_{t-1} N_{t-1}} \frac{X_{t-1} N_{t-1}}{X_t N_t} + \frac{I_t}{X_t N_t}$ which after some algebra becomes:

$$k_t = \frac{(1-\delta)}{(1+g)(1+n)} k_{t-1} + i_t$$

Differentiating it around the steady state, we get:

$$k\widehat{k}_t = \frac{(1-\delta)}{(1+g)(1+n)} k\widehat{k}_{t-1} + i\widehat{i}_t$$

Inserting this equation into equation A.13 we get:

$$v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[\sum_{i=1}^{Z} p_i^C c_i \widehat{c}_{i,t+s} + i\widehat{i}_{t+s} - p^L L \ \widehat{L}_{t+s} - p^K k \ \widehat{k}_{t+s} \right] \tag{A.14}$$

A.4 Connecting the level of productivity to the level of welfare

Define value added (for normalized variables in deviation from steady state) as:

$$\widehat{y}_t = \log y_t - \log y = \sum_{i=1}^{Z} \frac{P_i^C C_i N}{P^Y Y} \widehat{c}_{it} + \frac{P^I I}{P^Y Y} \widehat{i}_t = \sum_{i=1}^{Z} s_{c_i} \widehat{c}_{it} + s_i \widehat{i}_t \tag{A.15}$$

Inserting this equation into A.14, we get:

$$v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda p^Y y \left[\widehat{y}_t - s_L \ \widehat{L}_{t+s} - s_K \ \widehat{k}_{t+s} \right] \tag{A.16}$$

Using the definition of the normalized variable, this can be rewritten as:

$$v_t = v + \left(\lambda p^Y y\right) E_t \sum_{s=0}^{\infty} \beta^s \left[\left(\log \frac{Y_{t+s}}{N_{t+s} X_{t+s}} - \log y\right) - s_L (\log L_{t+s} - \log L) - s_K (\log \frac{K_{t+s}}{N_{t+s} X_{t+s}} - \log \ k) \right] \tag{A.17}$$

or:

$$v_t = \left(\lambda p^Y y\right) E_t \sum_{s=0}^{\infty} \beta^s \left[\log Y_{t+s} - s_L^V \log \ N_{t+s} L_{t+s} - s_K^V \log K_{t+s} \right] + f(t) \tag{A.18}$$

where:

$$\begin{aligned} f(t) \quad &= \quad -\frac{\lambda p^y y}{1-\beta} \left[\log y - s_L \log L - s_K \log k + \frac{\beta}{(1-\beta)} \left[g(1-s_K) + n(1-s_L-s_K) \right] \right] \\ &\quad -\frac{\lambda p^y y}{1-\beta} \left[(1-s_K) \log X_t + (1-s_L-s_K) \log N_t \right] \end{aligned}$$

Define aggregate productivity (in log level) as: $\log pr_t = \log Y_t - s_L \log N_t L_t - s_K \log K_t$. Notice that we are taking a definition with constant shares. Using this definition, the equation above can be rewritten as:

$$v_t - v = \left(\lambda p^Y y\right) E_t \sum_{s=0}^{\infty} \beta^s \log pr_{t+s} + f(t) \tag{A.19}$$

A.5 Connecting the aggregate Solow residual with the level of welfare

Now define: $\Phi_{t+s} = \sum_{i:0}^{Z} p_i^C c_i \log c_{i,t+s} + i \log i_{t+s} - p^L L \log L_{t+s} - p^K k \log k_{t+s}$ and note that for a variable x:

$$E_t \widehat{x}_{t+s} = E_t(\log x_{t+s} - \log x) = E_t \sum_{i=1}^{G} (\log x_{t+i} - \log x_{t+i-1}) + \log x_t - \log x$$

Using this property, equation A.14 can be rewritten as:

$$v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[E_t \sum_{z=1}^{s} (\Phi_{t+z} - \Phi_{t+z-1}) + \Phi_t - \Phi \right]$$

If we are willing to make the hypothesis that the period before the shock the system was in its steady state, so that: $\Phi_{t-1} = \Phi$, then the equation above can be rewritten as: $v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[E_t \sum_{i:0}^{s} \Delta \Phi_{t+i} \right]$ or alternatively:

$$v_t = v + \frac{\lambda}{(1-\beta)} E_t \sum_{s=0}^{\infty} \beta^s \Delta \Phi_{t+s}$$

Substituting back the definition of Φ_{t+s} , we get:

$$v_t = v + \frac{\lambda}{(1-\beta)} E_t \sum_{s=0}^{\infty} \beta^s \left(\sum_{i=1}^{Z} p_i^C c_i \Delta \log c_{i,t+s} + i \Delta \log i_{t+s} - p^L L \Delta \log L_{t+s} - p^K k \Delta \log k_{t+s} \right) \tag{A.20}$$

where Δ denotes difference over time. Define value added (at constant shares) as:[17]:

$$\Delta \log y_t = \sum_{i=1}^{Z} \frac{p_i^C c_i}{p^Y y} \Delta \log c_{i,t+s} + \frac{i}{p^Y y} \Delta \log i_t \tag{A.21}$$

Using the fact that nominal value added $P_t Y_t = \sum_{i:0}^{Z} P_{i,t}^C C_{i,t} N_t + P_t^I I_t$, it is also true that:

$$\Delta \log Y_t = \sum_{i=1}^{Z} \frac{P_i^C C_i N}{P^Y Y} \Delta \log (C_{i,t} N_t) + \frac{P^I I}{P^Y Y} \Delta \log I_t \tag{A.22}$$

Now, insert this into equation A.20 and factor out $p^Y y$ to obtain:

[17] Here we are departing slightly from convention, as value added is usually calculated with time varying shares.

$$v_t = v + \frac{\lambda p^Y y}{(1-\beta)} E_t \sum_{s=0}^{\infty} \beta^s \left(\Delta \log y_{t+s} - \frac{p^L L}{p^Y y} \Delta \log L_{t+s} - \frac{p^K k}{p^Y y} \Delta \log k_{t+s} \right) \tag{A.23}$$

Using the fact that:

$$\Delta \log y_t = \Delta \log(\frac{Y_t}{X_t N_t}) = \Delta \log Y_t - g - n$$

$$\Delta \log L_t = \Delta \log(N_t L_t \frac{1}{N_t}) = \Delta \log N_t L_t - n$$

$$\Delta \log k_t = \Delta \log(\frac{K_t}{X_t N_t}) = \Delta \log K_t - g - n$$

and noticing that: $\frac{p^L L}{p^Y y} = \frac{P^L LN}{P^y Y} = s_L$, $\frac{p^K k}{p^Y y} = \frac{P^K K}{P^y Y} = s_K$, we can rewrite equation A.23 as: $v_t = v + \frac{\lambda p^y y}{(1-\beta)} E_t \sum_{s=0}^{\infty} \beta^s \left(\Delta \log y_{t+s} - s_L^V \Delta \log L_{t+s} - s_K^V \Delta \log k_{t+s} \right)$

$$\begin{aligned} v_t \quad = \quad & v + \frac{\lambda p^Y y}{(1-\beta)} E_t \sum_{s=0}^{\infty} \beta^s \left[\Delta \log Y_{t+s} - s_L \Delta \log N_{t+s} L_{t+s} - s_K \Delta \log \ K_{t+s} \right] \\ & + \frac{\lambda p^Y y}{(1-\beta)} \sum_{s=0}^{\infty} \beta^s \left[g(1-s_K) + n(1-s_K-s_L) \right] \end{aligned}$$

Denote $\Delta \log pr_{t+s}$ the (modified) Solow productivity residual:

$$\Delta \log pr_{t+s} = \Delta \log Y_{t+s} - s_L \Delta \log N_{t+s} L_{t+s} - s_K \Delta \log \ K_{t+s}$$

Using this definition, we get:

$$v_t - v = \frac{\lambda p^Y y}{(1-\beta)} E_t \sum_{s=0}^{\infty} \beta^s \Delta \log pr_{t+s} + f_0$$

where:

$$f_0 = \frac{\lambda p^Y y}{(1-\beta)^2} \left[g(1-s_K) + n(1-s_K-s_L) \right]$$

Now suppose we are not willing to assume that: $\Phi_{t-1} = \Phi$ and we are back to the case:

$$v_t = v + E_t \sum_{s=1}^{\infty} \beta^s \lambda \left[\sum_{z=1}^{s} (\Phi_{t+z} - \Phi_{t+z-1}) + \Phi_t - \Phi \right]$$

which can be rewritten as:

$$v_t = v + \frac{\lambda}{(1-\beta)} E_t \sum_{s=1}^{\infty} \beta^s \Delta \Phi_{t+s} + \frac{\lambda}{(1-\beta)} (\Phi_t - \Phi) \tag{A.24}$$

If we assume that:

$$\sum_{i=1}^{G} p_i^C c_i \widehat{c}_{i,t+s} + i\widehat{i}_{t+s} = p^y y \widehat{y}$$

then $\Phi_t - \Phi = \left(p^y y \widehat{y}_t - p^L L \widehat{L}_t - p^K k \widehat{k}_t \right)$ and after some algebra:

$$\Phi_t - \Phi = p^y y \cdot \log pr_t - p^y y \left[(1 - s_K) \log X_t + (1 - s_L - s_K) \log N_t \right] - p^y y \left[\log y - s_L \log L - s_K \log k \right]$$

Substituting this result into equation A.24 and rearranging some terms, we get:

$$v_t - v = \frac{\lambda p^Y y}{(1-\beta)} E_t \sum_{s=1}^{\infty} \beta^s \Delta \log pr_{t+s} + \frac{\lambda p^Y y}{(1-\beta)} \log pr_t + f(t)$$

A.6 Connecting the aggregate Solow residual with the change in welfare

Take the difference between the expected level of intertemporal utility v_t defined in (A.14) and v_{t-1}.

$$\begin{aligned} \Delta v_t \quad = \quad & E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[\sum_{i=1}^{Z} p_i^C c_i \log c_{i,t+s} + i \log i_t - p^L L \log L_{t+s} - p^K k \log \ k_{t+s} \right] \\ & - E_{t-1} \sum_{s=0}^{\infty} \beta^s \lambda \left[\sum_{i=1}^{Z} p_i^C c_i \log c_{i,t+s-1} + i \log i_{t+s-1} - p^L L \log L_{t+s-1} - p^K k \log \ k_{t+s-1} \right] \end{aligned}$$

The right hand side, after adding and subtracting, for each variable x_{t+s}, $E_t x_{t+s}$, can be written as:

$$\begin{aligned} \Delta v_t \quad = \quad & E_t \sum_{s=0}^{\infty} \beta^s \lambda [\sum_{i=1}^{Z} p_i^C c_i \Delta \log c_{i,t+s} + i \Delta \log i_t - p^L L \Delta \log L_{t+s} - p^K k \Delta \log \ k_{t+s}] \\ & + \sum_{s=0}^{\infty} \beta^s \lambda [\sum_{i=1}^{Z} p_i^C c_i \left(E_t \log c_{i,t+s} - E_{t-1} \log c_{i,t+s} \right) + i \left(E_t \log i_{t+s} - E_{t-1} \log i_{t+s} \right) \\ & - p^L L E_t (\log L_{t+s} - E_{t-1} \log L_{t+s}) - p^K k \left(E_t \log k_{t+s} - E_{t-1} \log k_{t+s} \right)] \end{aligned}$$

By dividing both sides for v and using the fact that $E_t x_t = x_t$, we obtain:

$$\begin{aligned} \frac{\Delta v_t}{v} \quad = \quad & \frac{\lambda p^y y}{v} E_t \sum_{s=0}^{\infty} \beta^s \Delta \log pr_{t+s} + f_1 && \text{(A.25)} \\ & + \frac{\lambda p^y y}{v} \sum_{s=0}^{\infty} \beta^s \left[E_t \log pr_{t+s} - E_{t-1} \log pr_{t+s} \right] && \text{(A.26)} \end{aligned}$$

where $E_t \log pr_{t+s} - E_{t-1} \log pr_{t+s}$ represents the revision in expectations of the level of the productivity residual (normalized by population and Harrod neutral technological progress) based on the new information received between t-1 and t and:

$$f_1 = \frac{\lambda p^Y y}{v(1-\beta)} \left[g(1-s_K) + n(1-s_K-s_L) \right]$$

A.7 Summary and dividing by v

In order to present the results in a clearer fashion, notice that in steady state the FOC with respect to c_i can be rewritten as:

$$C(c_1,..,c_Z)^{-\sigma} \nu(\overline{L}-L) C_{c_i} = \lambda p_i^C$$

By multiplying both sides by c_i and summing over all G goods we get:

$$C(c_1,..,c_Z)^{-\sigma} \nu(\overline{L}-L) \sum_{i=1}^{Z} C_{c_i} c_i = \lambda \sum_{i=1}^{Z} p_i^C c_i$$

Using the fact that $C(c_1,..,c_G)$ is an homogenous function of degree 1 and applying Euler's theorem, we have: $\sum_{i=0}^{G} C_{c_i} c_i = C$ so that the expression above becomes:

$$C(c_1,..,c_Z)^{1-\sigma} \nu(\overline{L}-L) = \lambda \sum_{i=1}^{Z} p_i^C c_i \tag{A.27}$$

In steady state, we have: $v = \frac{C(c_1,..,c_G)^{1-\sigma} \nu(\overline{L}-L)}{(1-\sigma)(1-\beta)}$. Using this expression, together with equation A.27, we get:

$$\frac{\lambda}{v} = \frac{(1-\sigma)(1-\beta)}{\sum_{i=1}^{Z} p_i^C c_i}$$

Now let's use this result in the expression we have derived in the previous paragraph.

First, after denoting $s_C = \frac{\sum_{i=1}^{Z} p_i^C c_i}{p^y y}$ the level of welfare depends on the level of productivity according to the following formula:

$$\frac{v_t - v}{v} = \frac{(1-\sigma)(1-\beta)}{s_C} E_t \sum_{s=0}^{\infty} \beta^s \log pr_{t+s} + \Lambda(t) \tag{A.28}$$

where:

$$\begin{aligned} \Lambda(t) \quad = \quad & -\frac{(1-\sigma)}{s_C} \left[\log y - s_L \log L - s_K \log k + \frac{\beta}{(1-\beta)} \left[g(1-s_K) + n(1-s_L-s_K) \right] \right] \\ & -\frac{(1-\sigma)}{s_C} \left[(1-s_K) \log X_t + (1-s_L-s_K) \log N_t \right] \end{aligned}$$

Second, the level of welfare depends on the Solow residual according to the following formula:

$$\frac{v_t - v}{v} = \frac{(1-\sigma)}{s_C} E_t \sum_{s=1}^{\infty} \beta^s \Delta \log pr_{t+s} + \frac{(1-\sigma)}{s_C} \log pr_t + \Lambda(t)$$

If we assume that in t, the system is in steady state, this becomes:

$$\frac{v_t - v}{v} = \frac{(1-\sigma)}{s_C} E_t \sum_{s=0}^{\infty} \beta^s \Delta \log pr_{t+s} + \Lambda_0$$

where:

$$\Lambda_0 = \frac{(1-\sigma)}{(1-\beta) s_C} \left[g(1-s_K) + n(1-s_K-s_L) \right]$$

Third, the change in welfare depends on the Solow residual according to the following formula:

$$\frac{\Delta v_t}{v} = \frac{(1-\sigma)(1-\beta)}{s_C} E_t \sum_{s=0}^{\infty} \beta^s \Delta \log pr_{t+s} + \Lambda_1 \tag{A.29}$$

$$+ \frac{(1-\sigma)(1-\beta)}{s_C} \sum_{s=0}^{\infty} \beta^s \left[E_t \log pr_{t+s} - E_{t-1} \log pr_{t+s} \right] \tag{A.30}$$

$$\Lambda_1 = \frac{(1-\sigma)}{s_C} \left[g(1-s_K) + n(1-s_K-s_L) \right]$$

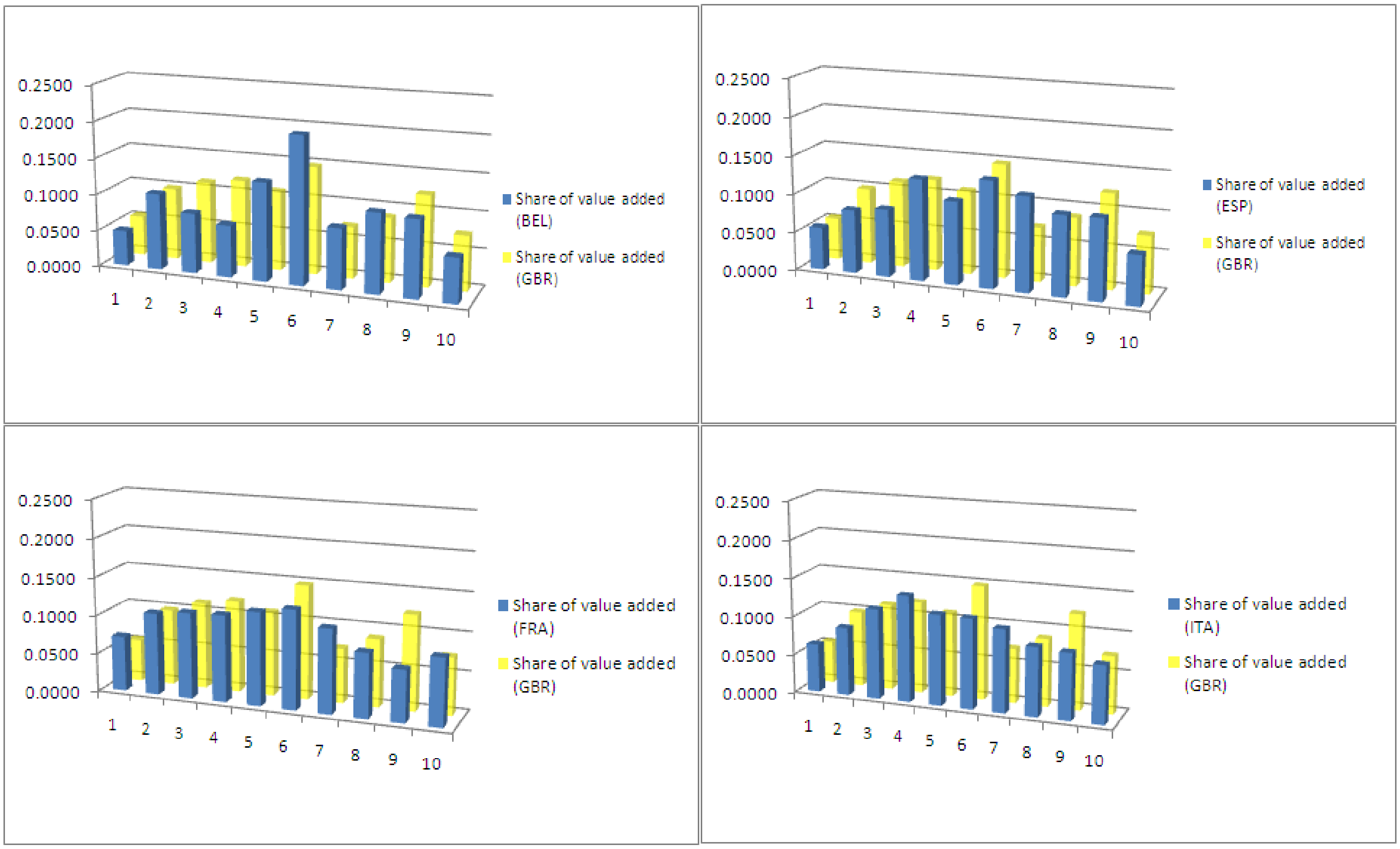

Figure 1: Accounting for differences with respect to Great Britain in aggregate productivity change: firm-level data

Table 1: Time series of the Solow residual

Dep. Variable = $\log pr_t^J$	(1) BEL	(2) ESP	(3) FRA	(4) ITA	(5) GBR
$\log pr_{t-1}^J$	0.790***	0.875***	0.694***	0.847***	0.739***
	(0.0957)	(0.0973)	(0.100)	(O.133)	(0.0984)
N	25	25	25	35	35
LM1 (p-value)	0.491	0.926	0.215	0.927	0.396
LM3 (p-value)	0.290	0.0166	0.4740	0.992	0.118

The table reports OLS estimates for the years up to 2005. A time trend is included in the regression. LM1 (LM3) is the Lagrange Multiplier tests for residual serial correlation up to the first (third) order. Standard errors are reported in parentheses. *** significant at less than 0.1 percent; ** significant at 1 percent; * significant at 5 percent.

Table 2: Accounting for aggregate productivity change between 1984 and 2004: aggregate and industry productivity change (annual rates)

	BEL	**ESP**	**FRA**	**ITA**	**GBR**
	$d\log pr$	$d\log pr$	$d\log pr$	$d\log pr$	$d\log pr$
	-0.0014	*-0.0024*	*0.0076*	*0.0027*	*0.0088*
Industry:					
Manufacturing	0.0068	0.0048	0.0112	0.0041	0.0103
Electricity, Gas, Water supply	0.0159	0.0327	0.0287	-0.0071	0.0149
Construction	0.0100	-0.0091	-0.0034	-0.0120	0.0000
Wholesale and Retail	-0.0203	-0.0069	0.0065	0.0051	0.0058
Finance, Insurance, Real estate	0.0007	-0.0009	-0.0103	-0.0030	-0.0132

Table 3: Accounting for aggregate productivity change between 1998 and 2005: small and large firms (annual rates)

	$d\log pr$**(aggregate)**	**group**	**Share(group)**	$d\log pr$**(group)**
BEL	0.0251	*LARGE FIRMS*	0.9241	0.0256
		SMALL FIRMS	0.0759	0.0182
ESP	0.0006	*LARGE FIRMS*	0.7044	0.0031
		SMALL FIRMS	0.2956	-0.0053
FRA	0.0293	*LARGE FIRMS*	0.7964	0.0294
		SMALL FIRMS	0.2036	0.0288
ITA	0.0057	*LARGE FIRMS*	0.7019	0.0046
		SMALL FIRMS	0.2981	0.0083
GBR	0.0523	*LARGE FIRMS*	0.9417	0.0519
		SMALL FIRMS	0.0583	0.0587

Table 4: Accounting for differences with respect to Great Britain in aggregate productivity change between 1984 and 2003: industry-level data (annual rates)

	Tot difference to GBR $d\log pr$	**Difference accounted by:**	
		differences in average $d\log pr$	*differences in industrial comp*
BEL	-0.0101	-0.0099	-0.0002
ESP	-0.0112	-0.0117	0.0005
FRA	-0.0011	0.0005	-0.0017
ITA	-0.0061	-0.0066	0.0005

Table 5: Accounting for differences with respect to Great Britain in aggregate productivity change between 1998 and 2005: firm-level data (annual rates)

BEL

Decile	$d \log pr$ (average)	Share value added (BEL)	Share (BEL)*$d \log pr$		Share(GBR)*$d \log pr$
1	-0.1593	0.0480	-0.0076		-0.0087
2	-0.0665	0.1045	-0.0069		-0.0066
3	-0.0321	0.0823	-0.0026		-0.0036
4	-0.0083	0.0713	-0.0006		-0.0010
5	0.0106	0.1343	0.0014		0.0012
6	0.0323	0.2006	0.0065		0.0048
7	0.0536	0.0834	0.0045		0.0038
8	0.0726	0.1089	0.0079		0.0064
9	0.1065	0.1056	0.0112		0.0131
10	0.1859	0.0611	0.0114		0.0139
		aggregate $d \log pr$=	0.0251	aggregate $d \log pr$ using GBR shares =	0.0232

ESP

Decile	$d \log pr$ (average)	Share value added (ESP)	Share (ESP)*$d \log pr$		Share(GBR)*$d \log pr$
1	-0.2116	0.0555	-0.0117		-0.0116
2	-0.1101	0.0826	-0.0091		-0.0109
3	-0.0671	0.0883	-0.0059		-0.0076
4	-0.0354	0.1316	-0.0047		-0.0042
5	-0.0121	0.1076	-0.0013		-0.0013
6	0.0112	0.1384	0.0015		0.0016
7	0.0319	0.1229	0.0039		0.0023
8	0.0562	0.1040	0.0058		0.0050
9	0.0928	0.1050	0.0097		0.0114
10	0.1907	0.0643	0.0123		0.0143
		aggregate $d \log pr$=	0.0006	aggregate $d \log pr$ using GBR shares =	-0.0011

FRA

Decile	$d \log pr$ (average)	Share value added (FRA)	Share (FRA)*$d \log pr$		Share(GBR)*$d \log pr$
1	-0.1552	0.0716	-0.0111		-0.0085
2	-0.0697	0.1063	-0.0074		-0.0069
3	-0.0334	0.1113	-0.0037		-0.0038
4	-0.0048	0.1129	-0.0005		-0.0006
5	0.0186	0.1212	0.0023		0.0020
6	0.0413	0.1285	0.0053		0.0061
7	0.0632	0.1091	0.0069		0.0045
8	0.0896	0.0837	0.0075		0.0079
9	0.1284	0.0675	0.0087		0.0159
10	0.2440	0.0878	0.0214		0.0182
		aggregate $d \log pr$=	0.0293	aggregate $d \log pr$ using GBR shares =	0.0348

ITA

Decile	$d \log pr$ (average)	Share value added (ITA)	Share (ITA)*$d \log pr$		Share(GBR)*$d \log pr$
1	-0.2138	0.0628	-0.0134		-0.0117
2	-0.0979	0.0886	-0.0087		-0.0097
3	-0.0565	0.1170	-0.0066		-0.0064
4	-0.0262	0.1383	-0.0036		-0.0031
5	-0.0031	0.1179	-0.0004		-0.0003
6	0.0189	0.1171	0.0022		0.0028
7	0.0418	0.1077	0.0045		0.0030
8	0.0682	0.0892	0.0061		0.0060
9	0.1094	0.0863	0.0094		0.0135
10	0.2148	0.0751	0.0161		0.0160
		aggregate $d \log pr$=	0.0057	aggregate $d \log pr$ using GBR shares =	0.0101

GBR

Decile	$d \log pr$ (average)	Share value added (GBR)	Share (GBR)*$d \log pr$
1	-0.1477	0.0549	-0.0081
2	-0.0418	0.0990	-0.0041
3	-0.0076	0.1126	-0.0009
4	0.0170	0.1195	0.0020
5	0.0354	0.1091	0.0039
6	0.0555	0.1477	0.0082
7	0.0766	0.0710	0.0054
8	0.1051	0.0881	0.0093
9	0.1430	0.1234	0.0176
10	0.2536	0.0747	0.0189
		aggregate $d \log pr$=	0.0523

Table 6: Estimate of the production function for Belgium using OLS.

industry	$\boldsymbol{\epsilon_k}$	se(ϵ_k)	ϵ_l	se(ϵ_l)	ϵ_m	se(ϵ_m)	$\boldsymbol{\theta}$	$\boldsymbol{\mu}$
15	.024254	.0021049	.1715899	.002782	.7892767	.0021862	.9851206	1.095934
16	.0950597	.0137134	.0665313	.0187802	.7669978	.026175	.9285887	1.110088
17	.0345663	.0026726	.2034413	.0039416	.7408124	.0036503	.97882	1.11444
18	.0098506	.0050477	.1718834	.0065111	.8142364	.0058267	.9959704	1.119567
19	.0366267	.021146	.2478886	.0275733	.7290825	.0223094	1.013598	1.037782
20	.0331624	.0032934	.1650923	.0050643	.7872586	.0044691	.9855132	1.12492
21	.018816	.0042258	.2189154	.0084079	.7431836	.0068755	.980915	1.116072
22	.0242819	.0024171	.2473774	.0046304	.7000399	.0044372	.9716992	1.185321
23	.014024	.0175733	.2790716	.0273318	.7549599	.0124866	1.048056	1.172826
24	.0350945	.0028137	.2011176	.0048504	.7674007	.0043999	1.003613	1.126789
25	.0327325	.0026066	.2045444	.0043409	.7536795	.0040567	.9909564	1.132583
26	.0297956	.0025126	.1893844	.0036685	.7797855	.0039222	.9989654	1.170156
27	.0424432	.003352	.1620249	.0052216	.7756518	.0044041	.9801198	1.179669
28	.0307327	.0018343	.248825	.0031259	.701847	.0029655	.9814047	1.169207
29	.0417338	.0026518	.2236885	.004274	.725044	.0044404	.9904663	1.147926
30	.0438384	.0097131	.2133444	.0152757	.7542191	.0179286	1.011402	1.1183
31	.0185072	.0038281	.2576855	.007376	.7055325	.0069817	.9817252	1.167024
32	.0293824	.0113148	.1988501	.0195012	.7742139	.0164682	1.002446	1.262404
33	.0285132	.0050092	.187353	.009275	.7525092	.0091761	.9683754	1.257869
34	.0152044	.0044613	.1854333	.0063027	.7913007	.0053243	.9919384	1.153288
35	.0283478	.0063816	.224206	.0111818	.7547176	.0114414	1.007271	1.216691
36	.0168709	.0027038	.1537802	.003637	.8301196	.0034038	1.000771	1.235662
37	.0496096	.0052469	.142829	.009983	.7814211	.0076006	.9738598	1.133132
Total	.029304	.0029448	.2042291	.0047425	.7533956	.0043346	.9869287	1.150046

Table 7: Estimate of the production function for Belgium using system GMM.

industry	ϵ_k	se(ϵ_k)	ϵ_l	se(ϵ_l)	ϵ_m	se(ϵ_m)	$\boldsymbol{\theta}$	$\boldsymbol{\mu}$
15	.0215719	.0143043	.116369	.0533856	.845057	.0234978	.9829979	1.173386
16	.1015663	.0310604	.0631688	.0307818	.7815633	.0627052	.9462985	1.131169
17	.0197039	.0122925	.2157265	.0192355	.7701956	.0248191	1.005626	1.158642
18	-.0056251	.0206674	.1726151	.035114	.8318266	.0329617	.9988165	1.143753
19	.0311772	.0317691	.2345751	.0333582	.7474033	.0138473	1.013156	1.06386
20	.0339828	.0176306	.1430129	.0319975	.8317935	.0344319	1.008789	1.188557
21	.0109747	.0141275	.2621731	.0495077	.7361645	.0470625	1.009312	1.105531
22	-.0020648	.01703	.2060769	.0415456	.7255507	.0572205	.9295627	1.228516
23	.0090164	.0240922	.275057	.0588486	.7610017	.0387986	1.045075	1.182212
24	-.0025681	.0212298	.2419967	.0459446	.7398928	.0336436	.9793214	1.086399
25	.0253915	.0109819	.2186054	.034797	.7245669	.0391779	.9685638	1.088834
26	.0415794	.0154214	.1784004	.0352567	.7706105	.0488047	.9905902	1.156388
27	.0359403	.0272913	.1275111	.0291271	.7962665	.0325451	.9597178	1.211021
28	.0082466	.0110756	.221101	.0223465	.7515167	.0249234	.9808643	1.251952
29	.003992	.0129839	.1750448	.0241039	.8069009	.027443	.9859377	1.277526
30	.0428727	.0109362	.2259799	.0234276	.7459196	.0248053	1.014772	1.105995
31	.0098459	.0318262	.2204551	.0485979	.7511831	.0353373	.9814841	1.242535
32	-.0129264	.0155392	.2198884	.0259918	.7900414	.0319096	.9970034	1.288212
33	.0143141	.0157171	.2116466	.0439033	.7581939	.0447299	.9841546	1.267371
34	.0058168	.0141078	.1667487	.0274858	.8067272	.0201717	.9792928	1.175772
35	.0199072	.0128357	.1912204	.0391206	.7724271	.0389743	.9835548	1.24524
36	-.0145004	.0134578	.1721131	.0259554	.8601587	.0200723	1.017771	1.280377
37	.0538229	.0186533	.1383617	.0440445	.7744551	.0277837	.9666397	1.123031
Total	.014119	.0154878	.1875977	.0356587	.7802108	.0328418	.9819276	1.191647

Table 8: Estimate of the production function for Belgium using system GMM: validating tests

industry	**hansen** (p-value)	**ar1** (p-value)	**ar2p** (p-value)
15	.1526701	2.48e-08	.2259859
16	1	.1564406	.7954623
17	.0493479	.0102783	.080242
18	.7306365	.0457279	.6914725
19	1	.9864044	.0796718
20	.6683506	.001477	.089917
21	.2565103	.0130565	.6296991
22	.1097888	.0124187	.9321187
23	1	.8665326	.1526774
24	.6437734	.0004697	.1085116
25	.5183361	.0621041	.2388145
26	.1234163	.0059662	.437268
27	.605441	.0003418	.3046295
28	.0032045	.0080472	.4424166
29	.0916351	.0000682	.5258806
30	1	.3186199	.2074303
31	.1405106	.0128848	.8066118
32	.9991825	.4230046	.0968406
33	.853533	.0147613	.6647028
34	.2278766	.0112883	.4655574
35	.9994887	.0619346	.1509752
36	.956588	.0000944	.2486518
37	.6836056	.054695	.4062182

Table 9: Estimate of the production function for Belgium using Olley Pakes.

industry	$\boldsymbol{\epsilon_k}$	ϵ_l	ϵ_m	$\boldsymbol{\theta}$	$\boldsymbol{\mu}$
15	.0053129	.1607034	.7857859	.9518021	1.091087
16	.006006	.0524176	.7730815	.8315051	1.118893
17	.0042158	.195162	.737206	.9365838	1.109014
18	.0054147	.1729344	.8007517	.9791009	1.101026
19	.0053613	.2503178	.7430528	.9987319	1.057668
20	.0111285	.1595692	.7815555	.9522532	1.116771
21	.0111529	.2168774	.7270212	.9550515	1.0918
22	.0097333	.2480117	.6909198	.9486648	1.169878
23	.0099718	.281923	.7470524	1.038947	1.160542
24	.0110562	.1813903	.766848	.9592946	1.125978
25	.0110247	.1982173	.7443261	.9535682	1.118527
26	.0143439	.189329	.757315	.9609879	1.136437
27	.0138856	.1633305	.7672731	.9444892	1.166926
28	.0129493	.2405701	.6961676	.949687	1.159746
29	.0136856	.2278383	.7157144	.9572383	1.133155
30	.0138737	.2379677	.7353308	.9871722	1.090294
31	.0145976	.2384386	.6918178	.944854	1.144339
32	.0144183	.2252213	.7305138	.9701534	1.191149
33	.0147442	.1848012	.7544926	.9540381	1.261184
34	.0146547	.1883912	.7944484	.9974943	1.157876
35	.0152902	.2039668	.7447227	.9639798	1.200578
36	.0152605	.1524164	.8277795	.9954565	1.232179
37	.0156083	.1322395	.7942417	.9420895	1.151723
Total	.0109288	.1985189	.7462118	.9556595	1.138884

Table 10: Decomposition of the change in aggregate productivity (estimates using System GMM)

country	$\mathbf{d}\log\mathbf{pr}$	$(\overline{\mu}-1)(d\log X + d\log M/Q)$	$R_\mu + R_M$	$d\log T$	Residual
BEL	.0352278	0.01446	0.00048	.0114122	0.00888
ESP	.0311934	0.02011	-0.00132	.0048491	0.00755
FRA	.0478567	-0.00055	0.00169	.0405101	0.00621
GBR	.0601621	-0.00084	0.00083	.0490316	0.01114
ITA	.0280874	0.00695	0.00025	.0141505	0.00674

Notes: The estimates of firm productivity are obtained by estimating a production function with year fixed effects using system GMM.

Table 11: Decomposition of the change in aggregate productivity. Additional results.

country	**Productivity growth**	Aggregate distortions	Reallocation	Technological change
BEL	1	0.5488	0.0182	0.4331
ESP	1	0.8505	-0.0558	0.2051
FRA	1	-0.0132	0.0406	0.9727
GBR	1	-0.0171	0.0169	1.0002
ITA	1	0.3256	0.0117	0.6629

Notes: The entries in the table represent the percentage of productivity growth accounted by aggregate distortions, reallocation and technical change after reallocating the residual in proportion to the size of each of these components.