

# Growth in the Shadow of Expropriation\*

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## Abstract

In this paper, we propose a tractable variant of the open economy neoclassical growth model that emphasizes political economy and contracting frictions. The political economy frictions involve a preference for immediate spending on the part of political incumbents when facing the prospect of losing office, while the contracting friction is a lack of commitment regarding foreign debt and expropriation. We show that political economy frictions slow an economy's convergence to the steady state by generating a high equilibrium tax rate on investment that declines slowly over time. While in the standard neoclassical growth model capital's share in production determines the speed of convergence, this role is played by political economy frictions in our open economy reformulation. Moreover, while political frictions shorten the horizon of the government, the government may still pursue a path of tax rates in which the first best investment is achieved in the long run, although the transition may be slow. The model rationalizes why openness has different implications for growth depending on the political environment, why institutions such as the treatment of capital income evolve over time, why governments in open countries that grow rapidly tend to accumulate net foreign assets rather than liabilities, and why foreign aid may not affect growth.

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# 1 Introduction

In this paper we present a tractable growth model that highlights the interaction of political economy frictions, tax policy, and capital flows in a small open economy. We augment the standard neoclassical growth model with two frictions. First, there is limited commitment on the part of the domestic government. Specifically, capital income is subject to ex-post expropriation and the government can default on external debt. Second, political parties with distinct objectives compete for power. We show that the combination of these two frictions generate several prominent features of developing economy growth experiences, including the fact that economies with relatively high growth rates tend to have governments that accumulate large net foreign asset positions and that governments with weak political institutions tend to grow more slowly.

The model assumes that political parties prefer spending to occur while in office. That is, political incumbency with the prospect of losing office distorts how politicians view inter-temporal tradeoffs. One motivation for this incumbency distortion is the insight of [Alesina and Tabellini \(1990\)](#) and [Persson and Svensson \(1989\)](#), which argue that political disagreement between potential incumbents makes parties prefer spending to occur while in office. Another interpretation is that corruption allows incumbents to consume a disproportionate share of spending. As in [Amador \(2004\)](#), we show that the political environment can be conveniently modeled as a sequence of incumbents that possess time-inconsistent preferences. We embed this political process in a small open economy in which the government can expropriate capital and default and study the dynamics of investment and external debt.

Specifically, we consider the path of taxes, consumption, investment, and sovereign debt, that maximizes the population's welfare subject to the constraint that each incumbent has the power to repudiate debt and expropriate capital. Deviation, however, leads to financial autarky and reversion to a high tax-low investment equilibrium. In this sense, we study self-enforcing equilibria in which allocations are constrained by the government's lack of commitment, building on the framework used by [Marcet and Marimon \(1992\)](#), [Thomas and Worrall \(1994\)](#), [Albuquerque and Hopenhayn \(2004\)](#), and [Aguiar et al. \(2009\)](#). These papers discuss how limited commitment can slow capital accumulation. A main result of the current paper is that the political economy frictions generate additional dynamics. In particular, we show that the degree to which political parties value incumbency has a first order (negative) effect on the speed of the economy's convergence to the steady state. In the standard closed economy version of the neoclassical growth model, the speed of convergence is governed in large part by the capital share parameter.

In our reformulation, the role of capital share is played by parameters reflecting the value of incumbency.

The intuition behind the dynamics begins with debt overhang. A country with a large external sovereign debt position has a greater temptation to default, and therefore cannot credibly promise to leave large investment positions un-expropriated. Growth therefore requires the country to pay down its debt, generating a trade off between the incumbent's desire to consume while in office against reducing foreign liabilities and increasing investment. In a highly distorted political environment, governments are unwilling to reduce their sovereign debt quickly, as the desire for immediate consumption outweighs the future benefits of less overhanging debt. In this manner, the model is able to reconcile the mixed results that countries have had with financial globalization. Countries with different underlying political environments will have different growth experiences after opening: some economies will borrow and stagnate, while others will experience net capital outflows and grow quickly.

Political friction in our model generates short-term impatience, but is distinct from a model of an impatient decision maker with time consistent preferences. For example, in our framework the degree of political distortions may not affect the long run capital stock. In particular, if the private agents discount at the world interest rate, the economy eventually reaches the first best level of capital for any finite level of political distortion. This reflects the fact that while incumbents disproportionately discount the near future, this relative impatience disappears as the horizon is extended far into the future. In the neoclassical growth model, a high geometric discount rate speeds conditional convergence, as the low savings rate is offset by a lower steady state capital stock, while political friction in our model slows conditional convergence. The level of political distortion will determine the level of steady state debt that supports the first best capital.

The mechanism in our paper is consistent with the empirical fact that fast growth is accompanied by reductions in net foreign liabilities, the so-called "allocation puzzle" of [Gourinchas and Jeanne \(2007\)](#) (see also [Aizenman et al., 2004](#) and [Prasad et al., 2006](#)). This allocation puzzle represents an important challenge to the standard open economy model which predicts that opening an economy to capital inflows will speed convergence, as the constraint that investment equals domestic savings is relaxed.<sup>1</sup> Our model rationalizes

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<sup>1</sup>Two important papers that also study the neoclassical growth model in an open economy setting are [Barro et al. \(1995\)](#), who introduce human capital accumulation and a credit market imperfection to obtain non-trivial dynamics, and [Ventura \(1997\)](#). In the open economy version of the neoclassical model studied by [Barro et al. \(1995\)](#), the debt to output ratio is constant along the transition path if the production function is Cobb-Douglas. More generally, their prediction is for an economy to unambiguously accumulate net liabilities as it accumulates capital. [Cohen and Sachs \(1986\)](#) also study growth in an open economy model with limited commitment. In their formulation, a fraction of capital serves as collateral, so more capital

the allocation puzzle as capital will not be invested in an economy with high debt due to the risk of expropriation. Limited commitment therefore provides an incentive for the government to pay down its external debt along the transition path, while political frictions determine how aggressively the government responds to this incentive.

Our model emphasizes *sovereign* debt overhang. In particular, external debt matters in the model to the extent that the government controls repayment or default. While the allocation puzzle has been framed in the literature in terms of aggregate net foreign assets (both public and private), the appropriate empirical measure for our mechanism is public net foreign assets. Figure 1 documents that the allocation puzzle is driven by the net foreign asset position of the public sector. Specifically, we plot growth in GDP per capita (relative to the U.S.) against the change in the ratio of the government's net foreign assets to GDP, where the net position is defined as international reserves minus public and publicly guaranteed external debt.<sup>2</sup> Figure 1 depicts a clear, and statistically significant, relationship between growth and the change in the government's external net assets.

We should emphasize that this relationship is not driven by fast growing governments borrowing heavily at the beginning of the sample period – the relationship between initial government assets and subsequent growth is weakly positive. Moreover, it is not simply that governments save during transitory booms and borrow during busts. As documented by [Kaminsky et al. \(2004\)](#), fiscal surpluses in developing economies are negatively correlated with income at business cycle frequencies. Figure 1 therefore reflects long run behavior.

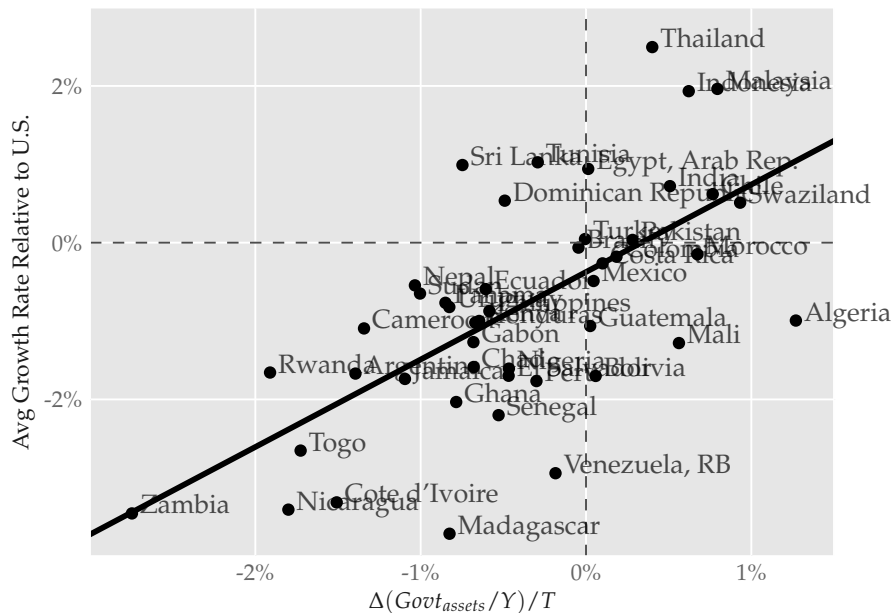
Figure 2 plots growth against the change in private net foreign assets, which is simply total net foreign assets minus public net foreign assets. For the private sector, positive growth is associated with greater net capital inflows on average (albeit weakly), consistent with standard theory. Thus, the puzzle is one regarding government assets, the focus of our model.

Similarly, our paper addresses the issue of “global imbalances” as it relates to the interaction of developing economies with world financial markets. An alternative explanation to ours is that developing economies have incomplete domestic financial markets and therefore higher precautionary savings, which leads to capital outflows (see [Willen, 2004](#)

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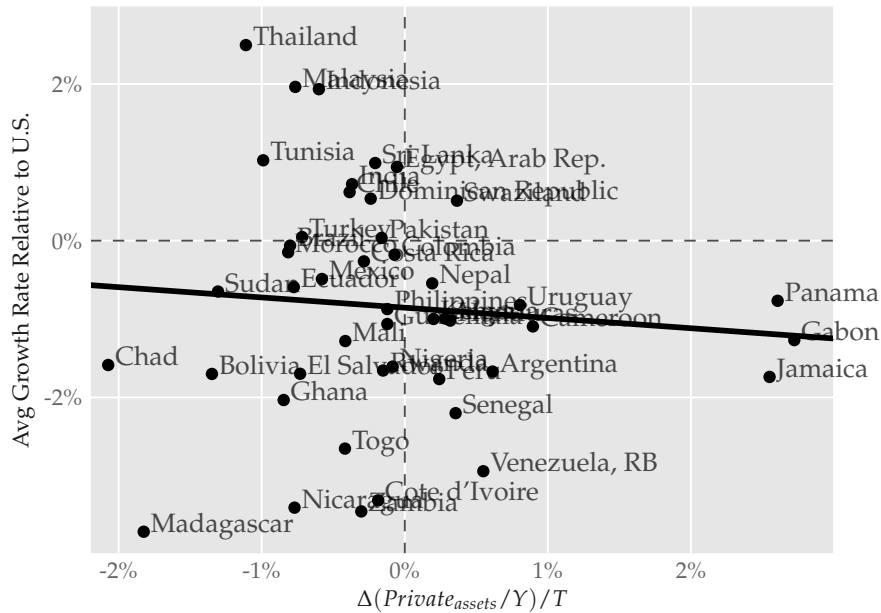
is accompanied by more debt, and therefore debt as a fraction of income does not decline as the economy grows. See [Castro \(2005\)](#) for a careful quantitative exploration of whether open economy models with incomplete markets and technology shocks can account for the patterns of development observed in the data.

<sup>2</sup>See the notes to the figures for data sources and sample selection. See also the online appendix for a discussion of an augmented model with exogenous growth.



**Figure 1:** This figure plots average annual growth in real GDP per capita relative to the U.S. against the change in ratio of public net foreign assets to GDP between 1970–2004.  $T$  represents the number of years:  $T = 34$ . Public net foreign assets are international reserves (excluding gold) minus public and publicly guaranteed external debt, both from WDI. Real GDP per capita is constant local currency GDP per capita from World Development Indicators (WDI). The sample includes countries with 1970 GDP per capita less than or equal to USD 10,000 in year 2000 dollars.

and [Mendoza et al., 2008](#)). However, this literature is silent on the heterogeneity across developing economies in terms of capital flows. For example, several Latin American economies have similar or even more volatile business cycle than South Korea ([Aguir and Gopinath, 2007](#)) and less developed financial markets ([Rajan and Zingales, 1998](#)), yet Latin America is not a strong exporter of capital (Figure 1). [Caballero et al. \(2008\)](#) also emphasize financial market weakness as generating capital outflows. In their model, exogenous growth in developing economies generates wealth but not assets, requiring external savings. Our model shares their focus on contracting frictions in developing economies, but seeks to understand the underlying growth process. As noted above, our paper shares the feature of [Marcet and Marimon \(1992\)](#) and [Thomas and Worrall \(1994\)](#) that reductions in debt support larger capital stocks. [Dooley et al. \(2004\)](#) view this mechanism through the lens of a financial swap arrangement, and perform a quantitative exercise that rationalizes China’s large foreign reserve position. These papers are silent on why some developing countries accumulate collateral and some do not, a primary question of this paper. Our paper also explores how the underlying political environment



**Figure 2:** This figure plots average annual growth in real GDP per capita relative to the U.S. against the change in the ratio of private net foreign assets to GDP between 1970–2004.  $T$  represents the number of years:  $T = 34$ . Private net foreign assets are total net foreign assets (Net foreign assets are gross foreign assets minus gross liabilities in current US dollars from EWN Mark II) minus public net foreign assets (from Figure 1). Real GDP per capita is constant local currency GDP per capita from World Development Indicators (WDI). The sample includes countries with 1970 GDP per capita less than or equal to USD 10,000 in year 2000 dollars.

affects the speed with which countries accumulate collateral or reduce debt.

A predominant explanation of the poor growth performance of developing countries is that weak institutions in general and poor government policies in particular tend to deter investment in capital and/or productivity enhancing technology.<sup>3</sup> A literature has developed that suggests that weak institutions generate capital outflows rather than inflows (see, for example, [Tornell and Velasco, 1992](#) and [Alfaro et al., 2008](#), who address the puzzle raised by [Lucas, 1990](#)). While it is no doubt true that world capital avoids countries with weak property rights, our model rationalizes why countries with superior economic performance are net exporters of capital.

Our paper also relates to the literature on optimal government taxation with limited commitment. Important papers in this literature are [Benhabib and Rustichini \(1997\)](#) and

<sup>3</sup>An important contribution in this regard is [Parente and Prescott \(2000\)](#). Similarly, a large literature links differences in the quality of institutions to differences in income per capita, with a particular emphasis on protections from governmental expropriation (for an influential series of papers along this line, see [Acemoglu et al., 2001, 2002](#) and [Acemoglu and Johnson, 2005](#)).

Phelan and Stacchetti (2001), who share our focus on self-enforcing equilibria supported by trigger strategies (a parallel literature has developed that focuses on Markov perfect equilibria, such as Klein and Rios-Rull, 2003, Klein et al., 2005, and Klein et al., 2008). In this literature, our paper is particularly related to Dominguez (2007), which shows in the environment of Benhabib and Rustichini (1997) that a government will reduce its debt in order to support the first best capital in the long run (see also Reis, 2008). Recently, Azzimonti (forthcoming) has shown how political polarization and government impatience can lead to high levels of investment taxes, slow growth, and low levels of output per capita in the context of a closed economy model with capital accumulation, partisan politics and a restriction to Markov strategies. Differently from her work, we are focused on the open economy implications of a political economy model. On the technical side, we analyze trigger strategies and reputational equilibria, as we think these are important elements to consider in any analysis of sovereign debt. Debt sustained by reputational equilibria has important implications for growth dynamics in our framework, a point we discuss in detail in section 5. Our work is also related to the recent paper by Acemoglu et al. (2009), which studies efficient equilibria in a political economy model with production. Differently from us, they analyze an economy without capital and closed to international financial markets.<sup>4</sup>

The remainder of the paper is organized as follows. Section 2 presents the environment. Sections 3 and 4 characterize the path of equilibrium taxes, investment, and output. Section 5 explores the quantitative implications of our model and compares our framework with other popular growth models, and section 6 concludes. The appendix contains all proofs. An online appendix features the following extensions of the model: (i) introducing exogenous growth and (ii) allowing capitalists welfare to be in the objective function of the incumbent governments.

## 2 Environment

In this section we describe the model environment (which is based on Aguiar et al., 2009). Time is discrete, indexed by  $t$ , and runs from 0 to infinity. There is a small open economy which produces a single good, whose world price is normalized to one. There is also an international financial market that buys and sells risk-free bonds with a constant return denoted by  $R = 1 + r$ .

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<sup>4</sup>See also Myerson (2010), who study the decision to liberalize the economy when the government cannot commit to not expropriate investments, and characterizes stationary equilibrium under non-expropriation constraints. Differently from him, we do not study the choice of liberalization regime, but instead, analyze transitional dynamics under lack of commitment and political economy frictions.

The economy is populated by capitalists, who own and operate capital, workers who provide labor, and a government. In our benchmark analysis, we assume that capitalists do not enter the government’s objective function, defined below. This assumption is convenient in that the government has no hard-wired qualms about expropriating capital income and transferring it to its preferred constituency. This assumption is not crucial to the results and we discuss the more general case with “insider” capitalists in the online appendix. The important assumption is that capitalists are under the threat of expropriation.

## 2.1 Firms

Domestic firms use capital together with labor to produce according to a strictly concave, constant returns to scale production function  $f(k, l)$ .<sup>5</sup> We assume that  $f(k, l)$  satisfies the usual Inada conditions. Capital is fully mobile internationally at the beginning of every period,<sup>6</sup> but after invested is sunk for one period. Capital depreciates at a rate  $d$ .

Labor is hired by the firms in a competitive domestic labor market which clears at an equilibrium wage  $w$ . The government taxes the firm profits at a rate  $\tau$ . Let  $\pi = f(k, l) - wl$  denote per capita profits before taxes and depreciation, and so  $(1 - \tau)\pi$  is after-tax profits. The firm rents capital at the rate  $r + d$ . Given an equilibrium path of wages and capital taxes, profit maximizing behavior of the representative firm implies:

$$(1 - \tau_t)f_k(k_t, l_t) = r + d \tag{1}$$

$$f_l(k_t, l_t) = w_t. \tag{2}$$

For future reference, we denote  $k^*$  as the first best capital given a mass one of labor:  $f_k(k^*, 1) = r + d$ . When convenient in what follows, we will drop the second argument and simply denote production  $f(k)$ .

It is convenient to limit the government’s maximal tax rate to  $\bar{\tau} > 0$ . We assume that this constraint does not bind along the equilibrium path. Nevertheless, as discussed in Section 2.5, this assumption allows us to characterize possible allocations off the equilibrium path.

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<sup>5</sup>The model focuses on transitional dynamics and assumes a constant technology for convenience. The model easily accommodates exogenous technical progress in which the economy transitions to a balanced growth path. See the online appendix for details.

<sup>6</sup>That is, capital will earn the same after tax return in the small open economy as in the international financial markets. See [Caselli and Feyrer \(2007\)](#) on evidence that returns to capital are quantitatively similar across countries.



## 2.2 Domestic workers and the government

Labor is supplied inelastically each period by a measure-one continuum of domestic workers (there is no international mobility of labor). The representative domestic worker values flows of per capita consumption according to a bounded from below utility function,  $u(c)$ . Domestic workers discount the future with a discount factor  $\beta \in (0, 1/R]$ . We assume that agents in the small open economy are at least as impatient as foreigners, which guarantees that the government does not accumulate assets to infinity. The representative agent's utility is

$$\sum_{t=0}^{\infty} \beta^t u(c_t). \quad (3)$$

with  $u' > 0, u'' \leq 0$ , and where we normalize  $u(0) \geq 0$ .

We assume that domestic workers have no direct access to international capital markets. In particular, we assume that the government can control the consumption/savings decisions of its constituents using lump sum transfers and time varying taxes or subsidies on domestic savings. This is equivalent in our set up to workers consuming their wages plus a transfer:  $c_t = w_t + T_t$ , where  $T_t$  is the transfer from the government.<sup>7</sup>

The government every period receives the income from the tax on profits and transfers resources to the workers subject to its budget constraint:

$$\tau_t \pi_t + b_{t+1} = Rb_t + T_t \quad (4)$$

where  $b_t$  is debt due in period  $t$ . The government and workers combined resource constraint is therefore:

$$c_t + Rb_t = b_{t+1} + \tau_t \pi_t + w_t. \quad (5)$$

Note that output is deterministic, and so a single, risk-free bond traded with the rest of the world is sufficient to insure the economy. However, as described in the next subsection, political incumbents face a risk of losing office, and this risk is not insurable.

## 2.3 Political Environment

There is a set  $I \equiv \{1, 2, \dots, N + 1\}$  of political parties, where  $N + 1$  is the number of parties. At any time, the government is under the control of an incumbent party that is chosen at the beginning of every period from set  $I$ . As described below, an incumbent party may lose (and regain) power over time. Our fundamental assumption is that the incumbent

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<sup>7</sup>This can be decentralized by consumers having access to a tax distorted bond.

strictly prefers consumption to occur while in power:

**Assumption 1** (Political Economy Friction). *A party enjoys a utility flow  $\tilde{\theta}u(c)$  when in power and a utility flow  $u(c)$  when not in power, where  $c$  is per capita consumption by the domestic workers and where  $\tilde{\theta} > 1$ .*

This specification captures that incumbents view inter-temporal comparisons differently than does the opposition. One motivation for this parameter is political disagreement regarding the type of expenditures, as in, for example, the classic paper of [Alesina and Tabellini \(1990\)](#). Specifically, suppose that the incumbent party selects the attributes of a public good that forms the basis of private consumption. If parties disagree about the desirable attributes of the consumption good, the utility stemming from a given level of spending will be greater for the party in power. We model such disagreement in a simple, reduced form way with the parameter  $\tilde{\theta}$ .<sup>8</sup> Alternatively, we can think of the incumbent capturing a disproportionate share of per capita consumption, perhaps through corruption or pork barrel spending.<sup>9</sup> Another interpretation is that incumbency itself brings with it a different viewpoint on inter-temporal trade offs, perhaps as a direct response to the responsibilities or temptations of power.<sup>10</sup>

The transfer of power is modeled as an exogenous Markov process. The fact that the transfer of power is exogenous can be considered a constraint on political contracts between the population (or other parties) and the incumbent. As will be clear, each incumbent will abide by the constrained efficient tax plan along the equilibrium path. However, doing so does not guarantee continued incumbency (although our results easily extend to the case where the incumbent loses office for sure if it deviates). That is, following the prescribed tax and debt plan does not rule out that other factors may lead to a change of government. We capture this with a simple parametrization that nests perpetual office holding, hard term limits, and the probabilistic voting model.

Let  $p$  denote the probability of an incumbent retaining office. Conditional on an incumbent losing office next period, each opposition party has an equal chance of winning office. Denoting  $q$  as the probability of regaining office, we have  $q \equiv (1 - p)/N$ . It may be the case that the incumbent has an advantage in maintaining office ( $p > q$ ), or that term

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<sup>8</sup>See [Battaglini and Coate \(2008\)](#) for a recent paper that incorporates pork-barrel spending in a dynamic model of fiscal policy. They obtain a reduced form representation that is similar to ours, except that  $\theta$  is also a function of the state of the economy.

<sup>9</sup>Suppose that when in power, a party receives a higher share  $\phi$  of  $c$ . Then, the marginal utility when in power is  $u'(\phi c)\phi$ , and our assumption would be similar to requiring that this marginal utility be increasing in  $\phi$ .

<sup>10</sup>More precisely, our framework accommodates such a direct incumbency effect, but incumbents are sophisticated enough to understand that their views will change again once they revert to opposition status.

limits or some or a similar institution places the incumbent at a disadvantage ( $p < q$ ). In particular,  $(p - q) \in [-1/N, 1]$ . We denote the probability that the incumbent at period  $t$  is also in office in period  $s > t$  by  $p_{t,s}$ . Given the Markov political process, we have  $p_{t,s+1} = p \times p_{t,s} + (1 - p_{t,s})q$ . Solving this difference equation starting from  $p_{t,t} = 1$ , we have for  $s \geq t$ :

$$p_{t,s} = \bar{p} + (1 - \bar{p})(p - q)^{s-t},$$

where  $\bar{p} = \lim_{s \rightarrow \infty} p_{t,s}$  is the unconditional probability of taking office.<sup>11</sup> For  $p < 1$ , we have  $\bar{p} = 1/(N + 1)$ , and if  $p = 1$  then  $\bar{p} = 1$  as well.

As we will see, the key to our mechanism is that incumbents have a different perspective on spending and saving decisions than non-incumbents. We now introduce notation that proves useful in the analysis and that captures the two aspects of incumbency. Let

$$\theta \equiv \frac{\tilde{\theta}}{\bar{p}\tilde{\theta} + 1 - \bar{p}}. \quad (6)$$

The parameter  $\theta$  is the ratio of how a political party views spending conditional on incumbency relative to how it values spending unconditional on incumbency. The greater this ratio, the more relative weight an incumbent puts on spending while in office, and the less inclined it is to delay spending. A second potential difference due to incumbency is that the probability of holding office next period may depend on current incumbency. Let

$$\delta \equiv p - q = \frac{p - \bar{p}}{1 - \bar{p}}. \quad (7)$$

We will refer to  $\delta$  as an incumbency advantage, as the larger is  $\delta$  the greater the advantage incumbency confers in retaining office. Note that an increase in  $p$  holding constant  $\bar{p}$  raises the relative value of incumbency, and, as we will see, a greater  $\delta$  is associated with slower growth. One might consider a relatively high conditional probability  $p$  makes an incumbent more patient. However, the important distinction is the difference between the conditional and unconditional probabilities— the greater is  $p - \bar{p}$ , the greater is the premium on acting while in office. Even though the current spell may be relatively long, it could also be the last one if  $\bar{p}$  is small.<sup>12</sup> Therefore a greater  $\delta$ , like  $\theta$ , is also a measure

<sup>11</sup>To obtain  $p_{t,t} = 1$  from the expression when  $p = q$ , use the convention that  $0^0 = 1$ .

<sup>12</sup>For example, [Alesina et al. \(1996\)](#) document that Asia and Latin America change governments at similar frequencies. However, Latin America is much more likely to have military coups and what [Alesina et al. \(1996\)](#) refer to as “major” government changes, while Asia rarely has a major government change. This suggests that  $p$  is similar in Asia and Latin America, but incumbency ( $\theta$  and  $\delta$ ) is more important in Latin America. Similarly, leadership changes are infrequent in Africa, but most changes are major, with more

of the distortion due to incumbency. Of course, a  $p = 1$  (that is,  $\delta = 1$  and  $\theta = 1$ ) is a special case, as there is no room for disagreement across potential incumbents given that the current incumbent is the only relevant political party.

Given a deterministic path of consumption, the utility of the incumbent in period  $t$  can now be expressed as:

$$\tilde{W}_t = \sum_{s=t}^{\infty} \beta^{s-t} p_{t,s} \tilde{\theta} u(c_s) + \sum_{s=t}^{\infty} \beta^{s-t} (1 - p_{t,s}) u(c_s). \quad (8)$$

We can simplify this expression by using the definition of  $p_{t,s}$  and introducing a normalized incumbent utility  $W_t$ :

$$W_t \equiv \frac{\tilde{W}_t}{\bar{p}(\tilde{\theta} + N)} = \sum_{s=t}^{\infty} \beta^{s-t} (\theta \delta^{s-t} + 1 - \delta^{s-t}) u(c_s). \quad (9)$$

As will be clear below, the scaling of the incumbent's utility has no effect on the equilibrium allocations, and so we work with  $W_t$ .

The preferences in equation (9) indicate that the current incumbent behaves as if it has a political survival hazard  $\delta$ , and then becomes a private agent once out of office, with the parameter  $\theta$  indicating how much it favors incumbency. To see how the incumbent values inter-temporal trades, consider discounting between the current period ( $s = t$ ) and the following period. Utility today is  $\theta u(c_t)$ , while tomorrow's utility is  $\beta(\delta\theta + 1 - \delta)$ , so the discount factor is  $\beta(\delta\theta + 1 - \delta)/\theta < \beta$ . However, this is not the same as a low geometric discount factor. To see this, consider discounting between period  $s$  and  $s + 1$  in the distant future. As  $s \rightarrow \infty$ ,  $p_{t,s}$  converges to the unconditional probability of incumbency  $\bar{p}$ . As today's incumbent is equally likely to be in office in  $s$  and  $s + 1$  for large  $s$ , it discounts between these periods using the private agents' discount factor  $\beta$ . In this manner, political incumbents discount between today and next period at a higher rate than they discount between two periods in the future. This implies that political incumbents behave similarly to a quasi-hyperbolic (or quasi-geometric) agent as in [Laibson \(1997\)](#). The comparison is exact when there is no incumbency advantage ( $\delta = 0$  and  $p = q = 1/(N + 1)$ ), so the conditional re-election probability always equals the unconditional election probability. In this parameterization, the incumbent discounts between today and tomorrow at  $\beta/\theta < \beta$ , and between any two periods after the current period at  $\beta$ .<sup>13</sup> As we increase  $\delta < 1$ , the distortion to the discount factor persists farther into the future. This framework is rich

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than half of the leadership changes resulting from military coups.

<sup>13</sup>The fact that political turnover can induce hyperbolic preferences for political incumbents was explored by [Amador \(2004\)](#).

enough to capture several cases. A situation where the country is ruled by a “dictator for life” who has no altruism for successive generations, can be analyzed by letting  $\theta \rightarrow \infty$ , reflecting the zero weight the dictator puts on aggregate consumption once it is out of power. Letting  $\theta \rightarrow 1$ , the government is benevolent and the political friction disappears.

## 2.4 Equilibrium Concept

The final key feature of the environment concerns the government’s lack of commitment. Specifically, tax policies and debt payments for any period represent promises that can be broken by the government. Given the one-period irreversibility of capital, there exists the possibility that the government can seize capital or capital income. Moreover, the government can decide not to make promised debt payments in any period.

We consider self-enforcing equilibria that are supported by a “punishment” equilibrium. Specifically, let  $\underline{W}(k)$  denote the payoff to the incumbent government after a deviation when capital is  $k$ , which we characterize in the next subsection. Self-enforcing implies that:

$$W_t \geq \underline{W}(k_t), \forall t, \quad (10)$$

where  $W_t$  is given by (9).

Our equilibrium concept assumes that political risk is not insurable. That is, sovereign debt or tax promises cannot be made contingent on the realization of the party in power, which we take as a realistic assumption.<sup>14</sup> We therefore look for equilibria under the following definition:

**Definition 1.** *A self-enforcing deterministic equilibrium is a deterministic sequence of consumption, capital, debt, tax rates and wages  $\{c_t, k_t, b_t, \tau_t, w_t\}$ , with  $\tau_t \leq \bar{\tau}$  for all  $t$  and such that (i) firms maximize profits given taxes and wages; (ii) the labor market clears; (iii) the resource constraint (5) and the associated no Ponzi condition hold given some initial debt  $b_0$ ; and (iv) the participation constraint (10) holds given deviation payoffs  $\underline{W}(k_t)$ .*

There may be many allocations that satisfy these equilibrium conditions. In the next section we discuss equilibrium selection. However, we first characterize the self enforcing equilibrium that yields the lowest payoff for the incumbent.

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<sup>14</sup>That is, foreigners contract with governments, not individual parties. One way to rationalize the absence of political insurance is to assume that foreign creditors cannot distinguish among the various domestic political parties (or factions within a party, and so on). International financial assets therefore cannot make promises contingent on political outcomes.

## 2.5 The punishment and the deviation payoff

Our definition of equilibrium is conditional on deviation payoffs  $\underline{W}(k)$ . As will be clear in the next section, we characterize the economy's dynamics for arbitrary  $\underline{W}(k)$ , subject to a concavity assumption. However, it is useful to characterize a deviation utility that delivers the lowest payoff to the government of any self-enforcing equilibrium. Towards obtaining this punishment payoff, we assume that after any deviation from the equilibrium allocation, the international financial markets shut down access to credit and assets forever. That is, if the government deviates on promised tax or debt payments, the government is forced into financial autarky.

Given that the government has access to neither borrowing nor savings after a deviation, we construct a punishment equilibrium of the game between investors and the government that has the following strategies. For any history following a deviation, the party in power sets the tax rate at its maximum  $\bar{\tau}$ , and investors invest  $\underline{k}$ , where  $\underline{k}$  solves:

$$(1 - \bar{\tau})f'(\underline{k}) = r + d,$$

where  $\underline{k} = 0$  if  $\bar{\tau} \geq 1$ . These strategies form an equilibrium. A party in power today cannot gain by deviating to a different tax rate, given that it is already taxing at the maximum rate and reducing taxes does not increase future investment. On the other hand, investors understand that they will be taxed at the maximum rate, and thus invest up to the point of indifference. Note that we allow domestic capitalists to invest overseas ("capital flight") in the periods after deviation, so they continue to discount returns at the world interest rate.

The following lemma establishes that the above allocation is the harshest punishment:<sup>15</sup>

**Lemma 1.** *The continuation equilibrium where  $\tau_t = \bar{\tau}$  after any history and the government is in financial autarky generates the lowest utility to the incumbent party of any self-enforcing equilibrium.*

To calculate the deviation utility, note that deviation triggers financial autarky and the lowest possible investment for all periods to follow. Therefore, the party in power at the time of deviation will find it optimal to tax the existing capital at the maximum possible

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<sup>15</sup>The result in the lemma, although intuitive, is not direct and requires a proof. In particular, one needs to rule out the type of equilibria first analyzed by Laibson (1997); namely, equilibria which are supported by cascading punishments from future players and result in unbounded continuation values. Reminiscent of Laibson's results, our assumption of a utility function bounded from below is sufficient to rule these out.

rate, and its deviation payoff will be given by  $\underline{W}(k)$ :

$$\underline{W}(k) = \theta u(\bar{c}(k)) + \beta \left( \frac{\delta(\theta - 1)}{1 - \beta\delta} + \frac{1}{1 - \beta} \right) u(\bar{c}(\underline{k})), \quad (11)$$

where  $\bar{c}(k) = f(k) - (1 - \bar{\tau})f'(k)k$ .

### 3 Efficient Allocations

There are in principle many equilibria of this economy. In this section we solve for the self-enforcing deterministic equilibrium that maximizes the utility of the population as of time 0 given an initial level of debt. That is, the population chooses its preferred fiscal policy subject to ensuring the cooperation of all future governments, which is a natural benchmark.<sup>16</sup>

As is standard in the Ramsey taxation literature, we first show that we can restrict attention to allocations, that is, to sequences of consumption and capital:  $\{c_t, k_t\}$ . To see this note that conditions (i), (ii) and (iii) of Definition 1 can be collapsed to a present value condition:

$$b_0 \leq \sum_{t=0}^{\infty} R^{-t} (f(k_t) - (r + d)k_t - c_t)$$

Importantly, for any allocation  $\{c_t, k_t\}$  that satisfies the above present value condition and  $k_t \leq \underline{k}$ , there exist a tax rate sequence  $\{\tau_t \leq \bar{\tau}\}$ , a wage sequence  $\{w_t\}$ , and a debt position  $\{b_t\}$  such that  $\{c_t, k_t, b_t, \tau_t, w_t\}$  is a competitive equilibrium (satisfies (i), (ii) and (iii)). That is, if an allocation satisfies the present value condition and also satisfies the participation constraint (10), then it is a self enforcing deterministic equilibrium.

We can then obtain the equilibrium allocation that maximizes the utility of the population at time zero, given an initial stock of debt  $b_0$ , by solving:

$$V(b_0) = \max_{\{c_t, k_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (\text{P})$$

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<sup>16</sup>An alternative equilibria is one in which the initial government selects the best self-enforcing fiscal policy from its perspective, where “initial” could be interpreted as the time the economy opens itself to capital flows. This equilibrium has the same dynamics as the one we study in the next subsection, starting from the second period.

subject to:

$$b_0 \leq \sum_{t=0}^{\infty} R^{-t} (f(k_t) - (r+d)k_t - c_t), \quad (12)$$

$$\underline{W}(k_t) \leq W_t, \quad \forall t \quad (13)$$

$$\underline{k} \leq k_t \quad (14)$$

The first constraint is the present value condition discussed previously; the second constraint is the participation constraint for the sequence of incumbents; and the last constraint, (14), guarantees that  $\tau_t \leq \bar{\tau}$ . Unless stated otherwise, in what follows we assume that this last constraint does not bind along the equilibrium path.<sup>17</sup>

Let  $\mu_0$  be the multiplier on the budget constraint (12). Given that  $\mu_0$  will be strictly positive, we let  $\lambda_t (R^{-t} \mu_0 / \theta)$  be the multiplier on the sequence of constraints on participation (13), where we have normalized to simplify expressions below. The necessary first order condition for the optimality of consumption is:

$$1 = u'(c_t) \left( \underbrace{\frac{\beta^t R^t}{\mu_0}}_{\text{impatience}} + \underbrace{\sum_{s=0}^t \beta^s R^s \frac{\lambda_{t-s}}{\theta}}_{\text{limited commitment}} + \underbrace{\sum_{s=0}^t \beta^s R^s \delta^s (\theta - 1) \frac{\lambda_{t-s}}{\theta}}_{\text{incumbency}} \right), \quad \forall t \geq 0. \quad (15)$$

This first order condition for consumption has three terms. The first term,  $(\beta R)^t / \mu_0$ , is the standard consumption tilting: agents prefer to delay, smooth, or bring forward consumption depending on whether  $\beta R \gtrless 1$ . The second term,  $\sum_{s=0}^t (\beta R)^s \frac{\lambda_{t-s}}{\theta}$ , reflects the fact that raising consumption in period  $t$  relaxes the participation constraints for periods  $t - s < t$  as well. This term highlights the efficiency of back-loading payments in one-sided limited commitment models: when  $\beta R = 1$ , this term is monotone and increasing in  $t$ , and thus will lead to an increasing path of consumption. The value of incumbency however, introduces one new term which is the focus of this paper: the discounted sum of  $(\theta - 1) \frac{\lambda_t}{\theta}$ . This term tells us that consumption during incumbency is special, as an increase in utility during incumbency relaxes the current incumbent's participation constraint by an extra  $(\theta - 1)$ .

The necessary condition for the optimality of the capital stock is:

$$\lambda_t = \frac{f'(k_t) - (r+d)}{\underline{W}'(k_t) / \theta}, \quad \forall t \geq 0 \quad (16)$$

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<sup>17</sup>In the appendix we describe the general solution taking into account that this constraint may bind.



The lack of commitment is also evident in this condition, when coupled with the firms' first order condition. Note that absent commitment problems ( $\lambda_t = 0$ ), capital would be at the first best, as taxing capital in this model is inefficient ex-ante. However, under lack of commitment, a zero tax may not be self-enforcing. When the participation constraint on the incumbent government is strictly binding ( $\lambda_t > 0$ ), then  $f'(k_t) > r + d$  and so the tax on capital is strictly positive. Nevertheless, the necessary conditions imply that  $\tau = 0$  will be sustained in the long run if private agents discount at the world interest rate:

**Proposition 1.** *If  $\beta R = 1$  and  $\theta < \infty$ , then  $k_t \rightarrow k^*$ .*

The proof of this proposition (see the appendix) relies on the fact that each time the participation constraint binds,  $\lambda_t > 0$  and we add a strictly positive term to the second term on the right hand side of equation (15). This generates a force for increasing consumption over time, which relaxes the government's participation constraint. There is a potentially countervailing force in that the current  $\lambda_t$  is weighted by more than the past, as  $\theta > 1$ . However, eventually the (infinite) sum dominates and consumption levels off at a point such that participation no longer binds at  $k^*$ .

As discussed above, a general feature of models with one-sided limited commitment is that the optimal contract "back loads" incentives when agents are patient (see, for example, Ray, 2002). However, if the agent that suffers from lack of commitment is impatient, this is not necessarily the case. For example, in the models of Aguiar et al. (2009) and Acemoglu et al. (2008), governmental impatience prevents the first best level of investment from being achieved in the long run. In our environment, we approach this first best level despite the fact that the incumbent government, which chooses the tax rate at every period, is discounting between today and tomorrow at a *higher rate* than that of the private agents. However, the critical point is that each incumbent discounts the distant future periods at the same rate  $\beta = 1/R$ . For this reason, each government is willing to support a path of investment that approaches the first best. This highlights that short term impatience of the incumbents is not sufficient to generate distortions in the long-run.

The second order conditions require that, in the neighborhood of the optimum, the right hand side of equation (16) be decreasing in  $k_t$ . We strengthen this by assuming that this holds globally:

**Assumption 2 (Convexity).** *Let  $H(k) \equiv \frac{f'(k_t) - r - d}{W'(k_t)/\theta}$ . The function  $H(k)$  is strictly decreasing in  $k$  for all  $k \in [\underline{k}, k^*]$ .*

With this assumption in hand, we can explore the dynamics of  $k$  by studying  $\lambda$ , as  $k$  is now monotonically (and inversely) related to  $\lambda$ . Assumption 2 also ensures that the

constraint set in problem (P) is convex, so conditions (15) and (16) are necessary and sufficient for optimality.<sup>18</sup> This assumption will always be satisfied for concave utility in the neighborhood of  $k^*$ . In the linear utility case, which we discuss in detail in the next section, this assumption holds under mild assumptions.<sup>19</sup>

## 4 Dynamics

This section derives the dynamics of capital and debt along the transition path to the steady state. The case of linear utility ( $u(c) = c$ ) provides simple closed form dynamics for the equilibrium, allowing us to analytically derive comparative statics with respect to political frictions. The results of the linear case are also relevant for more general utility functions. We show that the linear dynamics provide an upper bound on the speed of convergence in the neighborhood of the steady state for concave utility. In the next section, we explore the nonlinear case quantitatively, confirming that this upper bound is relevant quantitatively. Therefore, linear utility provides a convenient and relevant benchmark for growth dynamics with political frictions, and, as indicated throughout the analysis, many of the insights of the linear case carry over to the nonlinear model. When considering linear utility, we will ignore the non-negativity constraint on consumption (or else, the reader can assume that the analysis is in the neighborhood of the steady state of the economy, which will turn out to feature positive consumption levels).

In the case of linear utility, the first order condition for consumption becomes:

$$1 = \frac{\beta^t R^t}{\mu_0} + \sum_{s=0}^t \beta^s R^s \frac{\lambda_{t-s}}{\theta} + \sum_{s=0}^t \beta^s R^s \delta^s (\theta - 1) \frac{\lambda_{t-s}}{\theta}, \forall t \geq 0 \quad (17)$$

The initial period  $\lambda_0$  is therefore  $\lambda_0 = 1 - \mu_0^{-1}$ . As  $\mu_0$  is the multiplier on  $b_0$ , more debt in period 0 is associated (weakly) with a larger  $\mu_0$  and a larger  $\lambda_0$ . As can be seen, the multiplier on the resource constraint cannot be smaller than 1, which implies from the associated envelope condition that  $V'(b_0) = -\mu_0 \leq -1$ . This is intuitive as  $-1$  is the efficient rate of resource transfers between the small open economy and the foreigners in the

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<sup>18</sup>To see that Assumption 2 implies convexity of the planning problem, make the following change of variables in problem P: let  $h_t = \underline{W}(k_t)$  be our choice variable instead of capital, and define  $K(\underline{W}(k)) = k$  to be the inverse of  $\underline{W}(k)$ . Similarly, we make utility itself the choice variable and let  $c(u)$  denote the inverse utility function, that is, the consumption required to deliver the specified utility. In this way, the objective function and constraint (13) are linear in the choice variables. The budget constraint is convex if  $f(K(h)) - (r + d)K(h)$  is concave in  $h$ , which is the same requirement as Assumption 2.

<sup>19</sup>For linear utility, sufficient conditions are that  $\bar{\tau} = 1$ , or that the curvature of the production function,  $-\frac{f''(k)k}{f'(k)}$ , be non decreasing in  $k$ . The latter is satisfied for the usual Cobb-Douglas production function.

absence of a binding participation constraint (in an interior solution). The binding participation constraints distort this rate, making it increasingly costly to transfer resources to the foreigners as  $b_0$  increases.

Equation (17) pins down the dynamics of  $\lambda_t$ , the multiplier on the government's participation constraint. Recall that the dynamics of  $k_t$  can be recovered from  $\lambda_t$  through the function  $H$  (defined in Assumption 2). We now characterize the dynamics of  $\lambda_t$ :

**Proposition 2** (Linear Dynamics). *The multiplier  $\lambda_t$  that solves equation (17) satisfies the following difference equation:*

$$\lambda_{t+1} = (1 - \beta R)(1 - \beta R\delta) + \beta R \left(1 - \frac{1 - \delta}{\theta}\right) \lambda_t \quad \forall t \geq 1 \quad (18)$$

with  $\lambda_0 = 1 - \mu_0^{-1} \geq 0$  and  $\lambda_1 = 1 - \beta R + \beta R(1 - \delta) \left(\frac{\theta - 1}{\theta}\right) \lambda_0$ . The sequence of  $\lambda_t$  converges monotonically towards its steady state value  $\lambda_\infty$ :

$$\lambda_\infty = \frac{\theta(1 - \beta R)(1 - \beta R\delta)}{\theta(1 - \beta R) + (1 - \delta)\beta R}.$$

From the fact that  $\lambda_0 = 1 - \mu_0^{-1}$ , whether the government's participation constraint binds in the initial period depends on  $\mu_0$ , which is the multiplier on initial debt. If the economy starts off with low enough debt (or high enough assets), it can support  $k^*$  in the initial period. If  $\beta R = 1$ , from equation (18), it will stay at the first best thereafter. However, if initial debt is such that the first best is not sustainable immediately, then the economy will have non-trivial dynamics.<sup>20</sup> Similarly, if  $\beta R < 1$ , then equation (18) implies that  $\lambda_t > 0$  for  $t > 0$  regardless of initial debt, as consumption is front loaded due to impatience. In short, other than the case of patient agents starting off at the first best, the economy will experience non-trivial dynamics as it converges to the steady state. For the remainder of the analysis, we assume this is the case.

Figure 3 shows the transition mapping of  $\lambda_t$ . The diagram describes a situation where  $\beta R < 1$ .<sup>21</sup> Note that for  $\theta > 1$  the speed of convergence in the neighborhood of the steady state is finite, and governed by  $\beta R(1 - (1 - \delta)/\theta)$ . The greater the incumbency effect (greater  $\theta$  or  $\delta$ ), then the slower the convergence to the steady state. Note as well that when there is no political disagreement due to incumbency ( $\theta = 1$ ) or political turnover ( $\delta = 1$ ), then  $\lambda_1 = \lambda_\infty$  and the economy converges in one period.

The reason political frictions slow convergence is that each incumbent views the future differently. To see how this works, first note that there is an important distinction between

<sup>20</sup>We derive this threshold debt level explicitly in the next subsection.

<sup>21</sup>From (18), the transition for the case of  $\beta R = 1$  is a ray from the origin with slope  $1 - (1 - \delta)/\theta$ .

a low (geometric) discount factor and disagreement over the timing of spending. With geometric discounting ( $\theta = 1$ ), all incumbents agree on inter-temporal tradeoffs – a unit of consumption in period  $t + 1$  is worth  $\beta$  in period  $t$ , regardless of the current period. Reducing consumption in period  $t$  and increasing consumption by  $1/\beta$  in  $t + 1$  leaves all incumbents prior to  $t + 1$  indifferent and raises the utility of period  $t + 1$ 's government. This implies that moving consumption from  $t$  to  $t + 1$  leaves capital unchanged prior to  $t + 1$ , and sustains more capital in  $t + 1$ . If  $\beta R = 1$  and utility is linear, then it is optimal to delay consumption until capital is at the first best level in all periods after the initial period. In this case, all incumbents agree to postpone consumption to sustain first best capital after one period. If  $\beta R < 1$ , there are still no dynamics: It is optimal to bring (some) consumption forward to the initial period, and then maintain a constant level (below the first best) of capital after that.<sup>22</sup>

However, when  $\theta > 1$ , incumbents disagree about inter-temporal tradeoffs. Take the case of  $\delta = 0$ , so that each incumbent has quasi-hyperbolic preferences, discounting between today and tomorrow at  $\beta/\theta$  and then discounting at  $\beta$  thereafter. Set  $\beta R = 1$  as well, so the costs and benefits of inter-temporal tradeoffs are equal for private agents. The incumbent in period 0 discounts between  $t > 0$  and  $t + 1$  at  $\beta$ , and so is willing to delay spending from  $t$  to  $t + 1$  at the rate  $1/\beta$  in order to raise capital in  $t + 1$ . All incumbents prior to  $t$  are also willing to make this trade. However, the incumbent in period  $t$  is strictly worse off, as it discounts between  $t$  and  $t + 1$  at  $\beta/\theta < \beta$ . This implies that postponing consumption does not weakly increase capital in all periods, as it does when  $\theta = 1$ . Therefore, back loading incentives cannot sustain the first best capital immediately, despite the linear utility.

This explains why capital is not first best after the initial period, but not why it is increasing at a speed governed by  $\theta$ . To shed light on this question, consider the same perturbation: Reduce consumption in period  $t$  by one unit, and raise consumption in period  $t + 1$  by  $1/\beta$  units. It is costless to do this as  $1/\beta = R$ . All incumbents prior to  $t$  are indifferent, so  $k_s$  is unchanged for  $s < t$ . Utility of the incumbent in  $t + 1$  increases by  $\theta$ , and so from the participation constraint we can increase discounted period  $t + 1$

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<sup>22</sup>The linear case in standard models of expropriation has been studied in detail by [Thomas and Worrall \(1994\)](#) and [Albuquerque and Hopenhayn \(2004\)](#) for  $\beta R = 1$ . In those papers, non-trivial transition dynamics are generated because of the binding requirement that consumption must be positive. The results here make clear that the speed of convergence around the steady state in these models is infinity (independently of whether  $\beta R$  is equal to or less than one), and also that these linear models will immediately converge if they start with sufficiently low debt. It is possible to generate smoother dynamics in the above models by introducing risk aversion. However in Section 5 we argue that numerically, for a neoclassical technology and standard parameter values, the speed of convergence is determined primarily by  $\theta$  and  $(1 - \delta)$  even in the presence of risk aversion.

income by  $1 - (r + d)/f'(k_t)$ .<sup>23</sup> However, utility of the incumbent in  $t$  falls by  $-\theta + 1$ . The  $-\theta$  is the drop in period  $t$  consumption, and the plus 1 is the discounted value of the increased consumption in period  $t + 1$ . If  $\theta = 1$ , there is no change in utility, as discussed above. However, if  $\theta > 1$ , the next period's consumption is more heavily discounted, and the period  $t$  incumbent is worse off. From the participation constraint, we have that net income falls by  $(1 - 1/\theta)(1 - (r + d)/f'(k_t))$ .<sup>24</sup> Optimality requires that there is zero gain or loss from this perturbation, or that  $(1 - 1/\theta)(1 - (r + d)/f'(k_t)) = 1 - (r + d)/f'(k_{t+1})$ .<sup>25</sup> At equal capital levels, the fact that  $1 - 1/\theta < 1$  implies it pays to postpone spending at the margin for  $k_{t+1} < k^*$ . However, as  $k_t$  decreases and  $k_{t+1}$  increases, diminishing returns set in, and eventually the net gain of moving consumption is zero. For large  $\theta$ , it is very costly to have  $k_{t+1}$  much larger than  $k_t$ , and so growth is slow. Put another way, the greater is  $\theta$ , the more costly it is to move consumption away from incumbent  $t$ , and therefore the more costly it is to save or pay down debt quickly.

The same intuition goes through for  $\delta \neq 0$ , but now incumbents prior to  $t$  are no longer indifferent to trades between  $t$  and  $t + 1$  at the rate  $1/\beta$ . A  $\delta > 0$  (but less than one) further slows convergence, as the incumbent in  $t - 1$  prefers a unit of consumption in  $t$  to  $1/\beta$  units in  $t + 1$ .<sup>26</sup> The same goes for incumbents prior to  $t - 1$ . This raises the cost of delayed spending, and slows convergence. Keep in mind that a high  $\delta$  does not necessarily mean more turnover. Rather, it represents a greater incumbency advantage (see equation 7), which drives a wedge between the incumbent and non-incumbent's discount factors.

The above exercise points to the difference between  $\theta$  and the discount factor  $\beta$ . A value of  $\theta > 1$ , makes parties short term impatient, and creates continuous disagreement about the timing of expenditures, making the optimal allocation dynamic. A value of  $\beta < 1/R$ , also makes the incumbents more impatient, but as long as  $\theta = 1$ , this impatience does not create disagreement, and no dynamics are generated. Moreover, we see from Proposition 2 that impatience (a low  $\beta$ ) speeds convergence to the steady state. If agents are impatient, there is less disagreement across incumbents about spending in the future, as  $\beta$  increases in importance relative to  $\theta$ .

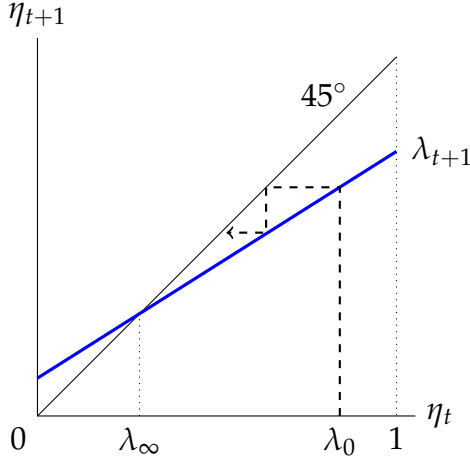
Perhaps the dichotomy between impatience and incumbency is starkest when the economy is shrinking. This will be the case if the economy starts with low enough debt and  $\beta R < 1$  (that is, the economy starts to the left of  $\lambda_\infty$  in Figure 3). A low  $\beta$  economy

<sup>23</sup>Here, we are assuming  $\bar{\tau} = 1$ , so the participation constraint implies  $\Delta c_{t+1} = \Delta f(k_{t+1}) \approx f'(k_{t+1})\Delta k_{t+1}$ . Setting  $\Delta c_{t+1} = 1/\beta$ , discounted income net of the rental rate is  $\beta(f'(k_{t+1}) - (r + d))\Delta k_{t+1} = 1 - (r + d)/f'(k_{t+1})$ .

<sup>24</sup>This follows from  $\theta\Delta c_t + \beta\Delta c_{t+1} = -\theta + 1 = \theta\Delta f(k_t)$ .

<sup>25</sup>This expression is of course equivalent to 18, as can be seen from the fact that  $\lambda_t = 1 - (r + d)/f'(k_t)$ .

<sup>26</sup>From (9), the  $t - 1$  incumbent's discount factor between  $t$  and  $t + 1$  is  $\beta(\theta\delta^2 + 1 - \delta^2)/(\theta\delta + 1 - \delta)$ , which is less than  $\beta$  for  $\delta \in (0, 1)$ .



**Figure 3:** Transition mapping for  $\eta_t$  when  $\beta R < 1$ . The blue line in the diagram represents the transition mapping as given by equation (18). The dashed line represents a possible equilibrium path for initial condition  $\lambda_0$ .

(holding  $\theta$  constant) will collapse relatively quickly to a low steady state. Conversely, a high  $\theta$  economy (holding  $\beta < 1/(1+r)$  constant) will experience a relatively long, slow decline.

The case of a shrinking economy also highlights the distinction between our model and one with a simple borrowing constraint. Borrowing constraints do not induce dynamics if capital starts above its steady state level (see Barro et al., 1995). However, our model has non-trivial dynamics whether the economy is growing or shrinking.

A direct implication of the convergence of  $\lambda_t$  in Proposition 2 is that the sequence of  $k_t$  converges to a steady state as well. Define  $\bar{\theta}$  to be such that  $\bar{\theta}(1 - \beta R)(1 - \beta R\delta)/(\bar{\theta}(1 - \beta R) + (1 - \delta)\beta R) = (f'(k) - (r + d))/\bar{c}'(k)$ . Then,

**Corollary 1 (Monotone Convergence).** *The sequence of capital,  $k_t$ , converges monotonically to its steady state level of capital,  $k_\infty$ . The value of  $k_\infty$  solves*

$$\frac{f'(k_\infty) - (r + d)}{\bar{c}'(k_\infty)} = \frac{\theta(1 - \beta R)(1 - \beta R\delta)}{\theta(1 - \beta R) + (1 - \delta)\beta R} \quad (19)$$

as long as  $\theta \leq \bar{\theta}$ , and equals  $\underline{k}$  otherwise.<sup>27,28</sup> If country A starts with a higher sovereign debt level than country B, then all else equal, the path of capital for country A will be (weakly) lower than that for country B.

Note that the value  $k_\infty$  is decreasing in  $\theta$  as long as  $\beta R < 1$  and  $\theta \leq \bar{\theta}$ . That is, a greater

<sup>27</sup>Recall that we have assumed that  $k_t > \underline{k}$  along the equilibrium path. That is, the constraint  $\tau_t \leq \bar{\tau}$  does not bind. If  $\frac{\theta(1-\beta R)(1-\beta R\delta)}{\theta(1-\beta R)+(1-\delta)\beta R} > \frac{f'(k)-(r+d)}{\bar{c}'(k)}$ , or  $\bar{\theta} < \theta$ , then the constraint will for sure bind as  $t \rightarrow \infty$ . In this case,  $k_\infty$  achieves the lower bound of  $\underline{k}$  and further increases in  $\theta$  do not affect  $k_\infty$ . See the appendix for a complete treatment.

<sup>28</sup>Note that  $\bar{\theta}$  is infinity when  $\bar{\tau} = 1$ .

incumbency effect when coupled with impatience leads to lower steady state levels of investment. All else equal, an increase in the incumbency advantage  $\delta$  also lowers  $k_\infty$ . In particular, for a given re-election probability  $p$ , the greater the number of competing opposition parties  $N$ , the lower  $k_\infty$ .<sup>29</sup>

To obtain a sense of how these results map into income growth rates, consider the case of  $y = k^\alpha$ ,  $\beta R = 1$ , and  $\bar{\tau} = 1$ . When  $\bar{\tau} = 1$ , the multiplier  $\lambda_t$  is the tax wedge  $\tau_t$ , as  $W'(k_t)/\theta = f'(k_t)$  and so 16 implies  $(1 - \lambda_t)f'(k_t) = r + d$ . Using  $f'(k_\infty) = r + d$  when  $\beta R = 1$  and  $f'(k) = \alpha y^{(\alpha-1)/\alpha}$  for Cobb-Douglas production, we have that  $\left(\frac{y_t}{y_\infty}\right)^{\frac{1-\alpha}{\alpha}} = 1 - \lambda_t$ . Using the fact that  $\ln(1 - \lambda_t) \approx -\lambda_t$  close to the first best steady state, it follows that  $\ln\left(\frac{y_t}{y_\infty}\right) \approx -\left(\frac{\alpha}{1-\alpha}\right)\lambda_t$  and with the law of motion for  $\lambda_t$ , this implies

$$\ln\left(\frac{y_{t+1}}{y_t}\right) \approx -\left(\frac{1-\delta}{\theta}\right)\ln\left(\frac{y_t}{y_\infty}\right). \quad (20)$$

This expression relates the growth rate between  $t$  and  $t + 1$  to the distance from the steady state. The usefulness of equation (20) is that it relates our political economy friction parameters to the rate of convergence. For perspective, the comparable speed of convergence in the standard Solow-Swan model is  $(1 - \alpha)d$ .<sup>30</sup> A comparison of this term with that in equation (20) highlights that we have replaced capital share with political economy frictions in the speed of convergence. The slow rate of convergence observed empirically, when viewed through the standard model, suggests a large capital share, on the order of 0.75 when using plausible values for other parameters (see Barro and Sala-I-Martin, 2004 p. 59). This has generated a literature on what is the appropriate notion of capital in the neoclassical model, such as Mankiw et al. (1992) which extends the notion of capital to include human capital. In our framework, slow convergence does not necessarily require a high capital share, but rather indicates large political economy frictions. For the empirical growth literature, this framework suggests an emphasis on political economy frictions in determining the speed of convergence, in addition to their possible effect on the steady state. We return to the comparison between our framework and alternative growth models in section 5.

Before proceeding to the dynamics for debt, we briefly discuss how the insights from the linear utility case studied above apply to the case of concave utility. The expressions for the steady state and the convergence properties of the transition are naturally more

<sup>29</sup>Note that for comparative statics for  $\delta$ , we accommodate the fact that an increase in  $\delta$  will also increase  $\theta$  as defined in (6), holding constant  $p$  and  $\tilde{\theta}$ .

<sup>30</sup>More precisely, the speed of convergence is  $(1 - \alpha)(g + n + d)$ , where  $g$  is the rate of exogenous technological progress and  $n$  is the population growth rate, both of which we have set to zero in our benchmark model. See Barro and Sala-I-Martin (2004).

complicated with concave utility. In the appendix, we discuss in detail the dynamics for concave utility when  $\delta = 0$ , a case for which one can represent the dynamic system in a two-dimensional phase diagram with a monotonic saddle path. We show there that the insights of the linear case carry over in a straightforward way. In particular, countries converge monotonically toward a steady state along a saddle path, with their initial capital determined by their initial debt positions (the concave counterpart of Corollary 1). Moreover, for general  $\delta < 1$ , the speed of convergence along the saddle path (in the neighborhood of the steady state) is bounded above by the speed derived in Proposition 2 (that is,  $\beta R \left(1 - \frac{1-\delta}{\theta}\right)$ ). Interestingly, as is the case for linear utility, this bound on the speed of convergence holds regardless of the functional form<sup>31</sup> of  $\underline{W}(k)$  – growth is always capped by the extent of political disagreement. We collect the key results for the case of concave utility in the following proposition:

**Proposition 3** (Concave Utility). *Let  $u(c)$  be such that  $u'(c) > 0$ ,  $u''(c) < 0$ , and the Inada conditions,  $\lim_{c \rightarrow 0} u'(c) = \infty$  and  $\lim_{c \rightarrow \infty} u'(c) = 0$ , hold. Then*

- (i) *When  $\beta R < 1$ , there is a unique steady state in which  $k_\infty < k^*$ . When  $\beta R = 1$ , there is a continuum of possible steady states, all of which have  $k = k^*$ .*
- (ii) *Suppose  $\delta = 0$ . Capital converges monotonically along a saddle path toward the steady state, with initial capital weakly decreasing in initial debt.*
- (iii) *In the neighborhood of the steady state, the speed of convergence along the saddle path is bounded above by  $\beta R \left(1 - \frac{1-\delta}{\theta}\right)$  for  $\theta > 1$  and  $\delta < 1$ .*

## 4.1 Debt Dynamics

Now that we have solved for the dynamics of  $\lambda$ ,  $k$ , and  $y$ , we turn to the dynamics of debt. We show that the path of capital is accompanied by opposite movements in the stock of debt. This relationship holds for general utility along a monotonic transition path for capital.

The sequence of binding participation constraints,  $W_t = \underline{W}(k_t)$  map the dynamics of capital into that of incumbent utility, given that  $\underline{W}(k)$  is strictly increasing in  $k$ . Therefore,  $W_t$  tracks the path of capital. We now show that the information contained in the infinite sequence of incumbent utility values is sufficient to recover the utility of the population at any time:

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<sup>31</sup>With the caveat that Assumption 2 still holds.



**Lemma 2.** *The utility of the population as of time  $t$ ,  $V_t = \sum_{s=0}^{\infty} \beta^s u(c_{t+s})$ , is given by:*

$$V_t = \theta^{-1} \left( W_t + \beta(1 - \delta) \left( 1 - \frac{1}{\theta} \right) \sum_{s=0}^{\infty} \beta^s \left( 1 - \frac{1 - \delta}{\theta} \right)^s W_{t+1+s} \right).$$

Given that the values  $k_t$  are monotonic and that  $W_t = \underline{W}(k)$  is an increasing function of  $k$ , it follows that:

**Proposition 4.** *If  $k_t$  is monotonically increasing (decreasing) along the transition to the steady state, then the utility of the population,  $V_t$ , is also monotonically increasing (decreasing).*

We have now shown that the discounted utility of the population and the sequence of incumbent utility move monotonically in the same direction towards their respective steady states. Given that  $W_t$  and  $V_t$  increase monotonically, it follows that outstanding sovereign debt decreases monotonically:

**Corollary 2.** *The stock of the economy's outstanding sovereign debt decreases (increases) monotonically to its steady state value if the sequence of  $k_t$  is increasing (decreasing).*

The above corollary closes the loop between growth and debt and brings us back to our original motivation. It states, quite generally, that capital accumulation will be accompanied with a reduction in the external debt of the government. Similarly, a country that shrinks, does so while their government accumulates sovereign liabilities. In section 5 we will explore this link between debt, capital, and growth quantitatively.

Steady state consumption and debt can be recovered from the fact that  $W_{\infty} = \underline{W}(k_{\infty})$ , where

$$W_{\infty} = \left( \frac{\theta - 1}{1 - \beta\delta} + \frac{1}{1 - \beta} \right) u(c_{\infty}), \quad (21)$$

and

$$\underline{W}(k_{\infty}) = \theta u(\bar{c}(k_{\infty})) + \beta \left( \frac{\delta(\theta - 1)}{1 - \beta\delta} + \frac{1}{1 - \beta} \right) u(\bar{c}(k)). \quad (22)$$

Equating the two defines steady state consumption as a function of steady state capital. Note that  $\underline{W}(k_{\infty}) > 0$  implies that  $c_{\infty} > 0$  when utility is linear, confirms our underlying assumption that in the neighborhood of the steady state consumption is positive, and thus, the non-negativity constraint can be ignored.

The steady state level of debt then follows from the fact that debt equals the present

discounted value of net payments to the foreign financial markets:

$$B_\infty = \left( \frac{1+r}{r} \right) (f(k_\infty) - (r+d)k_\infty - c_\infty). \quad (23)$$

Recall that for the case of  $\beta R = 1$ , we have assumed we start with enough debt that  $k_0 < k_\infty = k^*$  to generate interesting dynamics. This is equivalent to stating that  $b_0 > B_\infty$ , with  $B_\infty$  and  $c_\infty$  evaluated at  $k_\infty = k^*$ . From the above expressions, we can see how this level of debt depends on the political parameters. In particular, a higher value of  $\theta$  requires a greater steady state level of consumption to avoid expropriation at  $k = k^*$ .<sup>32</sup> From equation (23), this implies a lower level of debt. The same holds for more opposition parties or a lower re-election probability. Therefore, for  $\beta R = 1$ , the debt threshold at which  $k_0 < k^*$  is lower for economies with greater political economy frictions. The prediction that economies with greater political distortions sustain less debt in the steady state is consistent with the “debt intolerance” regularity found in the data. In particular, [Reinhart et al. \(2003\)](#) document that many advanced economies exhibit very high debt to income ratios without apparent difficulty, while developing economies have debt crises at much lower debt levels.

When  $\beta R < 1$ , we always have dynamics regardless of initial debt. Moreover, it also not generally the case that steady state debt is decreasing in political economy frictions. When  $\beta R = 1$ ,  $k_\infty = k^*$  for all  $\theta < \infty$  and  $\delta < 1$ . However, when  $\beta R < 1$ , a greater degree of political economy frictions, the lower the steady state capital stock, as shown in (19). Therefore, on the one hand, a higher  $\theta$  requires higher  $c_\infty$  and lower  $B_\infty$  for a given level of capital; while on the other hand, a higher  $\theta$  lowers the steady state capital stock when  $\beta R < 1$ . By varying the parameters, we can make either force dominant. Therefore, the response of steady state debt to the political economy parameters when  $\beta R < 1$  is ambiguous.

## 4.2 Aid versus Debt Forgiveness

Before concluding this section, we will use the model to discuss the role of two policies: foreign aid and debt forgiveness. Our set up delivers a laboratory that allows us to ask whether the introduction of foreign aid and debt relief changes the path of investment and growth. Although, similar in principle (they both represent a transfer from foreigners to the domestic agents), these two policies will end up having different effects on the behavior of the economy.

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<sup>32</sup>This follows from setting (21) equal to (22) and computing  $\partial c_\infty / \partial \theta$ .

From our previous analysis, we see that debt forgiveness, as given by a reduction in  $b_0$ , will affect the economy in the short run, but will not affect the steady state levels of investment and debt. That is, if  $b_0$  is reduced, then from Corollary 1, we know that the resulting path of capital will be higher, but the long run level of debt, capital, and output will not change as the economy converges to the same steady state. Debt forgiveness speeds convergence to the steady state, so has transitional growth effects but no long run effects.<sup>33</sup>

To be precise, we continue to select the equilibrium allocation that maximizes private agent utility, subsequent to debt forgiveness. That is, the problem remains (P), but evaluated at a lower  $b_0$ . More importantly, if the incumbent simply re-borrowed the forgiven debt it would be viewed as a deviation and trigger the punishment equilibrium. As debt is the only state variable in the problem, debt forgiveness simply means “jumping ahead” along the constrained efficient transition path.<sup>34</sup>

Another common policy aimed at helping developing countries is foreign aid. Among the different emerging market economies, several have received significant amounts of aid from abroad. In the data, however, the relationship between aid and growth seems, if anything, insignificant.<sup>35</sup>

We can easily evaluate unconditional aid in our framework. By unconditional, we mean that aid is not contingent on repaying debt or honoring tax promises. Unconditional aid and debt forgiveness both relax the budget constraint, but unconditional aid does not relax the incumbent government’s participation constraint. To see why this matters, consider an aid sequence announced in period  $t$  with present discounted value  $Y = \sum_{s=0}^{\infty} R^{-s} y_{t+s}$ . The aid sequence is deterministic and non-contingent on fiscal policy. The present value constraint on the resources of the government is:

$$b_0 - Y \leq \sum_{t=0}^{\infty} R^{-t} (f(k_t) - (r + d)k_t - c_t),$$

which is identical to the case of debt forgiveness by the amount  $Y$ . However, while debt forgiveness does not affect the deviation value after a default, the autarky value with aid

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<sup>33</sup>This is consistent with the empirical effect of debt relief on domestic stock market values, investment, and short run GDP growth rates. For a recent survey of the literature see [Arslanalp and Henry \(2006\)](#)

<sup>34</sup>The transitory effect of debt relief and the temptation to re-borrow the forgiven debt is relevant to recent experience in Africa. As Western donor countries consider debt forgiveness, the debtor African countries are simultaneously seeking new loans from China. See, for example, the Financial Times article “Donors press Congo over \$9bn China deal” on February 9 of 2009.

<sup>35</sup>See the original article by [Burnside and Dollar \(2000\)](#), [Easterly \(2003\)](#) for a survey, and [Rajan and Subramanian \(2008\)](#) for a more recent analysis.

is now given by the following:

$$\underline{W}(k_t) = \theta u(\bar{c}(k_t) + y_t) + \sum_{s=1}^{\infty} \beta^s u(\bar{c}(k) + y_{t+s})$$

for all  $t$ . That is, unconditional aid raises welfare in the event of a default as well as along the equilibrium path. The following proposition states that unconditional aid is dominated by debt forgiveness, and, in the case of linear utility, does not influence the path of capital (or growth):

**Proposition 5.** *Consider two alternative aid programs: (i) debt forgiveness of an amount  $Y$  versus (ii) an arbitrary stream of unconditional aid transfers with present value  $Y$ . Then*

- (a) *The recipient country agents always weakly prefer debt forgiveness. It strictly prefers debt forgiveness if either  $\beta R < 1$  or  $k < k^*$  after the aid (i.e., the country remains constrained after the aid is given).*
- (b) *If  $u(c) = c$ , then unconditional aid is immediately consumed and has no effect on growth. Specifically, let  $\{k_{t+s}\}_{s=0}^{\infty}$  be the optimal sequence of capital that solves the population's problem as of time  $t$  without the presence of aid. Then  $\{k_{t+s}\}_{s=0}^{\infty}$  is also an optimal sequence of capital of the economy with an aid sequence  $\{y_{t+s}\}_{t=0}^{\infty}$ . Moreover, if  $\{c_{t+s}\}_{s=0}^{\infty}$  is the pre-aid consumption sequence, then  $\tilde{c}_{t+s} = c_{t+s} + y_{t+s}$  is the post-aid consumption sequence.*

Aid without conditionality improves the utility of the population, as it represents a transfer that will be consumed by them, but is dominated by debt forgiveness. Debt forgiveness has the same effect as aid on the budget constraint, but in addition relaxes the participation constraint, as debt forgiveness is only valuable in the absence of default. In this manner, debt forgiveness is *conditional* aid, bringing with it the requirement that the benefits only accrue if the incumbents respect the optimal fiscal policy. It therefore assists the citizenry to constrain the government and sustain more investment, as well representing a transfer.<sup>36</sup> Note that the proposition holds regardless of how the aid is distributed over time. In particular, the timing of unconditional aid can be chosen to maximize its impact on welfare and it will still be dominated by debt forgiveness.

## 5 Quantitative Analysis

In this section, we explore the quantitative implications of our framework. We focus on the joint dynamics of income, consumption, and debt, motivated by the empirical facts

<sup>36</sup>See Scholl (2009) for a study of aid and conditionality in an environment with limited commitment.

discussed in the introduction. This section has two aims. First, we quantitatively relate our model to the large empirical growth literature. Second, we compare our framework to other growth models.

## 5.1 Calibration

Given that our framework builds on the neoclassical growth model, most of the parameters have accepted values. We assume  $y = Ak^\alpha$ , with  $A$  normalized so that the first best income ( $y^* = Ak^{*\alpha}$ ) is 1. We set the capital share parameter  $\alpha$  to one third. We set the flow utility function to be  $u(c) = \ln(c)$ .<sup>37</sup> Given that a period in our model corresponds to a term of incumbency, we set the period length to five years. The five-year interest rate is 20 percent, as is the capital depreciation rate. We set  $\beta = 1/R$ , so that agents discount at the world interest rate.

The novel parameters in our model govern the extent of political disagreement and turnover. We can use the model and insights from the empirical growth literature to obtain a plausible range for political disagreement. Empirical estimates of the speed of convergence vary depending on the identification strategy. Cross-sectional growth regressions suggest an annualized continuous time convergence rate of 0.02, or a five year rate of 0.10 (see [Barro and Sala-I-Martin, 2004](#) for a review). At the other extreme, fixed effect estimation using panel data suggest convergence rates as high as 0.10, or a five year rate of 0.39 (see, for example, [Caselli et al., 1996](#)). We will focus on the case where  $\delta = 0$  and where  $\theta = 3, 5$  and  $7$ .<sup>38</sup> This corresponds to convergence rates in our log utility model of 0.27, 0.16 and 0.11 respectively. In our comparative statics, we will treat  $\theta = 3$  as our benchmark.

The final parameter is the maximal tax rate  $\bar{\tau}$ , which governs the degree of expropriation after deviation. The tougher the punishment, the more debt can be sustained in equilibrium. We can use empirical debt to income ratios to calibrate this parameter. We first need to take a stand on what is the empirical counterpart of the model's external government debt. In the model, public debt consists of the net liabilities of the government's

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<sup>37</sup>Although this utility function is a standard one, it has the property of being unbounded below. We can alternatively define the utility function to be  $u(c) = \ln(c + c_0) - \ln(c_0)$  where  $c_0$  is a sufficiently small positive number such that its presence has no effect in the numerical computations but satisfies our requirement that utility be bounded below.

<sup>38</sup>The computational advantage of setting  $\delta = 0$  is that the continuation value of each incumbent is the private agent's value function, removing the need to carry the government's continuation payoff as a separate state variable. We know from equation (18) that, for the linear utility case, only the ratio  $(1 - \delta)/\theta$  matters for the speed of convergence, so we can set  $\delta = 0$  and adjust  $\theta$  accordingly. As shown in the online appendix, the (near) sufficiency of  $(1 - \delta)/\theta$  for convergence speeds can be confirmed by numerical analysis of the linearized system introduced in Proposition 3.

favored constituency (workers) owed to political outsiders (capital owners and foreigners). Foreign debt is therefore conceptually consistent between model and data, representing net claims between the government and political outsiders. Domestic public debt in the data is more problematic, as there are well known issues about treating domestic bonds as net wealth of domestic residents.<sup>39</sup> To the extent domestic public debt in the data includes debt workers as a group owe themselves, it is distinct from the model's measure of debt. Similarly, domestic capitalists' holdings of government bonds that are balanced by their non-capital-income tax liabilities also do not represent net claims against political insiders. This leaves the portion of domestic debt that is held by outsiders but backed by insider tax payments. As there is no good empirical measure of this subset of domestic debt, we assume that government bonds do not represent net wealth for capitalists and equate external government debt in the model with measured foreign public debt minus international reserves. In the empirical sample used in Figure 1, the median debt to GDP ratio is 25 percent with an average per capita growth rate of one percent. We therefore set  $\bar{\tau} = .6$ , which yields an external debt to income ratio of roughly 26 percent when our benchmark economy ( $\theta = 3$ ) is growing at 1 percent per year, and a steady state debt to income ratio of 9 percent.

## 5.2 Growth, Convergence, and Debt

We present our quantitative results in figures 4 through 6. Figure 4 plots the annualized growth rate against the log ratio of current income to steady state income. We do this for several values of  $\theta$ , including  $\theta = 1$  which represents no political economy frictions. Figure 4 reflects that a greater incumbency effect flattens the relationship of growth and income, slowing an economy's transition path. This is consistent with the discussion surrounding equation (20) for linear economies.

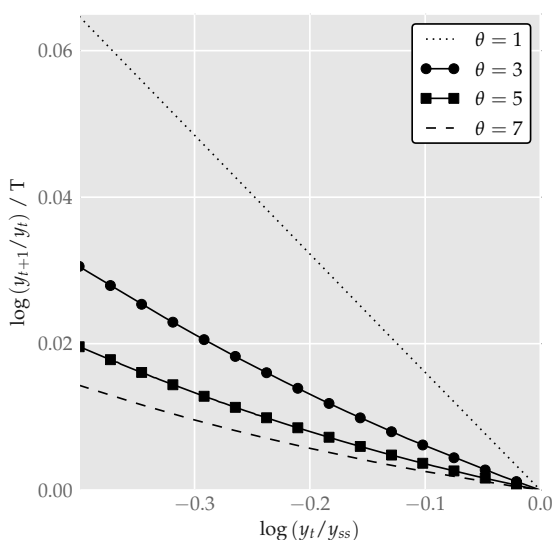
Figure 4 also suggests the absence of strong non-linearities in the rate of convergence. In fact, the slope of the lines are numerically close to  $-1/\theta$ , which is the analytical result for the linear case. Specifically, the slope evaluated at the steady state is  $-0.27$  and  $-0.11$  for  $\theta = 3$  and  $\theta = 7$ , respectively, while the linear model predicts slopes of  $-0.33$  and  $-0.14$ , respectively. Therefore, the dynamics derived analytically for the linear utility case are quantitatively similar to our calibrated non-linear utility case.

Moreover, Figure 4 suggests that as we look at a cross section of countries, economies at the same stage of development (relative to their individual steady states) will have different growth rates depending on the quality of their political institutions. There is a vast

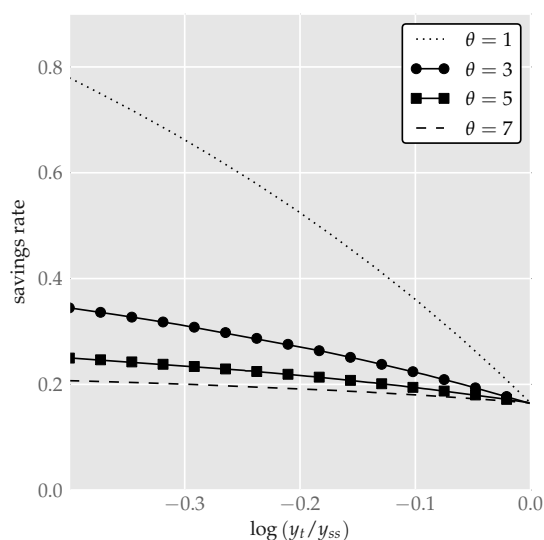
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<sup>39</sup>The classic reference is Barro (1974).

literature studying the effect of political institutions on growth. To relate our quantitative results to this literature, consider the results of [Knack and Keefer \(1995\)](#) (KK), which includes measures of institutional quality in a cross-sectional growth regression. KK use data from the *International Country Risk Guide* (ICRG) on expropriation risk, rule of law, repudiation of contracts by the government, corruption, and quality of the bureaucracy, summing these individual ICRG measures into one index. KK find that a one standard deviation increase in their measure of institutional quality (the difference between Honduras versus Costa Rica, or of Argentina versus Italy) increases annual growth by 1.2 percentage points.



**Figure 4:** Growth and Convergence: This figure plots annualized income growth rates versus distance from steady state for different values of  $\theta$ . The length of a period is 5 years:  $T = 5$ .



**Figure 5:** Saving Rate: This figure plots the ratio of aggregate savings to income along the transition to the steady state for different values of  $\theta$ . Aggregate savings is computed as the change in government net foreign assets plus investment. See text for details.

We can use our model to ask how much  $\theta$  must change to generate a 1.2 percent change in the growth rate. From Figure 4, it is clear that the answer depends on the distance from the steady state. We anchor our comparison at 2 percent growth rate, which is the mean growth rate in the typical Penn World Table sample used in cross-country growth regressions. For an economy with  $\theta = 3$ , a 2 percent growth rate corresponds to  $\ln(y_t/y_\infty) = -0.29$ . In the spirit of the cross-sectional growth regressions, we hold constant the distance from the steady state and ask how much  $\theta$  must increase to reduce growth rates by 1.2 percent. We find that  $\theta$  must increase to just above 7. Going the other

direction, an increase in the growth rate of 1.2 percent is consistent with moving from  $\theta = 3$  to  $\theta = 1.7$ .<sup>40</sup> Considering that KK produce slightly bigger estimates than those reported in [Barro and Sala-I-Martin \(2004\)](#), we can think of the two standard deviation range of  $[1.7, 7]$  as an upper bound on the true dispersion of political frictions.<sup>41</sup>

We now turn to the model's implications for debt dynamics. [Figure 5](#) plots the ratio of saving to income at each point along the transition to the steady state. Recall that the change in government debt represents net savings by political insiders in the model. To get to aggregate savings, we assume capitalists' savings equals domestic investment, an assumption consistent with the fact that private foreign liabilities are relatively small in developing economies. The saving rate is falling along the transition paths for all parameterizations, as both the return to capital is high for low levels of income and the fact that back loading spending is the optimal response to limited commitment. However, political frictions make it difficult to delay spending, which can be seen by the fact that  $s/y$  is lower as we increase  $\theta$ , all else equal. For the stage of development used above ( $\ln(y_t/y_\infty) = -.29$ ), a movement of  $\theta$  from 3 to 7 lowers the saving rate by 6 percentage points. For comparison, the mean and standard deviation of savings rates across the sample from [Figure 1](#) are 18 percent and 7 percent, respectively.

[Figure 6](#) plots the relationship between growth in per capita income and growth in the government's net foreign assets, the model's equivalent to [Figure 1](#). We see that for all parameterizations, there is a strong relationship between growth and the accumulation of net foreign assets. Quantitatively, there are only small differences in the depicted relationship between debt and growth for different parameterizations. Near the steady state, the smaller is  $\theta$ , the stronger is the relationship. However, as we move further away from the steady state, this pattern reverses itself. Recall that for large  $\theta$ , high growth occurs only if capital is very far from the steady state. At such low levels of capital, a small reduction in debt has a large impact on growth. However, each economy depicted in the figure converges to the same steady state. Therefore, near this steady state, we are comparing economies with similar levels of capital but different  $\theta$ . In this region, the high  $\theta$  economy requires a larger reduction in debt to achieve the same rate of growth. For growth rates between 0 and 1 percent, the slope is not substantially different from one for all  $\theta$ . Empirically, the trend line in [Figure 1](#) has a slope of 1.1, close to that implied by our calibrated model.

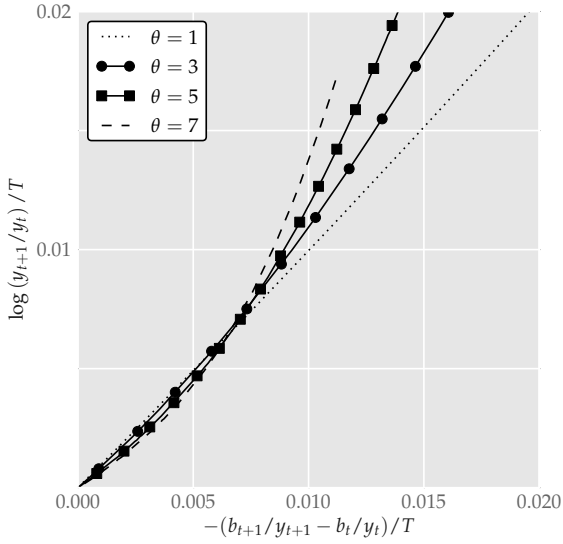
The fact that the various parameterizations predict a quantitatively similar relation-

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<sup>40</sup>If we perform the same calculation, but use  $\theta = 1$  to anchor the distance from the steady state, a 1.2 percentage point decline in the growth rate is consistent with moving to  $\theta = 3$ .

<sup>41</sup>Specifically, Chapter 12.3 of [Barro and Sala-I-Martin \(2004\)](#) reports that a one standard deviation movement in a "rule of law" index is associated with a decline in growth of 0.5 points.





**Figure 6:** Growth in Income and Government Net Foreign Assets: This figure plots growth in per capita income against the change in the ratio of net external assets to income for different values of  $\theta$ . The length of a period is 5 years:  $T = 5$ .

ship between growth and the change in net foreign assets is important for interpreting Figure 1. As each country in Figure 1 potentially has a unique  $\theta$ , fitting a common trend is valid only if they share a common slope. Otherwise, a cross-sectional scatter plot from a single growth period could yield any arbitrary pattern, depending on how the countries were distributed in regard to initial income. The results of this section indicate that all countries must traverse a similar path in the face of limited commitment – an increase in capital must be accompanied by a reduction in debt to avoid complete expropriation. How fast a country makes this transition, however, depends critically on the extent of political frictions, which speaks to the heterogeneous growth experiences for the economies of Figure 1.

### 5.3 Alternative Models

We now compare our framework to alternative growth models. This allows us to identify how limited commitment and the incumbency effect jointly distort growth dynamics relative to familiar benchmark models. The closed economy neoclassical growth model is an important benchmark, both in its own right and for the fact it nests a variety of alternative models. For example, a relevant comparison for our framework is a growth model without capital taxation, but one populated by agents with time inconsistent preferences (and no commitment technology). Barro (1999) explores such a framework, endowing agents with a quasi-hyperbolic discount factor a la Laibson (1997), and shows that a competitive equilibrium of the closed-economy neoclassical model with such consumers is observationally equivalent to the standard growth model in which agents have a lower (geomet-

ric) discount factor.<sup>42</sup> Barro (1999) considers a continuous time model, but the paper’s insight carries over to a discrete time framework. In particular, if private agents discount between this period and next at  $\beta/\theta$ , and between future periods at  $\beta$ , the competitive equilibrium is equivalent to the standard growth model with  $\tilde{\beta} = \beta/(\theta + (1 - \theta)\beta)$ . Similarly, a neoclassical model in which political frictions induce a higher, constant tax rate on capital has the same conditional convergence properties as an undistorted model, where “conditional” refers to controlling for the (distorted) steady state. Such a constant tax policy, for example, is the equilibrium outcome of the closed-form political game studied by Azzimonti (forthcoming).

Two important open economy models also can be viewed through the lens of the standard closed-economy growth model. Barro et al. (1995) (BMS) present an open economy growth model in which physical capital can be financed with foreign debt, but human capital must be self-financed. BMS shares our interest in borrowing constraints, but models them as a constant fraction of income (or a constant fraction of the capital stock). BMS show that such a model is equivalent to the closed economy growth model with a lower capital share.

Similarly, Marcet and Marimon (1992) (MM) present a “Full Information/Limited Enforcement” model which also focuses on limited commitment using an endogenous borrowing constraint, but does not have political economy frictions. Given the limited commitment environment, it shares prominent features with Thomas and Worrall (1994) as well. A deterministic version of the MM model can be obtained in our framework by setting  $\theta = 1$  and letting the deviation equilibrium coincide with the closed economy neoclassical growth model. Comparison with this useful benchmark allows us to highlight the importance of political economy frictions in the presence of limited commitment. It turns out that in the absence of shocks, the MM allocation is equivalent to the closed economy growth model.

We therefore compare our benchmark model to two versions of the closed economy neoclassical growth model. The first version uses the same private agent discount factor  $\beta$  as in our benchmark model (i.e. a five-year discount factor of  $\beta = 1/1.2$ ). This corresponds to the open-economy limited commitment model with no political frictions studied by Marcet and Marimon (1992). The second version lowers the five-year discount factor to  $\beta/(\theta + (1 - \theta)\beta) = 0.63$ . As noted above, this corresponds to a laissez-faire competitive equilibrium in which the private agents have the same preferences as the po-

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<sup>42</sup>More precisely, Barro (1999) considers a competitive equilibrium in which agents have continuous policy functions and log utility. There are other competitive equilibria with discontinuous policy functions, as discussed by Krusell and Smith (2003).

litical incumbents in our benchmark model. In the interests of space, we do not present the neoclassical growth model with a lower capital share (the BMS model), as it is well understood that lowering the capital share speeds convergence (this point is discussed extensively in [Barro and Sala-I-Martin, 2004](#)).

Recall that the punishment equilibrium in our benchmark model is one which allows for capital flight. That is, after deviation, the government cannot borrow or save externally, but private capitalists can invest abroad in response to the high tax rate on capital income. This is a harsher punishment than that of [Marcet and Marimon \(1992\)](#), in which the deviation equilibrium is the closed economy neoclassical growth model. For a better comparison, we consider the corresponding variation to our benchmark framework. Specifically, we assume that after deviation, the economy is in financial autarky, but accumulates physical capital through domestic savings. This alternative punishment equilibrium naturally has quantitative implications (particularly regarding how much debt can be sustained in equilibrium), but as made clear in the analytical results, the key qualitative features of our model do not require explicit modeling of the deviation payoff beyond the concavity condition in Assumption 2. As in section 2.5, we assume the economy resorts to a Markov perfect equilibrium after deviation. However, for  $\theta > 1$ , the disagreement between incumbents sustain many such equilibria, as discussed in detail in a related context by [Krusell and Smith \(2003\)](#). We follow [Barro \(1999\)](#) and [Harris and Laibson \(2001\)](#) and consider the equilibrium in which policies are differentiable functions of the state variable (capital). To compute this equilibrium we use the polynomial approximation algorithm of [Judd \(2004\)](#).<sup>43</sup> For completeness, we also present results for this deviation equilibrium.

Figure 7 reproduces Figure 4 for the alternative models. The figure plots growth in per capita income against log distance from the steady state, indicating each models' predictions for the speed of convergence.<sup>44</sup>

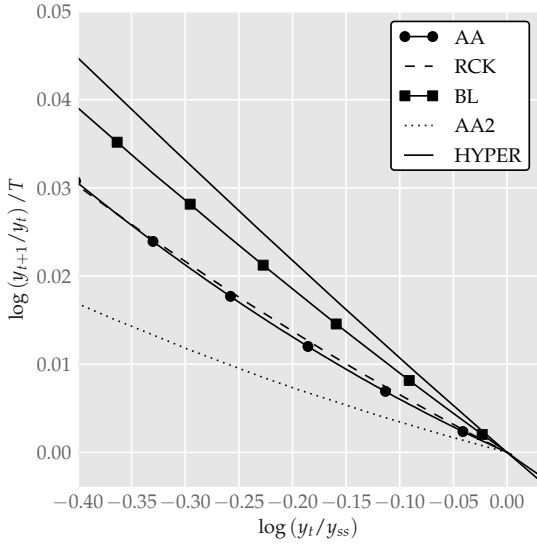
The benchmark "AA" model converges at roughly the same speed as the closed economy growth model ("RCK"), despite the fact that the benchmark model is an open economy. This reflects that the benchmark model's political frictions slow convergence relative to the case of  $\theta = 1$  (see figure 4). Moreover, we have calibrated the benchmark model to converge at a 5-year rate of 0.27 near the steady state, which is similar to that of the neoclassical growth model.

Recall that the neoclassical growth model coincides with the open economy model

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<sup>43</sup>Judd also discusses issues of local and global uniqueness of such "smooth" equilibria.

<sup>44</sup>As all the alternative models, with the exception of "AA2," are closed economy models, we do not discuss debt dynamics. We do not present results for [Barro et al. \(1995\)](#), but the nature of the borrowing constraint in that model implies that debt is always a constant fraction of income. The model of [Cohen and Sachs \(1986\)](#) also predicts a constant debt to income ratio once the borrowing constraint binds.



**Figure 7:** Growth and Convergence for Alternative Models. This figures plot annualized income growth rates versus distance from steady state for alternative models: (i) “AA” refers to the Benchmark calibration of Section 5.2 ( $\theta = 3$ ); (ii) “RCK” (Ramsey-Cass-Koopmans Neoclassical Growth Model) refers to the neoclassical growth model; (iii) “BL” (Barro’s Ramsey Meets Laibson) Competitive equilibrium of RCK with time inconsistent private agents, or RCK with  $\tilde{\beta} = \frac{\beta}{\theta + (1-\theta)\beta} = 0.63$ ; (iv) “HYPER” (differentiable) Markov Perfect Equilibrium of the closed economy growth model with a quasi-hyperbolic government; (v) “AA2” AA model but using HYPER as deviation utility. The length of a period is 5 years:  $T = 5$ .

of [Marcet and Marimon \(1992\)](#), which is a limited commitment model without political frictions. However, as noted above, MM’s deviation equilibrium is different then our benchmark. Model “AA2” is the benchmark model altered so the punishment equilibrium is consistent with MM’s. Figure 7 indicates that AA2 converges much slower than MM’s model (i.e., RCK), indicating that all else equal, political economy frictions slow convergence.

A major focus of the present analysis is that political economy frictions (absent commitment) slow convergence. However, it is important how one models political economy frictions. If the political economy frictions are such that the model collapses to the growth model with higher impatience (lower discount factor), we see from Figure 7 that this *speeds* conditional convergence (line “BL”). It is true that impatience slows growth by lowering the saving/investment rate, but it also lowers the steady state as well. Conditional on this distorted steady state, the economy converges faster. The fact that greater impatience speeds conditional convergence in the neoclassical growth model is discussed by [Barro and Sala-I-Martin \(2004\)](#).

Recall as well that the lower discount factor can be interpreted as the standard growth model populated by quasi-hyperbolic consumers, as in [Barro \(1999\)](#). This raises the question of why our benchmark model behaves so differently than Barro’s version. One possibility is that Barro studies the competitive equilibrium, while our framework emphasizes capital taxation. However, model “HYPER” is the MPE of the extension of Barro’s model in which quasi-hyperbolic governments tax capital. Figure 7 indicates that the convergence properties of this model is if anything slightly faster, as steady state capital is even

more distorted. Rather, the reason for the difference is that our mechanism emphasizes external debt and reputation in the presence of political frictions. The threat of losing access to international financial markets and reverting to a high tax, low income equilibrium supports equilibria with higher capital stocks than that of the closed economy quasi-hyperbolic model.<sup>45</sup> In fact, as noted previously, the economy converges to the first best capital in the long run, despite the high short-term impatience of each government. The savings rate of such an economy is low (as depicted in figure 5), but it will eventually pay down its debt and accumulate a large stock of capital. The combination of low savings but high steady state capital translates into slow rates of convergence.<sup>46</sup>

This feature highlights a benefit of openness in a model of political frictions. Note that in our model, the benefits of financial openness are not the usual faster transition, as in the neoclassical growth model, as limited commitment prevents a large inflow of capital. Rather, openness allows the economy to sustain a higher steady state income due to the accumulation of net foreign assets and the threat of exclusion. The steady state welfare gains from openness may therefore be higher than the transitional gains in the neoclassical growth model, which are quantitatively small as emphasized by [Gourinchas and Jeanne \(2006\)](#).<sup>47</sup> We should emphasize that limited commitment is not sufficient for this gain from openness. Recall that the limited commitment environment of [Marcet and Marimon \(1992\)](#) coincides with the closed economy growth model in the absence of shocks, an application of the result of [Bulow and Rogoff \(1989\)](#), making openness irrelevant. However, in the presence of political frictions ( $\theta > 1$ ), the dynamics of the equilibrium differ from that of the corresponding closed economy model, as access to debt mitigates the time consistency problem along the equilibrium path.

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<sup>45</sup>Note that we follow the original papers and consider Markov equilibria of the closed economy hyperbolic models. To our knowledge, no one has considered reputational equilibria in the closed economy setting.

<sup>46</sup> Even if  $\beta R < 1$ , so the economy does not converge to the first best, access to international credit markets and reputational considerations sustains a higher (albeit distorted) steady state level of capital than the closed economy MPE counterpart. While the equation (18) indicates that lowering  $\beta$  speeds convergence in our framework, it remains the case that convergence is still slower in our framework than in its closed economy counterpart.

<sup>47</sup>In our framework, openness with zero debt weakly dominates autarky, so it is always optimal to open one's economy. Financial openness expands the budget set relative to continued financial autarky, starting from zero external debt. Moreover, because deviation leads to financial autarky, no other constraint is affected. It immediately follows that financial openness, all else equal, will (weakly) raise the welfare of the population (or the initial decision maker).

## 6 Conclusion

In this paper, we presented a tractable variation on the neoclassical growth model that explains why small open economies have dramatically different growth outcomes, and the ones that grow fast do so while increasing their net foreign asset position. Figures 1 and 2 indicated that this pattern was driven by a net reduction in public debt combined with an inflow of private capital in fast growing economies, and the reverse in shrinking economies, facts consistent with the model developed in this paper. This paper focused on the negative relationship between sovereign debt and growth induced by political economy frictions. In an earlier paper (Aguiar et al., 2009), we explored how debt overhang can exacerbate volatility as well. This raises the intriguing possibility that political economy frictions and the associated debt dynamics may jointly explain the negative relationship between volatility and growth observed in the data, a question we leave for future research.

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## Appendix: Proofs

This appendix presents proofs of statements made in the body of the paper. We begin with Proposition 1, postponing proof of lemma 1 until after the proof of lemma 2. The proof of proposition 3 is contained in the final subsection of the appendix, in which we discuss the dynamics with concave utility more generally.

For convenience, we restate the problem (P):

$$V(b_0) = \max_{\{c_t, k_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (\text{AP})$$

subject to:

$$b_0 \leq \sum_{t=0}^{\infty} R^{-t} (f(k_t) - (r + d)k_t - c_t), \quad (\text{A12})$$

$$\underline{W}(k_t) \leq \sum_{s=t}^{\infty} \beta^{s-t} \delta^{s-t} \theta u(c_s) + \sum_{s=t}^{\infty} \beta^{s-t} (1 - \delta^{s-t}) u(c_s), \quad \forall t \quad (\text{A13})$$

$$\underline{k} \leq k_t \quad (\text{A14})$$

Let  $\mu_0$  be the multiplier on the budget constraint (A12),  $\lambda_t (R^{-t} \mu_0 / \theta)$  be the multiplier on the sequence of constraints on participation (A13) and  $\phi_t R^{-t}$  be the multiplier on (A14).

For  $c_t, k_t$  to be an optimal allocation, there exist non-negative multipliers such that: (i)

the following first order conditions hold:

$$\frac{1}{u'(c_t)} = \frac{(\beta R)^t}{\mu_0} + \sum_{s=0}^t (\beta R)^s (\delta^s (\theta - 1) + 1) \frac{\lambda_{t-s}}{\theta} \quad (\text{A15})$$

$$\frac{\lambda_t}{\theta} \underline{W}'(k_t) = f'(k_t) - (r + d) + \phi_t, \quad \forall t \geq 0; \quad (\text{A16})$$

(ii) the constraints (A12)-(A14) hold; and (iii) the associated complementary slackness conditions hold. Given the convexity of the problem (Assumption 2), these conditions are also sufficient.

## Proof of Proposition 1

To prove the proposition, suppose that  $k_t$  does not converge to  $k^*$ . Define  $T_\epsilon = \{t | k_t < k^* - \epsilon\}$ . It follows that for some  $\epsilon > 0$ ,  $T_\epsilon$  has infinite members. Then from (A15):

$$\frac{1}{u'(c_t)} = \frac{1}{\mu_0} + \sum_{s=0}^t (\delta^{t-s} (\theta - 1) + 1) \frac{\lambda_s}{\theta} \geq \frac{1}{\mu_0} + \sum_{s \in T_\epsilon, s \leq t} \frac{\lambda_s}{\theta} \geq \frac{1}{\mu_0} + \sum_{s \in T_\epsilon, s \leq t} C_\epsilon$$

where  $C_\epsilon \equiv (f'(k^* - \epsilon) - (r + d)) / (\underline{W}'(k^* - \epsilon) / \theta) > 0$ , and the inequalities reflect  $\lambda_s, \phi_s \geq 0$  for all  $s$  and  $\lambda_s \geq C_\epsilon$  for  $s \in T_\epsilon$ . It follows then that  $1/u'(c_t)$  diverges to infinity, and thus  $u(c_t)$  converges to its maximum. But this implies that the participation constraints will stop binding at some finite  $t_0$ , which leads to  $\lambda_s$  that are zero for all  $s > t_0$ , a contradiction.

## Proof of Proposition 2

From (17) evaluated at  $t$ , we have:

$$1 = \frac{\beta^t R^t}{\mu_0} + \sum_{s=0}^t \beta^s R^s \frac{\lambda_{t-s}}{\theta} + \sum_{s=0}^t \beta^s R^s \delta^s (\theta - 1) \frac{\lambda_{t-s}}{\theta} \quad \forall t \geq 0. \quad (\text{A17})$$

Evaluated at  $t = 0$ , we have  $\lambda_0 = 1 - 1/\mu_0$ . At  $t = 1$ , we have  $\lambda_1 = 1 - \beta R + \beta R(1 - \delta)(\theta - 1)\lambda_0$ . For  $t > 1$ , we let  $L$  represent the lag operator and write (A17) as

$$\begin{aligned} 1 &= \frac{\beta^t R^t}{\mu_0} + \frac{1}{\theta} \sum_{s=0}^t \beta^s R^s (1 + \delta^s (\theta - 1)) L^s \lambda_t \\ &= \frac{\beta^t R^t}{\mu_0} + \frac{1}{\theta} \left( \frac{1}{1 - \beta R L} + \frac{\theta - 1}{1 - \beta R \delta L} \right) \lambda_t \\ &= \frac{\beta^t R^t}{\mu_0} + \frac{1}{\theta} \left( \frac{1 - \beta R \delta L + (\theta - 1)(1 - \beta R L)}{(1 - \beta R L)(1 - \beta R \delta L)} \right) \lambda_t, \end{aligned}$$

where the step from the first to the second line uses the fact that  $\lambda_t = 0$  for  $t < 0$ . Multiplying through and rearranging yields equation (18) from the text:

$$\lambda_{t+1} = (1 - \beta R)(1 - \beta R\delta) + \beta R \left(1 - \frac{1 - \delta}{\theta}\right) \lambda_t \forall t \geq 1.$$

The steady state value can be computed in the usual way. Given that the slope of (18) is positive and less than one, convergence and monotonicity follow.

## Proof of Corollary 1

Proposition 2 characterized the dynamics of  $\lambda_t$ . One can then use the first order condition for capital to derive the associated dynamics for  $k_t$ . For any given value of  $\lambda_t$ , define  $K(\lambda_t)$  to be the solution to:

$$\lambda_t = \frac{f'(K(\lambda_t)) - (r + d)}{\underline{W}'(K(\lambda_t))/\theta} = \frac{f'(K(\lambda_t)) - (r + d)}{\bar{c}'(K(\lambda_t))}.$$

The convexity assumption (Assumption 2) guarantees that the above has a unique solution, and that  $K(\lambda_t)$  is strictly decreasing in  $\lambda_t$ . Now, let  $\bar{\lambda}$  be such that  $K(\bar{\lambda}) = \underline{k}$ . Then the optimal path for  $k_t$  will be:

$$k_t = \begin{cases} K(\lambda_t) & ; \text{for } \lambda_t < \bar{\lambda} \\ \underline{k} & ; \text{otherwise} \end{cases}$$

Given that  $\lambda_t$  is monotone, this implies that the path for  $k_t$  will also be monotone. Define  $\bar{\theta}$  to be the value such that:

$$\bar{\lambda} = \frac{f'(\underline{k}) - (r + d)}{\bar{c}'(\underline{k})} = \frac{\bar{\theta}(1 - \delta\beta R)(1 - \beta R)}{\bar{\theta}(1 - \beta R) + \beta R(1 - \delta)}.$$

Hence, the long run level of capital will be:

$$k_\infty = \begin{cases} K(\lambda_\infty) & ; \text{for } \theta < \bar{\theta} \\ \underline{k} & ; \text{otherwise} \end{cases}$$

This proves the first part of Corollary 1. For the second part, note that higher debt implies a (weakly) higher multiplier  $\mu_0$ , and a higher  $\lambda_0 = 1 - 1/\mu_0$ . Given that  $\lambda_1$  and  $\lambda_t$  are monotonic in previous values, it follows that the entire path of  $\lambda_t$  increases with  $\mu_0$  and debt. That is, a higher level of debt leads to a lower level of capital at each point in time.

## Proof of Lemma 2

Using the definitions, we have

$$\begin{aligned} V_t &= u_t + \beta V_{t+1} \\ W_t &= \theta u_t + \beta \delta W_{t+1} + \beta(1 - \delta) V_{t+1}. \end{aligned}$$

Eliminating  $u_t$  from the above and re-arranging:

$$\begin{aligned} \theta \left( V_t - \beta \left( 1 - \frac{1 - \delta}{\theta} \right) V_{t+1} \right) &= W_t - \beta \delta W_{t+1} \\ \theta \left( 1 - \beta \left( 1 - \frac{1 - \delta}{\theta} \right) F \right) V_t &= (1 - \beta \delta F) W_t, \end{aligned}$$

where  $F$  is the forward operator. Solving for  $V_t$  and eliminating explosive solutions:

$$\begin{aligned} \theta V_t &= \left( \frac{1 - \beta \delta F}{1 - \beta \left( 1 - \frac{1 - \delta}{\theta} \right) F} \right) W_t \\ &= W_t + \beta(1 - \delta) \left( 1 - \frac{1}{\theta} \right) \sum_{i=0}^{\infty} \beta^i \left( 1 - \frac{1 - \delta}{\theta} \right)^i W_{t+1+i}. \end{aligned}$$

Dividing through by  $\theta$  yields the expression in the lemma.

## Proof of Lemma 1

Define  $\underline{W}(k)$  to be the incumbent's value function if it deviates given capital  $k$ . We can write this as

$$\underline{W}(k) = \theta u(\bar{c}(k)) + \beta \delta \underline{W} + \beta(1 - \delta) \underline{V}, \quad (24)$$

where  $\underline{W}$  is the continuation value under the punishment if the incumbent retains power next period, and  $\underline{V}$  is the continuation value if it loses power. We normalize  $t = 0$  to be the time of the deviation, so we have  $\underline{W} = W_1$  and  $\underline{V} = V_1$ . From Lemma 2, we have:

$$\theta \underline{V} = \theta V_1 = W_1 + \beta(1 - \delta) \left( 1 - \frac{1}{\theta} \right) \sum_{i=0}^{\infty} \beta^i \left( 1 - \frac{1 - \delta}{\theta} \right)^i W_{1+i}.$$

As the punishment must be self-enforcing, we have  $W_t \geq \theta u(\bar{c}(k_t)) + \beta \delta \underline{W} + \beta(1 - \delta) \underline{V}$ , at each  $t$ . Note that a second deviation is punished in the same way as the first. The fact that  $\underline{W}(k)$  is the worst possible punishment implies that this maximizes the set of possible self-enforcing allocations, from which we are selecting the one with minimum

utility. Substituting in the participation constraint in the above expression yields:

$$\begin{aligned}\theta \underline{V} &\geq \theta u(\bar{c}(k_1)) + \beta \delta \underline{W} + \beta(1 - \delta) \underline{V} \\ &+ \beta(1 - \delta) \left(1 - \frac{1}{\theta}\right) \sum_{i=0}^{\infty} \beta^i \left(1 - \frac{1 - \delta}{\theta}\right)^i (\theta u(\bar{c}(k_{1+i})) + \beta \delta \underline{W} + \beta(1 - \delta) \underline{V}) \\ &\geq \left(\frac{1 - \beta \delta}{1 - \beta \left(1 - \frac{1 - \delta}{\theta}\right)}\right) (\theta u(\bar{c}(k)) + \beta \delta \underline{W} + \beta(1 - \delta) \underline{V}),\end{aligned}$$

where the last inequality uses the fact that  $k_t \geq k$ , for all  $t$ . Rearranging, we have

$$\underline{V} \geq \frac{(1 - \beta \delta) (u(\bar{c}(k)) + \beta \delta \underline{W})}{\theta(1 - \beta) + \beta^2 \delta(1 - \delta)}. \quad (25)$$

Recall that  $W_1 = \underline{W}$ . Participation at  $t = 1$  requires  $W_1 \geq \theta u(\bar{c}(k_1)) + \beta \delta \underline{W} + \beta(1 - \delta) \underline{V}$ , or using the fact that  $k_1 \geq k$ :

$$\underline{W} \geq \theta u(\bar{c}(k)) + \beta \delta \underline{W} + \beta(1 - \delta) \underline{V}.$$

Substituting (25) in for  $\underline{V}$  and rearranging yields:

$$\underline{W} \geq \left(\frac{\theta - 1}{1 - \beta \delta} + \frac{1}{1 - \beta}\right) u(\bar{c}(k)).$$

Substituting back into (25), we have

$$\underline{V} \geq \frac{u(\bar{c}(k))}{1 - \beta}.$$

The left hand sides of these last two inequalities are the government's and private agent's utility, respectively, from the Nash equilibrium repeated ad infinitum. As repeated Nash is a self enforcing equilibrium and bounds from below the punishment payoff, it is the self-enforcing equilibrium that yields the lowest utility for the deviating government.

## Proof of Proposition 4

The proof of this proposition follows directly from Lemma 2, the fact that  $k_t$  is monotone, and that  $\underline{W}(k)$  is an increasing function of  $k$ .

## Proof of Corollary 2

Suppose that  $k_t$  is increasing. Let  $B_t = \sum_{s=t}^{\infty} R^{s-t} (f(k_s) - (r + d)k_s - c_s)$  denote the stock of debt outstanding in period  $t$ . Suppose, to generate a contradiction, that  $B_{T+1} > B_T$  for some  $T \geq 1$ . Let  $\{c_t, k_t\}$  denote the equilibrium allocation. Now consider the alternative allocation:  $\tilde{c}_t = c_t$  and  $\tilde{k}_t = k_t$  for  $t < T$ , and  $\tilde{c}_t = c_{t+1}$  and  $\tilde{k}_t = k_{t+1}$  for

$t \geq T$ . That is, starting with period  $T$ , we move up the allocation one period. As  $\tilde{V}_0 - V_0 = \beta^T(\tilde{V}_T - V_T) = \beta^T(V_{T+1} - V_T) > 0$ , the objective function has increased and where the last inequality follows from the monotonicity of  $V_t$ . Similarly,  $\tilde{B}_0 - B_0 = R^{-T}(\tilde{B}_T - B_T) = R^{-T}(B_{T+1} - B_T) > 0$ , the budget constraint is relaxed, where the last inequality follows from the premise  $B_{T+1} > B_T$ . For  $t \geq T$ , we have  $\tilde{W}_t = W_{t+1} \geq \underline{W}(k_{t+1}) = \underline{W}(\tilde{k}_t)$ , so participation holds for period  $T$  and after. For  $t < T$ , note that  $W_t = \sum_{s=t}^{T-1} \beta^{s-t} [\delta^{s-t}(\theta - 1) + 1] u_s + \beta^T \delta^T W_T + \beta^T (1 - \delta^T t) V_{T+1}$ . As  $\tilde{W}_T > W_T$  and  $\tilde{V}_T > V_T$ , we have  $\tilde{W}_t > W_t$  for all  $t < T$ . As  $\tilde{k}_t = k_t$  for  $t < T$ , our new allocation satisfies the participation constraints of the governments along the path. Therefore, we have found a feasible allocation that is a strict improvement, a contradiction of optimality. A similar argument works for a decreasing path of  $k_t$ .

## Proof of Proposition 5

By construction, the budget constraint implications of both policies are the same. Moreover, the deviation utility is unchanged for debt relief, while strictly higher for unconditional aid. Therefore, debt relief can be viewed as a relaxed version of the problem with unconditional aid, and so weakly dominates. If the participation constraint binds in the solution with debt relief, this allocation is unattainable with unconditional aid. Given the convexity of the problem, welfare will be strictly higher with debt relief in this case. The second part of the proposition follows from a simple change of variable in the original problem (P). Let  $\{k_t, c_t\}_{t=0}^{\infty}$  denote the efficient allocation without aid, and  $\{\tilde{k}_t, \tilde{c}_t\}_{t=0}^{\infty}$  denote the efficient allocation given a sequence of unconditional aid payments  $\{y_t\}$ . Note that the resource constraint in the presence of aid is  $b_0 \leq \sum_t R^{-t} (f(\tilde{k}_t) - (r + d)\tilde{k}_t + y_t - \tilde{c}_t)$ . Define  $\hat{c}_t = \tilde{c}_t - y_t$ , so that the resource constraint can now be written  $b_0 \leq \sum_t R^{-t} (f(\tilde{k}_t) - (r + d)\tilde{k}_t - \hat{c}_t)$ , which is observationally equivalent to the non-aid problem. As the participation constraint is linear in  $\tilde{c}_t$  and unconditional aid, we can subtract the discounted stream of  $y_t$  from both sides replace  $\tilde{c}_t - y_t$  with  $\hat{c}_t$ , thereby eliminating  $y_t$  from the participation constraints. The objective function is also linear in  $\tilde{c}_t$ , so we can replace  $\sum \beta^t \tilde{c}_t$  with  $\sum \beta^t \hat{c}_t$  without changing the solution to the problem. With this change of variable, the problem with aid can be stated in terms of  $\hat{c}_t$ , without the presence of  $y_t$ . Therefore, the solution  $\{\hat{c}_t, \tilde{k}_t\}_{t=0}^{\infty}$  will coincide with the non-aid allocation  $\{c_t, k_t\}$ . That is  $\tilde{k}_t = k_t$  and  $\hat{c}_t = c_t$  for all  $t$ . From the definition of  $\hat{c}_t$ , we therefore have  $\tilde{c}_t = c_t + y_t$ , as stated in the proposition.

## Analysis of Nonlinear Dynamics and Proof of Proposition 3

We now characterize dynamics when utility is concave. For the case of  $\delta = 0$ , we can depict the dynamics in a two dimensional phase diagram. We then discuss a linearized system for the case of general  $\delta < 1$ . For concave utility, we impose the Inada conditions:  $\lim_{c \rightarrow 0} u'(c) = \infty$  and  $\lim_{c \rightarrow \infty} u'(c) = 0$ .

The non-linear system for general  $\delta$  is:

$$\begin{aligned} W_t &= \theta u_t + \beta\delta W_{t+1} + \beta(1 - \delta)V_{t+1} \\ V_t &= u_t + \beta V_{t+1} \\ c'(u_t) &= \frac{(\beta R)^t}{\mu_0} + \sum_{s=0}^t (\beta R)^s \frac{\lambda_{t-s}}{\theta} + \sum_{s=0}^t (\beta R)^s \delta^s (\theta - 1) \frac{\lambda_{t-s}}{\theta} \\ \lambda_t &= \frac{f'(k_t) - (r + d)}{\underline{W}'(k_t)/\theta} = H(k_t), \end{aligned}$$

where  $c(u)$  is the inverse utility function. For convenience, we assume that the constraint  $k_t \geq \underline{k}$  is not binding, a point we discuss below. A sequence  $\{u_t, k_t, V_t, W_t, \lambda_t\}$  plus a multiplier  $\mu_0$  that satisfies this system, the constraints (A12)–(A13) with the right hand side of (A13) replaced by  $W_t$ , complementary slackness, plus the boundary conditions  $\lim_{s \rightarrow \infty} \beta^s V_{t+s} = 0$  and  $\lim_{s \rightarrow \infty} \beta^s W_{t+s} = 0$ , will be an optimal allocation. To see this, note that the latter two equations are the same as (A15) and (A16). The solutions to the first two difference equations that satisfy the boundary conditions yield the correct value function for incumbent utility, so constraint (A13) is equivalent to  $\underline{W}(k_t) \leq W_t$ .

It will be convenient to introduce the following notation:

$$\begin{aligned} \Lambda_t &\equiv \frac{(\beta R)^t}{\mu_0} + \beta R \sum_{s=0}^{t-1} (\beta R)^s \frac{\lambda_{t-1-s}}{\theta} \\ \Phi_t &\equiv \beta R \delta \sum_{s=0}^{t-1} (\beta R)^s \delta^s (\theta - 1) \frac{\lambda_{t-s}}{\theta}. \end{aligned}$$

With this, we can write the first order condition for consumption as

$$c'(u_t) = \Lambda_t + \Phi_t + \lambda_t.$$

We have  $\Lambda_0 = 1/\mu_0$  and  $\Phi_0 = 0$ , with

$$\Lambda_{t+1} = \beta R \left( \Lambda_t + \frac{\lambda_t}{\theta} \right) \quad (26)$$

$$\Phi_{t+1} = \beta R \delta \left( \Phi_t + (\theta - 1) \frac{\lambda_t}{\theta} \right). \quad (27)$$

Letting a prime denote next period's value, we can write the dynamic system the



characterizes the equilibrium allocation as the set of first order difference equations:

$$\begin{aligned}
-W' &= \frac{\theta - 1 + \delta}{\beta\delta}u + \frac{1 - \delta}{\beta\delta}V - \frac{1}{\beta\delta}W \\
V' &= \frac{1}{\beta}V - \frac{1}{\beta}u \\
\Lambda' &= \beta R \left( \Lambda + \frac{\lambda}{\theta} \right) \\
\Phi' &= \beta R \delta \left( \Phi + (\theta - 1) \frac{\lambda}{\theta} \right);
\end{aligned} \tag{D}$$

the first order conditions::

$$\begin{aligned}
c'(u) &= \Lambda + \Phi + \lambda \\
\lambda &= H(k);
\end{aligned}$$

the complementary slackness condition for  $\lambda \geq 0$ :<sup>48</sup>

$$\lambda(W - \underline{W}(k)) = 0;$$

and the boundary conditions

$$\begin{aligned}
\lim_{s \rightarrow \infty} \beta^s V_{t+s} &= 0 \\
\lim_{s \rightarrow \infty} \beta^s W_{t+s} &= 0.
\end{aligned}$$

For convenience, we invert the first order condition  $c'(u) = \Lambda + \Phi + \lambda$  and introduce the function  $U : \mathbb{R}_+ \rightarrow [\underline{u}, \bar{u}]$ :

$$u = U(\Lambda + \Phi + \lambda) = c'^{-1}(\Lambda + \Phi + \lambda). \tag{28}$$

We can state the following, which is part (i) of Proposition 3:

**Lemma 3.** *When  $\beta R < 1$ , there is a unique steady state for the system (D) for which  $k_\infty < k^*$ . When  $\beta R = 1$ , there is a continuum of possible steady states which all have  $k_\infty = k^*$ .*

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<sup>48</sup>We omit the slackness condition on the budget constraint, as this constraint will always hold with equality at an optimal allocation.

*Proof.* Equations (D) imply:

$$\begin{aligned}\lambda_\infty &= \frac{(1 - \beta R)\theta}{\beta R} \Lambda_\infty \\ \Phi_\infty &= \frac{\beta R \delta (\theta - 1)}{(1 - \beta R \delta)\theta} \lambda_\infty \\ W_\infty &= \left( \frac{\theta - 1}{1 - \beta \delta} + \frac{1}{1 - \beta} \right) u_\infty \\ V_\infty &= \frac{u_\infty}{1 - \beta}.\end{aligned}$$

Case (i)  $\beta R < 1$ : Suppose, to generate a contradiction, that  $\lambda_\infty = 0$ , so  $W_\infty \geq \underline{W}(k^*)$ . From (A26)-(A27), we have  $\Lambda, \Phi \rightarrow 0$ . The first order condition for  $u$  implies that  $c'(u_\infty) = 1/u'(c_\infty) = 0$ , or that  $c_\infty = 0$ , which contradicts  $W_\infty \geq \underline{W}(k^*)$ . This establishes that  $\lambda_\infty > 0$ . From the slackness condition, we then have  $W_\infty = \underline{W}(k_\infty)$ . Using the other identities to substitute, we can write this as:

$$\begin{aligned}U \left( \left( \frac{\beta R}{(1 - \beta R)\theta} + \frac{\beta R \delta (\theta - 1)}{(1 - \beta R \delta)\theta} + 1 \right) H(k_\infty) \right) = \\ \left( \frac{\theta - 1}{1 - \beta \delta} + \frac{1}{1 - \beta} \right)^{-1} \underline{W}(k_\infty).\end{aligned}$$

The left hand side is strictly decreasing in  $k_\infty$  and the right hand side is strictly increasing in  $k_\infty$ . When  $\underline{k} = 0$ , the right hand side is zero at  $k_\infty = 0$ , while the left hand side is strictly positive at zero as  $H(0) > 0$ , so a unique steady state  $k$  exists for  $\underline{k}$  sufficiently small. If  $\underline{k}$  is such that the left hand side is less than the right hand side at  $k_\infty = \underline{k}$ , then  $k_\infty = \underline{k}$  is the steady state. The remaining variables can be uniquely derived from  $k_\infty$ .

Case (ii)  $\beta R = 1$ : From the above steady state relationships, we have  $\lambda = 0$ , so  $k_\infty = k^*$ . Note that any  $\Lambda_\infty$  such that  $u_\infty = U(\Lambda_\infty)$  is large enough to sustain  $k^*$  can be a steady state. In particular, if  $\mu_0$  is such that  $k_0 = k^*$ ,  $\Lambda_t = \Lambda_0 = 1/\mu_0$  for all  $t$  and the system stays there indefinitely. That is, let  $\bar{\mu}_0$  be such that  $\left( \frac{\theta - 1}{1 - \beta \delta} + \frac{1}{1 - \beta} \right) U \left( \frac{1}{\bar{\mu}_0} \right) = \underline{W}(k^*)$ . Then any  $\mu_0 \leq \bar{\mu}_0$  is a steady state.  $\square$

We now characterize the dynamics away from the steady state for the case of  $\delta = 0$ . From (A27), we have  $\Phi_t = \Phi_0 = 0$  when  $\delta = 0$ . This allows us to reduce the dimensionality of the system. In particular, we will reduce the system to two variables:  $\Lambda$  and  $V$ . The variable  $\Lambda_t$  is the discounted sum of multipliers through  $t - 1$  plus the multiplier on the initial budget constraint. This term reflects the extent that consumption in period  $t$  relaxes the participation constraints for prior incumbents. In this manner, it represents promises made to prior incumbents and will serve as our state variable. The variable  $V_t$  is the present discounted value of private agent utility going forward from  $t$ .

From the definitions of  $W$  and  $V$ , we have  $W_t = (\theta - 1)u_t + V_t$  when  $\delta = 0$ . We can use this plus the complementary slackness condition to replace  $\lambda$  with a function  $L(\Lambda, V)$ . To this end, note that  $W = (\theta - 1)U(\Lambda + \lambda) + V$ . Define  $L(\Lambda, V) = 0$  if  $(\theta - 1)U(\Lambda) + V >$

$\underline{W}(k^*)$ . When  $W \leq \underline{W}(k^*)$ , we have  $W = \underline{W}(k)$ . In this case, we can define  $L(\Lambda, V)$  as the  $\lambda$  that solves:

$$(\theta - 1)U(\Lambda + \lambda) + V = \underline{W}(H^{-1}(\lambda)),$$

where we have inverted  $\lambda = H(k)$  to map  $k$  into  $\lambda$ , which is possible as  $H(k)$  is strictly decreasing by Assumption 2. To see that there is a unique solution to this equation, the left hand side is strictly increasing in  $\lambda$  while the right hand side is strictly decreasing. As we are considering the case when  $(\theta - 1)U + V \leq \underline{W}(k^*)$ , by assumption the left hand side is less than or equal to the right hand side at  $\lambda = 0$ . Note that  $L$  is continuous in both arguments. Moreover, straightforward manipulations show that  $L$  is non-increasing in both arguments, and strictly decreasing when  $(\theta - 1)U(\Lambda) + V \leq \underline{W}(k^*)$ , but that  $\Lambda + L(\Lambda, V)$  is strictly increasing in  $\Lambda$ .

With this function in hand, our dynamic system can be written:

$$\begin{aligned} V' &= -\frac{U(\Lambda + L(\Lambda, V))}{\beta} + \frac{V}{\beta} \\ \Lambda' &= \beta R(\Lambda + L(\Lambda, V)). \end{aligned} \quad (D')$$

For both equations, the right hand side is strictly increasing in  $\Lambda$  and  $V$ , so  $V'$  and  $\Lambda'$  are uniquely defined given  $(\Lambda, V)$ . We depict the dynamics using a phase diagram in figure 8. Panel (a) is the case  $\beta R = 1$  and panel (b) treats  $\beta R < 1$ . The gray shaded area corresponds to points such that  $(\theta - 1)U(\Lambda) + V \geq \underline{W}(k^*)$ . This area has a downward sloping border in the  $V \times \Lambda$  plane as  $L$  is strictly increasing in both arguments when  $(\theta - 1)U(\Lambda) + V = \underline{W}(k^*)$ .

The two bold lines correspond to points for which  $V' = V$  and  $\Lambda' = \Lambda$ , respectively. From the first equation of  $D'$ , we see that  $V' = V$  if  $V = U(\Lambda + L(\Lambda, V))/(1 - \beta)$ . In the region where  $L(\Lambda, V) = 0$ , this equation is satisfied along a upward sloping locus in the  $V \times \Lambda$  plane, as  $U$  is a strictly increasing function. Outside this region, use can appeal to the fact that  $\Lambda + L(\Lambda, V)$  is increasing in  $\Lambda$  to show that the locus is upward sloping in the unshaded region as well. From  $(D')$ , we see that for a given  $V$ , as we increase  $\Lambda$  to the right of this line, then  $V' > V$ , and vice versa for  $\Lambda$  to the left of this line. These dynamics are represented by the vertical arrows in the phase diagram.

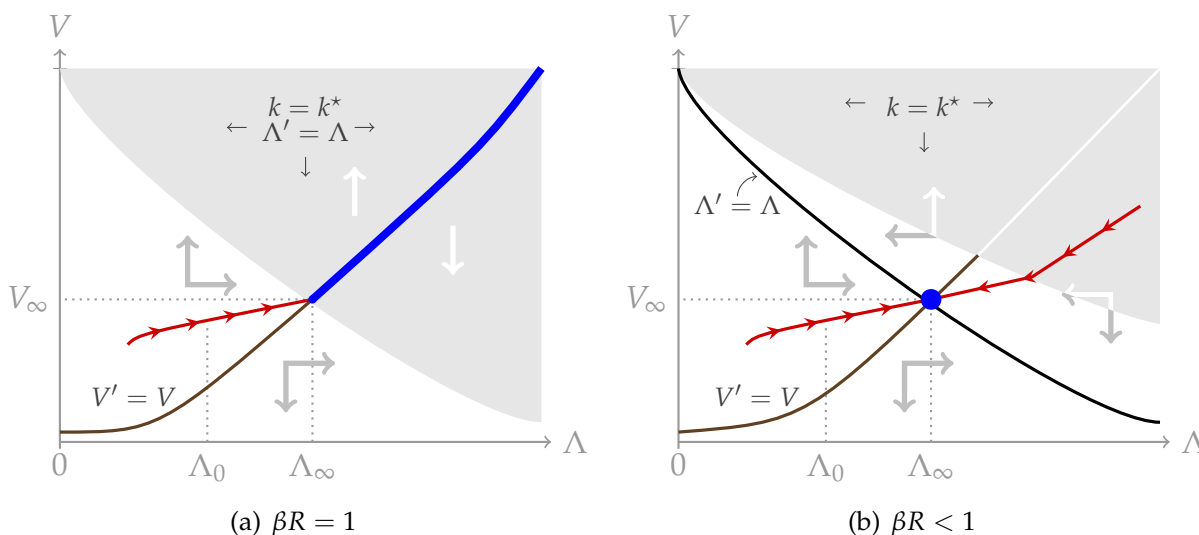
In panel (a),  $\Lambda' = \Lambda$  when  $\lambda = 0$ . Therefore,  $\Lambda' = \Lambda$  at all points in the gray shaded region. In panel (b), when  $\beta R < 1$ ,  $\Lambda' = \Lambda$  along the downward sloping line. This locus is defined by  $L(\Lambda, V) = (1 - \beta R)\Lambda$ . This coincides with the gray region at  $\Lambda = 0$ , and is strictly below it for  $\Lambda > 0$ . In this region,  $L$  is decreasing in both arguments, so the locus is downward sloping in the  $V \times \Lambda$  plane. As we increase  $V$  for a given  $\Lambda$  starting from a point on this locus,  $\Lambda' < \Lambda$  outside the gray region and constant otherwise. The reverse is true below the locus. These dynamics are represented by the horizontal arrows in the unshaded region of the phase diagram.

The steady state is represented by the intersection of the two loci, which exists by lemma 3. The dynamics imply saddle path stability. As  $\lim_{s \rightarrow \infty} \beta^s V_{t+s} = 0$  is a condition of optimality, the equilibrium allocation follows the saddle path. In panel (a), when  $\beta R = 1$ , there exists a continuum of steady states to the right of  $\Lambda_\infty$  corresponding to cases in which the system begins with low enough debt that we immediately have  $\lambda_0 = 0$  and

there are no further dynamics. In this case  $k_t = k^*$  and  $c_t$  is constant for all  $t$ . When we begin with enough debt that  $\Lambda_0 < \Lambda_\infty$ , the system converges along a saddle path to the  $\Lambda_\infty$ . During this transition, the investment wedge ( $\lambda = L(\Lambda, V)$ ) monotonically declines to zero. This is the case depicted by  $\Lambda_0$  in the figure.

When  $\beta R < 1$  (panel (b)), there exists a unique steady state at which capital is distorted away from  $k^*$ , and for any initial condition the dynamics are monotonic towards this steady state. If  $b_0$  is such that  $\Lambda_0 < \Lambda_\infty$  (the case depicted by  $\Lambda_0$  in the figure),  $V$  and  $\Lambda$  increase over time, and so  $\lambda$  declines over time and  $k$  increases; while if initial debt is sufficiently low ( $\Lambda_0 > \Lambda_\infty$ ), then  $k$  (weakly) decreases over time.

Note that in both panels, convergence is monotonic toward the steady state, which implies that capital also converges monotonically. We therefore can appeal to proposition 4 and corollary 2 for the transition dynamics of private agents' utility and external debt. Moreover, as  $\Lambda_0 = 1/\mu_0$ , the greater is  $\mu_0$  the lower the initial  $\Lambda_0$  and  $V$ . This implies initial capital is decreasing in  $\mu_0$ , and strictly decreasing if we begin with enough debt that  $\lambda_0 = L(\Lambda_0, V_0) > 0$ . As  $\mu_0$  is the multiplier on the resource constraint, this implies that initial capital is weakly decreasing in initial debt. We have now established part (ii) of Proposition 3. We turn next to part (iii), the speed of convergence.



**Figure 8:** Phase diagram for  $\delta = 0$ . The shaded region represents points for which  $k = k^*$ . The upward sloping bold line represents  $V' = V$ . In panel (a), the shaded region also represents points for which  $\Lambda' = \Lambda$ . In panel (b),  $\Lambda' = \Lambda$  for points along the downward sloping bold line. The intersection of the  $V' = V$  line with the shaded region represents steady states in panel (a). In panel (b), the intersection of the two lines defines the unique steady state. The (red) line marked with arrows represents the stable manifold (saddle path). The point  $\Lambda_0$  depicts one possible initial value, which is determined by initial debt.

## Speed of Convergence

We explore the speed of convergence by studying the first order dynamics in the neighborhood of the steady state. To do so, we linearize the dynamic system and solve for the

speed of convergence along the saddle path. Note that in the case of  $\beta R = 1$ , this means we are using the dynamics from “below” (the unshaded region in Figure 8). We consider general  $\delta \in [-1/N, 1]$  and linearize the system (D).<sup>49</sup> Letting  $\hat{x} = x - x_\infty$ , we have the four equation linearized system:

$$\begin{aligned}\hat{k}' &= \frac{1}{\beta\delta} \left[ -\frac{\theta-1+\delta}{W'(k_\infty)} \frac{H'(k_\infty)}{c''(u_\infty)} + 1 \right] \hat{k} - \frac{\theta-1+\delta}{\beta\delta W'(k_\infty)} \frac{1}{c''(u_\infty)} \hat{\Lambda} - \frac{\theta-1+\delta}{\beta\delta W'(k_\infty)} \frac{1}{c''(u_\infty)} \hat{\Phi} - \frac{1-\delta}{\beta\delta W'(k_\infty)} \hat{V}' \\ \hat{V}' &= \frac{1}{\beta} \hat{V}' - \frac{1}{\beta} \frac{H'(k_\infty)}{c''(u_\infty)} \hat{k} - \frac{1}{\beta} \frac{1}{c''(u_\infty)} \hat{\Lambda} - \frac{1}{\beta} \frac{1}{c''(u_\infty)} \hat{\Phi} \\ \hat{\Phi}' &= \beta R \delta \frac{\theta-1}{\theta} H'(k_\infty) \hat{k} + \beta R \delta \hat{\Phi} \\ \hat{\Lambda}' &= \beta R \frac{1}{\theta} H'(k_\infty) \hat{k} + \beta R \hat{\Lambda}\end{aligned}$$

Let  $\kappa = \frac{W'(k_\infty)c''(u_\infty)}{H'(k_\infty)}$ , which captures the nonlinearity of utility if  $c'' > 0$ . Note that  $\kappa \leq 0$  given that  $H'(k) \leq 0$ . We renormalize  $\Lambda$  and  $\Phi$  to be  $\Lambda/H'(k_\infty)$  and  $\Phi/H'(k^{ss})$ , and write the linear system in matrix form:

$$\begin{bmatrix} \hat{k}' \\ \hat{V}' \\ \hat{\Phi}' \\ \hat{\Lambda}' \end{bmatrix} = \begin{bmatrix} -\frac{1}{\beta\delta} \left[ (\theta-1+\delta) \frac{1}{\kappa} - 1 \right] & -\frac{1-\delta}{\beta\delta W'(k^{ss})} & -\frac{\theta-1+\delta}{\beta\delta\kappa} & -\frac{\theta-1+\delta}{\beta\delta\kappa} \\ -\frac{W'(k^{ss})}{\beta\kappa} & \frac{1}{\beta} & -\frac{W'(k^{ss})}{\beta\kappa} & -\frac{W'(k^{ss})}{\beta\kappa} \\ \beta R \delta \frac{\theta-1}{\theta} & 0 & \beta R \delta & 0 \\ \beta R \frac{1}{\theta} & 0 & 0 & \beta R \end{bmatrix} \times \begin{bmatrix} \hat{k} \\ \hat{V}' \\ \hat{\Phi} \\ \hat{\Lambda} \end{bmatrix}$$

The characteristic equation is:

$$\begin{aligned}Ch(x) \equiv & x(x\theta - \beta R(\theta - 1 + \delta))(-\theta + x\beta(\theta - 1 + \delta)) \\ & + (x - \beta R)(x\beta - 1)(x - \beta R\delta)(x\beta\delta - 1)\theta\kappa = 0\end{aligned}$$

### The linear case as a limiting case

Note that when  $\kappa \rightarrow 0$  (so that the utility becomes linear), the roots of the characteristic equation are:

$$x \in \left\{ 0, \frac{\theta}{\beta(\theta - 1 + \delta)}, \beta R \left( 1 - \frac{1 - \delta}{\theta} \right) \right\}$$

where the last one is the one we found in the linear case and corresponds to the highest eigenvalue less than one.

<sup>49</sup>By considering dynamics around the steady state, we are assuming that the convergence results derived for  $\delta = 0$  extend to arbitrary  $\delta$ .

### Proof of Proposition 3 Part (iii)

To prove part (iii) of Proposition 3, we need to show that for all  $\kappa < 0$  there is always a root of the characteristic equation that is less than one but higher than  $\beta R \left(1 - \frac{1-\delta}{\theta}\right)$ . Towards this goal, note that for  $\kappa < 0$ ,  $\theta > 1$ , and  $\delta < 1$ , we have:  $Ch(0) < 0$ ;  $Ch\left(\beta R \left(1 - \frac{1-\delta}{\theta}\right)\right) > 0$ ;  $Ch(\beta R) < 0$ ;  $Ch(1) < 0$ ;  $Ch\left(\frac{\theta}{\beta(\theta-1+\delta)}\right) > 0$ ; and  $\lim_{x \rightarrow \infty} Ch(x) = -\infty$ . Therefore, by continuity of the polynomial, we have two roots less than one, one between 0 and  $\beta R \left(1 - \frac{1-\delta}{\theta}\right)$ , and the other between  $\beta R \left(1 - \frac{1-\delta}{\theta}\right)$  and  $\beta R$ . There are also two roots greater than one, the first between 1 and  $\frac{\theta}{\beta(\theta-1+\delta)}$  and the other greater than  $\frac{\theta}{\beta(\theta-1+\delta)}$ . Note that the largest root less than one is between  $\beta R \left(1 - \frac{1-\delta}{\theta}\right)$  and  $\beta R$ . Thus the system is saddle path stable in the neighborhood of the steady state, with the the speed of convergence of the system bounded above by  $-\log\left(\beta R \left(1 - \frac{1-\delta}{\theta}\right)\right)$ .