Consumption and Labor Income Risk, Aggregation and Business Cycles*

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Abstract

This paper discusses the role of household heterogeneity in a model in which idiosyncratic consumption and labor income risks manifest as a distortion to the intra-temporal optimal condition of the representative agent. Aggregation over the undistorted labor-leisure condition of each household leads to a *wedge* between the Marginal Rate of Substitution, between consumption and leisure, (MRS) and the Marginal Product of Labor (MPL), through the lens of the representative agent model. I use household survey data to assess the properties of this aggregation wedge. I find that it is consistent with the systematic deviation between the MRS and the MPL observed in aggregate data, the so-called "Labor Wedge", for both the long-run and the business cycles. Additionally, I explore the quantitative implications of the model assuming imperfect insurability against idiosyncratic shocks. I show that cyclical changes in the distribution of household productivity and in the degree of risk sharing lead to cyclical changes in the aggregation wedge, as if the representative agent faced higher labor taxes - or was lazier- during recessions. The model also exhibits novel dynamics for labor, relative to the benchmark RBC, because of the various labor supply elasticities induced by the individual productivities and the wealth distribution.

1 Introduction

The neoclassical growth model predicts that the Marginal Rate of Substitution between consumption and leisure (MRS) should be equal to the Marginal Product of Labor (MPL), after adjusting for labor and consumption taxes and absent frictions. This condition, however, doesn't hold for

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aggregate data on consumption, output and hours worked. There is a wedge between the MRS and the MPL, the so-called "Labor Wedge", that has proven to be important for understanding differences in the level of labor supply between Europe and the US, and for the post-war cyclical fluctuations of hours worked in the US[1]. These short and long run properties of the "Labor Wedge" have received the attention of recent research in macroeconomics. Most of it has focused on alternative specifications of the standard model that can disconnect wages from aggregate productivity and replicate the fact that hours worked behave as if labor taxes were higher during recessions. However, aggregate data shows that most of the distortion appears on the household side as the MRS of the representative agent seems detached from wages, at both high and low frequencies[2].

This paper presents a theory of the labor wedge based on aggregation issues that can speak both to the long-run and business cycles properties of the labor wedge through a MRS distortion. I show that when households are subject to non-insurable productivity shocks, the aggregation over each household's labor supply decision is not perfect. In this case, the distribution of consumption and individual productivities manifest as a preference shock for leisure or labor taxes, through the lens of the representative agent model. If both distributions were invariant, this wedge arising from aggregation would be constant. However, changes in the distribution of idiosyncratic shocks and in the degree of risk-sharing among households can generate movements in the aggregation wedge, which can be wrongly attributed to changes in preferences or labor market distortions.

In order to make this point, I solve a model with uninsurable idiosyncratic productivity shocks, whose variance is counter-cyclical, and aggregate productivity shocks. For the benchmark specification, households cannot perfectly insure against consumption risk because they have access only to capital and cannot borrow. This model delivers a wedge between the MRS and the MPL, for aggregate series, with the same cyclical properties of the Labor Wedge as observed in the data. The model also exhibits novel dynamics for labor, when compared to the representative agent RBC model. Aggregate hours take longer to recover after a negative aggregate shock, just as if workers faced higher labor taxes during bad times.

The key mechanism that delivers the countercyclical labor tax for the representative agent is the various labor supply elasticities induced by the household-specific labor productivity and the wealth distribution. When a negative shock hits the economy, people at the borrowing constraint supply the same, or more, labor, despite the fact that wages are depressed. On the other hand, agents who are not at the constraint and have accumulated more assets because of precautionary motives will supply less labor than they will do in a model with perfect insurance. Upon shock, aggregate labor is less responsive than in the standard model because of the first effect. However, aggregate hours remain subdued after some quarters, as if labor taxes were higher, because of the weaker income effect of the those agents who have accumulated precautionary wealth. The

[1] See Prescott(2004),Charki, Kehoe and McGrattan (2007) and Shimer (2009,2010). The related literature section goes in more detail over recent papers regarding this issue.

[2] In the Appendix, I show a decomposition of the "Labor Wedge" between the household and the firm side.

jobless recovery of this economy is consistent with higher labor distortions from the perspective of the representative agent model. The assumption of financial market incompleteness is crucial for this result. If households have access to a full set of state-contingent Arrow-Debreu securities, all agents will have the same income effect after changes in aggregate productivity. The aggregation wedge in this case will be only a function of the distribution of productivity shocks as agents will be no longer subject to idiosyncratic consumption risk. Moreover, the implied distortion for the aggregate labor supply under a complete market specification will be counterfactual under the assumption that the variance of idiosyncratic productivities increases in recessions.

I test the aggregation hypothesis by directly calculating the aggregation wedge using household micro-level data for the US. Because I use data on consumption, I don't need to assume a particular financial structure in this calculation, so this result only relies on the broad conditions that ensure that the labor-leisure condition holds and the assumption regarding the stochastic process for idiosyncratic shocks. Using the CEX and CPS surveys, I calculate cross-sectional moments for the distribution of wages and consumption, and use them to construct a time series for the aggregation wedge implied by the sum of the undistorted labor-leisure condition for each worker. I find that this calculation replicates the observed decline of the labor wedge since the 1980s. I also find that the cyclical component of the aggregation wedge is positively correlated with the cylical changes of the observed labor wedge and negatively correlated with output and hours worked. The aggregation wedge under the assumption of complete markets, which ignores any changes in the distribution of consumption, is uncorrelated with the cyclical fluctuations of employment and output.

Regarding the solution method of the model, this paper uses a novel technique recently proposed by Preston and Roca (2007) and Ockatan and DenHaan (2009), which uses direct aggregation rather than the "bounded rationality" technique proposed by Krusell-Smith (1997). The decision of each agent is approximated using perturbation methods after replacing the occasionally binding borrowing constraint by a penalty function. Then, the policy function of each household is directly aggregated in order to find the law of motion for aggregate variables. I find that, given the nature of the aggregation problem, a first-order approximation doesn't suffice and a second-order approximation is needed in order to capture the novel dynamics implied by the heterogeneity of the model.

The remainder of this paper is organized as follows. In the next section I review the relevant literature and highlight the contribution of this paper. In the third and fourth sections, I present the model and the aggregation problem arising from the household heterogeneity. In section 5 the results of the model are summarized. Section 6 discusses a direct measure of the aggregate wedge without assuming a particular financial structure. For this part, I calculate directly the labor wedge implied by aggregation using micro data at the household level and compare it to the actual labor wedge implied by the aggregate data. The final section is for conclusions and a discussion of additional research questions.

2 Earlier Research

This work is related to three different strands of the literature: one that documents the short and long run properties of the labor wedge, and different stochastic properties or modeling assumptions that can account for these facts; the second, which studies conditions for perfect aggregation, and the third one that discusses how to solve models with idiosyncratic shocks and aggregate uncertainty.

Regarding the fist one, the basic references are Prescott (2004), Chari, Kehoe and McGrattan (2007) and Shimer (2009,2010). Shimer devotes an entire book to document the properties of the labor wedge and explore alternative specifications of the standard model that can account for it. Broadly speaking, the labor wedge has motivated two types of departures from the standard specification: the first one has emphasized the need of additional aggregate shocks for understanding business cycles, while the second has highlighted labor market frictions, such as search frictions and wage rigidity, as a key element in order to explain why wages could be disconnected from aggregate productivity.

Works by Mertens and Ravn (2008), Gali and Rabanal (2004) and Smets and Wouters (2007), claim that other sources of aggregate shocks, besides productivity shocks, are important to understand fluctuations in output and labor in the US. Mertens and Ravn (2008) argue that changes in the tax regime played an important role for the 1982 recession. Gali and Rabanal (2004) estimate the effect of various types of aggregate shocks using a bayesian estimation and find that preference shocks are the most important source for cyclical variations in output and hours worked. Smets and Wouters (2007) reach a similar conclusion using shocks to mark-up wages, in a model in which workers have monopolistic power over their own labor. Adding new shocks increases, mechanically, the prediction power of any model. These papers match better the data by assuming additional aggregate shocks, but proposed mechanisms that are less informative or appealing about the very nature of business cycles. According to these papers, recessions are result of higher taxes or less willingness to work either because agents are lazier or because workers decide to cut hours to drive up wages. In contrast to these papers, the model here proposed doesn't rely on the assumption of additional aggregate shocks. Moreover, the aggregation problem here studied can be understood as one explanation to existence of such preference shocks to the representative agent.

The alternative approach has highlighted the role of labor market frictions, using as a baseline the Diamond-Mortensen-Pisarides Model[3]. The key challenge for this literature is that search fric-

[3]Other works have highlighted the role of other labor market imperfections. Cole and Ohanian (2002) focus on industry collusions and collective bargaining to explain the weak recovery of hours during the great depression.

tions resemble an adjustment cost to labor, which slows down the process of job creation during a boom. This creates a pro-cyclical friction, counterfactual to the observed cyclical pattern of the labor wedge[4]. One solution to this problem is to add wage rigidities to the benchmark search model. Hall (2005,2009) shows that wage rigidity has an important effect in the volatility of unemployment in the presence of aggregate productivity shocks. Blanchard and Gali (2006) consider a model in which wages move less than proportionally to productivity shocks and there is countercyclical labor wedge. Shimer(2010) also discusses the effect of wage rigidity in the context of a model with searching and matching frictions. One problem with these models is that the degree of wage rigidity they assumed doesn't have a clear counterpart in the data[5]. Relative to these papers, this model presents a theory for the labor wedge without assuming any frictions in the labor market. Moreover, the aggregation wedge here proposed speaks directly to the observed distortion of the MRS of the representative agent, distortion that all of these papers are silent about.

Another literature that is relevant for this paper deals with issues of aggregation. The seminal works by Gorman (1953, 1959, 1976,1980) introduce the concept of perfect aggregation and define basic conditions under which the optimization problem of heterogeneous agents can be summarized by the optimization problem of a single-representative household. Other papers such as Rubistein (1974), Constantinides (1982), Atkeson and Ogaki (1986), among others, also discuss aggregation issues. Maliar and Maliar (2003) make the point that household heterogeneity collapses into parameters of the representative agent model, modifying its stochastic properties. The model here presented is close to this work, but I extend their analysis by departing from the complete markets environment. As mentioned earlier, allowing from market incompleteness will be important for delivering a counter-cyclical distortion.

The model here presented is also close to Chang and Kim (2007), which shows that the aggregation over individual labor supply decisions on the extensive margin manifests as labor wedge. Here, I rather focus on the intensive margin of labor supply. Furthermore, in the Chang and Kim's environment it is not possible to find an expression for the aggregation wedge. In their model, the labor-leisure condition doesn't hold at the household level, given the indivisibility of the labor decision. Hence, at the aggregate level, the labor-leisure expression is just the sum of inequalities that don't perfectly add up to the analogous condition of the representative agent model. In my model, I can back up an expression for the aggregation wedge and test it using household data.

Finally, there is a growing literature on how to solve models with idiosyncratic and aggregate shocks. The solution method used here derives from the works of Preston and Roca (2009) and Deen Haan and Ockatan(2010). Another paper that has used a similar approach is Kim, Kollmann

[4]In a recent working paper Arellano, Bai and Kehoe (2010) solve a model in which financial frictions and cyclical changes in the uncertainty at the firm level can generate a counter-cyclical labor wedge. In this model all the distortions will affect the relation between wages and aggregate productivity, while keeping the MRS equal to the wage. The two key elements of this model is that labor is predetermined and firms have the option to default once the uncertainty is resolved

[5]See Pissarides (2009) and Haefke, Sonntag and Van Rens(2007).

and Kim (2010)[6]. It is well known that models with heterogeneous agents and aggregate shocks are hard to solve because the entire history of shocks is relevant to the equilibrium allocation. The seminal paper by Krusell and Smith (1997) uses a bounded rationality algorithm assuming that agents only track a finite set of moments of the wealth distribution in order to forecast future prices, which are key objects in their optimization problem. Here I use a different approach. The solution technique is as follows, 1) A law of motion for aggregate state variables is proposed. 2) Based on this law of motion, each agent's problem is solved. 3) The policy rule of each individual is aggregated in order to get a policy function for aggregate variables. 4) The coefficients of the aggregate law of motions are updated until there is convergence. Section 5.1 explains the details of the solution algorithm.

3 Model Framework

There is a large number of consumers who share identical preferences over a final consumption good, restricted to be non-storable, and labor disutility. Each agent faces an idiosyncratic shock to labor productivity, ε, and chooses how much to consume, save and work each period. There are aggregate productivity shock Z, so an event $s \in R_{+} \times R_{+}$ is defined over the possible states of idiosyncratic and aggregate shocks (ε, z). A history s^t is the collection of all realizations up to period t.

3.1 Households

Household i maximizes its utility defined over preferences that are separable between consumption and working disutility[7]

$$\max_{\left\{c_t^i(s^t), h_t^i(s^t), a_{t+1}^i(s^t)\right\}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^i\left(s^t\right)^{1-\gamma}}{1-\gamma} - A \frac{h_t^i\left(s^t\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right)$$

Each household faces a budget constraint that dictates that consumption and investment have to be equal to the labor and capital income.

$$c_t^i\left(s^t\right) + a_{t+1}^i\left(s^t\right) = W_t\left(s^t\right) \varepsilon_t^i\left(s^t\right) h_t^i\left(s^t\right) + \left(R_t\left(s^t\right) + 1 - \delta\right) a_t^i\left(s^t\right)$$

[6] Den Haan (2010) compares the performance of different methods for solving models with idiosyncratic and aggregate shocks. The benchmark model of reference, though, is with exogenous labor supply.

[7] Hall (2009) discusses research regarding the consumption-hours complementarity. In general, the evidence seems to suggest that this complementarity is low, see Hall and Milgron (2008). Non-separable preferences will also lead to an aggregation wedge, but the separable case is the natural starting point as the macroeconomic literature has focused on this type of preferences to document the existence and properties of the labor wedge.

For the benchmark specification, financial markets are incomplete. Households can trade only physical capital and there is a borrowing constraint that forces agents to have non-negative wealth

$$a_{t+1}^{i}\left(s^{t}\right)\geq 0$$

Because financial markets are not perfect, the consumption, investment and hours worked allocation depends on the entire history of shocks, hence the distribution of wealth is a part of the set of state variables. The optimal conditions on the household side are summarized by the following first-order optimal equations:

$$c_{t}^{i}\left(s^{t}\right)^{-\gamma}=\lambda_{t}^{i}\left(s^{t}\right) \tag{1}$$

$$Ah_{t}^{i}\left(s^{t}\right)^{\frac{1}{\eta}}=W_{t}\left(s^{t}\right)\varepsilon_{t}^{i}\left(s^{t}\right)c_{t}^{i}\left(s^{t}\right)^{-\gamma} \tag{2}$$

$$c_{t}^{i}\left(s^{t}\right)^{-\gamma}=E_{t}\left[\beta c_{t}^{i}\left(s^{t},s_{t+1}\right)^{-\gamma}\left(R_{t+1}\left(s^{t},s_{t+1}\right)+1-\delta\right)\right]+\mu_{t}^{i}\left(s^{t}\right) \tag{3}$$

where $\lambda_{t}^{i}\left(s^{t}\right)$ and $\mu_{t}^{i}\left(s^{t}\right)$ are the Lagrange multipliers of the budget and the borrowing constraint, respectively. Note that the utility function is isoelastic, so the intertemporal elasticity of substitution $(1/\gamma)$ is constant, as well as the Frish-Elasticity of labor supply η. Equation (2) represents the labor-leisure condition and determines how many hours each household wants to supply given a prevailing wage and a level of wealth (marginal utility of consumption). This intra-temporal optimal condition holds period-by-period and in any state s. Equation (3) is the standard Euler equation and determines the inter-temporal savings decision. Agents who are not constrained by the borrowing limit will have the same marginal utility growth rate, which is determined by the return on physical capital. Constrained households would like to borrow against future labor income but have no other choice than to work more in order to smooth consumption.

3.2 Production

There is a representative firm that hires labor and rents capital from households and use them to produce the final good. The labor market and the rental market are competitive and the firm has a constant returns to scale technology, so there are no profits on this economy. The optimization problem of the firm can be break down into a sequence of statistic decisions described as follows:

$$\max_{\{K_{t}(s^{t}),H_{t}(s^{t})\}}Z_{t}\left(s^{t}\right)K_{t}\left(s^{t}\right)^{\alpha}H_{t}\left(s^{t}\right)^{1-\alpha}-W_{t}\left(s^{t}\right)H_{t}\left(s^{t}\right)-R_{t}\left(s^{t}\right)K_{t}\left(s^{t}\right)$$

The optimal demand for capital and labor are determined by the standard optimal conditions

$$
\begin{aligned}
MPL_t &= \alpha \frac{Y_t\left(s^t\right)}{H_t\left(s^t\right)} = W_t\left(s^t\right) \\
MPK_t &= (1-\alpha)\frac{Y_t\left(s^t\right)}{K_t\left(s^t\right)} = R_t\left(s^t\right)
\end{aligned}
$$

3.3 Stochastic Processes

I will follow the literature by assuming the following stochastic processes for both the idiosyncratic and aggregate productivity shocks

$$
\log\left(Z_{t+1}\right) = \rho_z \log\left(Z_t\right) + \epsilon_{zt+1} \tag{4}
$$

$$
\log\left(\varepsilon_{t+1}\right) = \rho_\varepsilon \log\left(\varepsilon_t\right) + \epsilon_{\varepsilon t+1} \tag{5}
$$

where $\epsilon_z \sim N\left(o, \sigma_z^2\right)$ and $\epsilon_z \sim N\left(o, \sigma_{\varepsilon t}^2\right)$. In Section 5.2, I will discuss the basic calibration for the parameters governing these two stochastic processes. For the moment, it is worth to mention that I will allow the variance of the idiosyncratic shock to be time-varying, with the purpose of generate cyclical changes in the distribution of household-specific shocks. More specifically, I will assume that the variance of idiosyncratic shocks gets larger during recessions. This assumption is based on empirical findings by Storesletten, Telmer and Yaron (2004) and it has been used in the finance literature[8]. The role of this cyclical changes in the variance of the household-specific shocks will be clear after I discuss the aggregation properties of the model.

3.4 Equilibrium

An equilibrium in this economy consists of an allocation over consumption, hours worked and assets for each household$\left\{\left[c_t^i\left(s^t\right), h_t^i\left(s^t\right), a_{t+1}^i\left(s^t\right)\right]_{i=1}^N\right\}_{t=0}^\infty$, and capital and labor demand for the representative firm$\left\{K_t\left(s^t\right), H_t\left(s^t\right)\right\}_{t=0}^\infty$, such that given prices $\left\{w_t\left(s^t\right), r_t\left(s^t\right)\right\}_{t=0}^\infty$,initial conditions and stochastic processes (4) and (5), this allocation is optimal, as defined by the first-order conditions before mentioned, and markets clear:

[8]Chien and Lustig (2010) use this fact to explain cyclical variations of the risk-premium. See also Storesletten, Telmer and Yaron (2007).

$$\begin{aligned}\sum_i a_t^i\left(s^t\right) &= K_t\left(s^t\right)\\ \sum_i h_t^i\left(s^t\right)\varepsilon_t^i\left(s^t\right) &= H_t\left(s^t\right)\end{aligned}$$

There is no analytical solution for this problem and the equilibrium allocation is nontrivial because the entire wealth distribution is part of the equilibrium. We will need to resort to a numerical approximation to this problem, but before it, let's analyze some of the aggregation issues related to this model

4 Aggregation

After aggregating over all households,the model implies a risk-sharing rule defined by the following expression:

$$\hat{c}_t^i\left(s^t\right) = \frac{c_t^i\left(s^t\right)}{C_t\left(s^t\right)} = \frac{\lambda_t^i\left(s^t\right)^{-1/\gamma}}{\sum_i \lambda_t^i\left(s^t\right)^{-1/\gamma}}$$

where $C_t\left(s^t\right) = \sum_i c_t^i\left(s^t\right)$. The dynamics of the lagrange multipliers, which can be thought as Pareto-Weights, are determined by the sequence of idiosyncratic and aggregate shocks and, for an specific household are governed by how many times the constraint has been binding. There is also a labor-sharing rule between any two households "i" and "j" determined by the ratio of the two labor-leisure conditions (2)

$$\frac{h_t^i\left(s^t\right)}{h_t^j\left(s^t\right)} = \left(\frac{\varepsilon_t^i\left(s^t\right)c_t^i\left(s^t\right)^{-\gamma}}{\varepsilon^j\left(s^t\right)c_t^j\left(s^t\right)^{-\gamma}}\right)^\eta \tag{6}$$

The relative supply of two households is a function of their relative productivity and their ratio of marginal utility of consumption. If markets were complete and households were not subject to idiosyncratic consumption risk, the ratio of marginal utilities should be constant, an without loss of generality equal to one. Then, only relative labor income shocks should matter for differences in labor supply. Values different from one for the Frisch Elasticity of Labor Supply, translates into non-linear responses in the relative supply of various households. This will have important implications for aggregate hours worked. Let's defined this variable as:

$$L_t\left(s^t\right) = \sum_i h_t^i\left(s^t\right)$$

If we aggregate the labor leisure condition of each agent, we end up with the following expression:

$$\sum_i Ah_t^i\left(s^t\right)^{\frac{1}{\eta}} = \sum_i W_t\left(s^t\right)\varepsilon_t^i\left(s^t\right)c_t^i\left(s^t\right)^{-\gamma} \tag{7}$$

This condition can be written in terms of aggregate variables, but there is imperfect aggregation for the aggregate labor-leisure condition: this expression will include an extra term spawn by the heterogeneity of the model:

$$\frac{AL_t\left(s^t\right)^{\frac{1}{\eta}}}{C_t\left(s^t\right)^{-\gamma}} = W_t\left(s^t\right)\left[\sum_i\left(\frac{\varepsilon_t^i\left(s^t\right)}{\hat{c}_t^i\left(s^t\right)^{\gamma}}\right)^{\eta}\right]^{\frac{1}{\eta}}$$

This extra-term shows that because agents are not perfectly insured against both consumption and leisure risk, the labor supply decision cannot be collapsed into the optimization problem of a single household. I will label this extra term the "Aggregation Wedge" and through the lens of the representative agent model it will be equal to the ratio of the MRS and the MPL:

$$\text{Aggregation Wedge} = \frac{MRS_t}{MPL_t} = \left[\sum_i\left(\frac{\varepsilon_t^i\left(s^t\right)}{\hat{c}_t^i\left(s^t\right)^{\gamma}}\right)^{\eta}\right]^{\frac{1}{\eta}}$$

This expression is defined by the $n-th$ moment of the joint distribution between the labor income idiosyncratic shock and the household-level consumption.

Note that there is a particular parameterization that will make the aggregation wedge disappear. When the IES is infinite $(\gamma = 0)$ and the Frish Elasticity is equal to one $(\eta = 1)$, the labor-leisure condition at the aggregate will resemble the optimal condition at the household level. This case corresponds to a quasi-linear specification of the utility function, in which the marginal utility of consumption is the same for all agents and the labor-leisure equation is linear. For this particular case, all agents share the same wealth effect and the aggregation problem dissappears[9].

There are several ways, yet isomorphic, in which this aggregation wedge can be interpreted from the perspective of the standard model. One is that this wedge can be mapped into the disutility of work parameter. Changes in the aggregation wedge will be interpreted as changes in preferences by the representative worker. A decrease in the aggregation wedge will be equivalent to a higher disutility of work and will be characterized as periods where the representative household is lazier. An alternative way to interpret this aggregation wedge is to map this term into taxes to the representative household. Movements in the aggregation wedge will be interpreted by an

[9]Different specifications for the household-speficit shocks can also lead to perfect aggregation result under different parameters of the utility function. For example, in the Krusell-Smith (1998) model, the idiosyncratic shocks, ε_t^i,are defined as employment status. In that case $L_t = \sum_i h_t^i \varepsilon_t^i$. For this case, it can be shown that assuming log preferences in both consumption and leisure, $\log(c_t) + A\log(1-h)$, will imply perfect aggregation regarding the labor-leisure condition up to a constant.

economist that ignores the underlying heterogeneity of the model as changes in labor taxes. Regardless of the interpretation, the central question is whether this aggregation wedge is such that it manifests as if the representative worker gets discourage to work during recessions. The next section discusses this issue.

4.1 The Aggregation Wedge

If both the distribution of wealth and household-specific productivity were invariant, the aggregation wedge will be unvarying too, and the aggregate labor-leisure condition will misspecified only up to a constant. In this model, however, the distribution of productivity changes along the cycle because the variance of the idiosyncratic component gets larger during recessions. Moreover, the borrowing constraint implies that the number of constrained households varies depending on the aggregate state of the economy. There is no analytical solution for the expression governing the labor wedge implied by aggregation. Nonetheless, we can learn about some of the properties of this wedge by analyzing two particular cases for which there is a closed-form expression for this term. For both complete markets and financial autarky, the consumption allocation doesn't depend on the entire history of shocks and consequently the aggregation wedge can be expresssed only in terms of the underlying distribution of productitivities[10].

4.1.1 Complete Markets - Perfect Risk Sharing

Under complete markets households will have acess to a complete set of state-contingent Arrow-Debreu securities and will be able to fully insure againts idiosyncratic shocks. In this case, the budget constraint of household i will be defined by:

$$c_t^i\left(s^t\right)+a_{t+1}^i\left(s^t\right)+\sum_{s_{t+1}} b_{t+1}^i\left(s^t,s_{t+1}\right)Q_t\left(s^t,s_{t+1}\right)=$$

$$W_t\left(s^t\right)\varepsilon_t^i\left(s^t\right)h_t^i\left(s^t\right)+\left(R_t\left(s^t\right)+1-\delta\right)a_t^i\left(s^t\right)+b_t^i\left(s^{t-1},s_t\right)$$

where $Q_t\left(s^t,s_{t+1}\right)$ is the price of a security that delivers one unit of consumption in state s_{t+1} and $b_{t+1}^i\left(s^t,s_{t+1}\right)$ is the quantity of the same security. Let's re-state the Aggregation Wedge (AW) and use the tax representation:

$$AW_t=(1-t_t)=\left[\sum_i\left(\frac{\varepsilon_t^i\left(s^t\right)}{\hat{c}_t^i\left(s^t\right)^{-\gamma}}\right)^{\eta}\right]^{\frac{1}{\eta}}$$

[10]Heathcote, Storesletten and Violante (2007) use a similar analysis to study the welfare implications of different market arrangements. They focus on steady-state welfare changes in a model in which the degree of insurability against idiosyncratic wage risk ranges also from complete markets to financial autarky.

where t_t mimics a labor tax. With this financial arrangement agents will be perfectly insure against specific shocks and $\hat{c}_t^i\left(s^t\right)$ will be constant over time (we can make it equal to one without loss of generality), hence, the aggregation wedge reduces to:

$$AW_t^{CM} = \left(1 - t_t^{CM}\right) = \left[\sum_i \varepsilon_t^{i\eta}\right]^{\frac{1}{\eta}}$$

Thanks to the assumption of log-normality of ε, this expression can be further simplified to:

$$t_t^{CM} = \frac{1-\eta}{2} V_t\left(\varepsilon\right)$$

which is a function only of the variance of the idiosyncratic productivity. Note that the commodity space of the model not only includes the final good but also each individual's hours worked (individual leisure). Prices for individual labor (leisure) are not identical, as household-specific productivity translates into different wages. In addition, agents don't consume each other's leisure. The absence of this market and the differences in wages explain the aggregation problem in the complete markets setup, even though there is no idiosyncratic consumption risk.

For a Frisch Elasticity parameter η higher than one, the tax-form representation of the aggregation wedge implies a negative tax (subside) for labor. This result is just a consequence of the labor-sharing rule. Under complete markets the high productive agents will work more relative to households with low productivity shocks. For values of η greater than one, the additional effort by highly productive workers will more than offset the fewer hours worked by unproductive agents. A higher variance of the productivity distribution, which implies more spreading of the possible realization of shocks, will translate into more hours worked at the aggregate level. In the complete-markets environment, the only relevant heterogeneity is on the dimension of household differences in productivity. Note that the tax form representation of the aggregation wedge is linear in the variance term. If recession are periods when there is more spreading of idiosyncratic shocks (larger variance), the aggregation wedge under complete markets will imply a pro-cyclical distortion.

4.1.2 Financial Autarky - No Risk Sharing

Under the assumption of financial autarky (no risk sharing) consumption is determined each period by labor income. The budget constraint reduces to:

$$c_t^i\left(s^t\right) = W_t\left(s^t\right) \varepsilon_t^i\left(s^t\right) h_t^i\left(s^t\right)$$

which implies that either agents have no assets at all or they keep their net wealth constant by having $a_{t+1}^i\left(s^t\right) = \left(R_t\left(s^t\right) + 1 - \delta\right) a_t^i\left(s^t\right)$ .Under this finantial structure, it is easy to show that the consumption (Pareto-Weight) of each household is just a function of its own labor productivity shock, $\hat{c}_t^i(s^t) = \varepsilon_t^i(s^t)^{(1+\eta)/(1+\eta\gamma)}$. High-productive agents will consume more and work less. The Labor Wedge implied by this financial arrangement is given by:

$$LW_t^{FA} = \left(1 - t_t^{FA}\right) = \left[\sum_i \varepsilon_{i,t}^{\frac{1+\eta}{1+\eta\gamma}}\right]^{\frac{1}{\eta}}$$

which under the assumption of log-normality reduces to:

$$\tau_t^{FA} = \frac{(1-\gamma)}{1+\eta\gamma}\tau_t^c$$

From this expression we can see that if agents have log preferences in consumption there is no wedge for the financial autarky case. For the log case, the income and substitution effect of changes in wages cancel out perfectly. Highly productive agents will have more incentives to work, but the income effect will make them to work the same amount of hours. This will be also the case for low-productive workes. Hence, the heterogeneity in labor productivity doesn't translate into heterogeneity in labor supply. Moreover, the labor supply is completely inelastic and aggregation is perfect. The number of hours worked is the same in good and bad times.

With a lower intertemporal elasticity of substitution ($1/\gamma < 1$) there will be a positive labor tax for the representative worker as a result of the aggregation wedge. The total elasticity of labor respect to wages turns negative, $\eta(1-\gamma)/(1+\eta\gamma)$, so low-productive agents work more than high productive ones. At the aggregate level, this breaks the connection between wages and hours supply. The income effect is stronger than the subtitution effect.

Relative to the complete markets case, financial autarky has the opposite effect for aggregate labor supply. In the complete markets case, in which all households have the same marginal utility of consumption, higher productivity draws are associated with more hours worked. In the financial autarky case, incentives are reversed: low-productive households will work more and consume less. The distribution of hours worked under financial autarky shifts to the left, relative to the distribution under complete markets. At the aggregate level, average hours worked are lower for the financial autarky case, relative to complete markets, as if labor taxes for the representative worker were higher.

4.1.3 Borrowing Constraint - Partial Risk Sharing

As I mentioned earlier, it is not possible to find an analytically an expression for the aggregation wedge when there is some, but not perfect, risk sharing. Nonetheless, we know that the borrowing constraint in the benchmark model implies that some agents cannot smooth consumption unless they work harder, as in the financial autarky case. During bad times, the amount of agents at the constraint will increase and these agents will have no other alternative that work more. Even in times when wages are depressed, households at the constraint will work the same, or even more. For these agents, the implied labor supply elasticity is zero or negative. On contrary, agents that are not at the constraint will act as if they were more risk averse. They will accumulate more assets due to a precautionary savings motive. Higher asset holdings by unconstrained households will translate into a lower income effect in periods of depressed wages. The substitution effect will be more important relative to the income effect, and unconstrained households will respond stronger to lower wages by cutting more their labor supply. This feature will help the model to deliver a counter-cyclical tax wedge, as the representative agent gets discourage to work during bad times. Transiting from an allocation with more risk sharing, such as complete markets, in which the aggregate representation implies a negative tax, towards one with less (or little) risk sharing, as the financial autarky case, will manifest in a reduced-form model as an increase in labor taxes.

5 Quantitative Analysis

This section discusses the quantitative properties of the model. I present first the solution method, then the benchmark calibration and I finish the section with the impulse-response functions and basic second moment statistics of the model.

5.1 Solution Method

The solution method I use is a combination of direct aggregation with perturbation methods. Perturbation methods cannot be used when dealing with occasionally binding constraints as the borrowing constraint of the model. Hence, as a first step we have to replace the borrowing constraint for a penalty function. The penalty function penalizes agents that are close to the borrowing constraint, preventing them from borrowing. Rotemberg and Woodford (1999) uses a penalty function in the context of a model on monetary policy rules. This approach has been recently used to solve models with heterogeneous agents: see Preston and Roca (2007). The penalty function I use here is based on Kim, Kollmann and Kim (2010). I modify the utility function to incorporate a new term that will prevent agents to hold negative wealth. The new instant utility function is:

$$u\left(c_t^i\left(s^t\right), h_t^i\left(s^t\right), a_{t+1}^i\left(s^t\right)\right) = \frac{c_t^i\left(s^t\right)^{1-\gamma}}{1-\gamma} - A\frac{h_t^i\left(s^t\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \phi\left[\log\left(\frac{a_{t+1}^i\left(s^t\right)}{\bar{a}}\right) - \frac{a_{t+1}^i\left(s^t\right)-\bar{a}}{\bar{a}}\right]$$

where $\bar{a}$ is the asset holdings of a household in the non-stochastic steady state. This modification doesn't alter the steady-state of the model so the choose of ϕ, the "barrier parameter", doesn't affect any of the targeted moments of the model in steady state. Note that when asset holdings approach zero, the utility of agents tends to be infinitely negative. In the Euler equation, the penalty function causes the marginal utility of saving to be quite large (potentially infinite) when capital holdings are close to the zero bound. This two features emulate the effect of the borrowing constraint and can be rationalized as if credit becomes too costly for people who are close to have zero net wealth.

After replacing the occasionally binding borrowing constraint with the penalty function, the optimal first-order conditions for the household can be redefined as:

$$Ah_t^i\left(s^t\right)^{\frac{1}{\eta}} = W_t\left(s^t\right)\varepsilon_t^i\left(s^t\right)c_t^i\left(s^t\right)^{-\gamma}$$

$$c_t^i\left(s^t\right)^{-\gamma} = E_t\left[\beta c_t^i\left(s^t, s_{t+1}\right)^{-\gamma}\left(R_{t+1}\left(s^t, s_{t+1}\right)+1-\delta\right)\right] + \phi\left[\frac{1}{a_{t+1}^i\left(s^t\right)} - \frac{1}{\bar{a}}\right]$$

The last equation helps to pin down the asset level of any agent. This penalty function not only allows to replace the borrowing constraint but also helps to induce stationarity into the model, as adjustment costs functions to asset holdings do for international models. Other penalty functions are suitable for replacing the borrowing constraint but leave unadressed the problem of stationarity. The specification here used serves both.

Thanks to the use of the penalty function, the household problem can be approximated around a non-stochastic steady state, using perturbation methods. Note that the approximation will be around a degenerate wealth distribution in which all agents hold the same amount of assets $\bar{a}$ in steady state. This level will be equal to the amount of aggregate capital, which is determined, as in the standard model, by the discount factor, the depreciation rate and the curvature of the production function.

A first-order approximation to each household's problem will be not enough to capture the aggregation problem discussed in the previous section. Imposing linearity in the intra-temporal optimal condition will guarantee that there will be perfect aggregation. The Aggregation Wedge is precisely result of the non-linear specification of the labor-leisure. We will need to resort to a second order approximation. For the second-order approximation, the decision rules of each individual depends on all the aggregate and household-specific state variables and their cross moments.The policy function for asset holdings is of the following form

$$
\begin{aligned}
a_{t+1}^{i} \quad = \quad & \theta_0 + \theta_1 a_t^i + \theta_2 \varepsilon_t^i + \theta_3 Z_t + \theta_4 K_t + \theta_5 M_{aet} \\
& + \theta_6 M_{a_t^2} + \theta_7 e_t^i + \theta_8 e_t^{iz} + \theta_9 a_t^{i2} + \theta_{10} a_t^i \varepsilon_t^i \\
& + \theta_{11} \varepsilon_t^{i2} + \theta_{12} Z_t a_t^i + \theta_{13} z_t \varepsilon_t^i + \theta_{14} Z_t^2 + \theta_{15} K_t a_t^i \\
& + \theta_{16} K_t \varepsilon_t^i + \theta_{17} K_t Z_t + \theta_{18} K_t^2 + \theta_{19} a_t^i M_{aet} \\
& + \theta_{20} \varepsilon_t^i M_{aet} + \theta_{21} Z_t M_{aet} + \theta K_t M_{aet} + \theta_{23} M_{aet}^2 \\
& + \theta_{24} a_t^i M_{at^2} + \theta_{25} \varepsilon_t^i M_{a^2} + \theta_{26} Z_t M_{at^2} + \theta_{27} K_t M_{at^2} \\
& + \theta_{28} M_{aet} M_{at^2} + \theta_{29} M_{at^2}^2 + \theta_{30} e_t^{i2} + \theta_{31} e_t^i e_t^{iz} \\
& + \theta_{32} e_t^{iz2} + \theta_{33} a_t^i e_t^i + \theta_{34} a_t^i e_t^{iz} + \theta_{35} \varepsilon_t^i e_t^i \\
& + \theta_{36} \varepsilon_t^i e_t^{iz} + \theta_{37} Z_t e_t^i + \theta_{38} Z_t e_t^{iz} + \theta_{39} K_t e_t^i \\
& + \theta_{40} K_t e_t^{iz} + \theta_{41} M_{aet} e_t^i + \theta_{42} M_{aet} e_t^{iz} + \theta_{43} M_{at^2} e_t^i \\
& + \theta_{44} M_{at^2} e_t^{iz}
\end{aligned}
$$

which in short notation, I will write as:

$$a_{t+1}^{i} = F\left(a_{i,t}, e_{i,t}, e_{i,t}^{z}, \varepsilon_{i,t}, Z_t, K_t, M_{aet}, M_{a_t^2}, \Theta\right)$$

The state vector of each household's decision rule includes idiosyncratic variables $\left\{a_t^i, e_t^i, e_t^{iz}, \varepsilon_t^i\right\}$ and the vector of aggregate variables $\{Z_t, K_t\}$ and cross-sectional moments $\left\{M_{aet}, M_{a_t^2}\right\}$,where

$$M_{aet} = \sum_i a_t^i \varepsilon_t^i$$

is the first moment of the joint distribution of asset holdings and

$$M_{a_t^2} = \sum_i a_t^{i2}$$

is the cross-sectional variance of assets.

Given that each household's decision rule includes a cross-term of assets and labor productivity, once we aggregate all the decision rules, we need to keep track of the cross-sectional moments of asset holdings and idiosyncratic shocks. After summing over all the individual policy functions for next period's capital, we end up with an expression for the law of motion of aggregate capital as follows:

$$
\begin{aligned}
K_{t+1} \quad = \quad & \pi_{0,K} + \pi_{1,K} K_t + \pi_{2,K} Z_t + \pi_{3,K} M_{aet} + \pi_{4,K} M_{a^2 t} \\
& + \pi_{5,K} K_t^2 + \pi_{6,K} Z_t^2 + \pi_{7,K} K_t Z_t + \pi_{8,K} K_t M_{ae,t} + \pi_{9,K} K_t M_{a^2 t} \\
& + \pi_{10,K} Z_t M_{aet} + \pi_{11,K} Z_t M_{a^2 t} + \pi_{12,K} M_{aet} M_{a^2} + \pi_{13,K} M_{aet}^2 + \pi_{14,K} M_{a^2 t}^2
\end{aligned}
$$

which in short notation, I write as:

$$
K_{t+1} = \digamma_K \left(Z_t, K_t, M_{aet}, M_{a_t^2}, \Pi \right)
$$

The law of motion for aggregate capital only depends on aggregates and the cross-sectional moments. All the idiosyncratic terms wash out. The same procedure can be done for other variables. Obviously, in order to solve each household's problem, we need a law of motion for aggregate variables, as prices are a key element to the individual optimization problem. The solution procedure, then, follows these steps:

1. We guess a law of motion for aggregate variables:

$$
K_{t+1} = \digamma_K \left(Z_t, K_t, M_{aet}, M_{a_t^2}, \Pi^1 \right)
$$

$$
M_{aet+1} = \digamma_{M_{ae}} \left(Z_t, K_t, M_{aet}, M_{a_t^2}, \Pi^1 \right)
$$

$$
M_{a_{t+1}^2} = \digamma_{M_{a^2}} \left(Z_t, K_t, M_{aet}, M_{a_t^2}, \Pi^1 \right)
$$

$$
H_t = \digamma_H \left(Z_t, K_t, M_{aet}, M_{a_t^2}, \Pi^1 \right)
$$

$$
L_t = \digamma_L \left(Z_t, K_t, M_{aet}, M_{a_t^2}, \Pi^1 \right)
$$

2. Based on these law of motion, we find an optimal decision rule for each household

$$
a_{t+1}^i = \digamma \left(a_{i,t}, e_{i,t}, e_{i,t}^{z}, \varepsilon_{i,t}, Z_t, K_t, M_{aet}, M_{a_t^2}, \Theta^1 \right)
$$

$$
h_t^i = \digamma \left(a_{i,t}, e_{i,t}, e_{i,t}^{z}, \varepsilon_{i,t}, Z_t, K_t, M_{aet}, M_{a_t^2}, \Theta^1 \right)
$$

3. Aggregation of the individual-level policy functions imply a new law of motion for the aggregates:

$$
\sum_i a_{t+1}^i = K_{t+1} = \digamma_K \left(Z_t, K_t, M_{aet}, M_{a_t^2}, \Pi^2 \right)
$$

$$\sum_i a_{t+1}^i \varepsilon_{t+1}^i = M_{aet} = \digamma_{M_{ae}}\left(Z_t, K_t, M_{aet}, M_{a_t^2}, \Pi^2\right)$$

$$\sum_i a_{t+1}^{i2} = M_{a_{t+1}^2} = \digamma_{M_{a^2}}\left(Z_t, K_t, M_{aet}, M_{a_t^2}, \Pi^2\right)$$

$$\begin{aligned}\sum_i h_t^i \varepsilon_t^i &= H_t = \digamma_H\left(Z_t, K_t, M_{aet}, M_{a_t^2}, \Pi^2\right) \\ \sum_i h_t^i &= L_t = \digamma_L\left(Z_t, K_t, M_{aet}, M_{a_t^2}, \Pi^2\right)\end{aligned}$$

The set of coefficients Π^1 is updated based on Θ^1 and transforms into Π^2. From here, one can iterate until coefficients converge $(\Pi = \Theta)$[11].

Once the aggregate law of motion variable is found, we can simulate the aggregate variables which will capture the role of heterogeneity in the model through the dynamics of the cross sectional moments of idiosyncratic productivities and asset holdings.

5.2 Basic calibration

I borrow most parameter values from the RBC literature. The baseline parameterization includes a discount factor β of 0.98, a depreciation rate δ of 10% in annual terms, a curvature α in the production function of 1/3, and a disutility of work A that will induce agents to work one-third of their available time in the steady-state. The intertemporal elasticity of substitution is one, so agents have log preferences in consumption and the model is consistent with balanced growth path. I set the Frisch labor supply elasticity parameter η to 1.5. There is a tension between the elasticities required by macroeconomic models in order to match the volatility of hours and the empirical evidence regarding estimates of the Frisch Elasticity at the micro level. Macroeconomic models often rely on elasticities well above one, in the range 2-4. The seminal paper my MaCurdy (1981) suggests elasticities of labor supply for males between 0 and 0.4 and a more recent estimate by Pistaferri (2003), using data for italian households, suggest a number around 0.7. Rogerson and Wallenius (2009) discuss a life-cycle model in which household-level elasticities are low but the implied elasticities for aggregates are much higher. Our benchmark number is higher than the empirical level estimates and below the standard number in the macroeconomic literature. The benchmark value of 1.5 seems consistent with estimates by Domeij and Floden (2006) that show that micro-level empirical estimates could be downard bias because of the presence of borrowing constraints. Most empircal studies assume that the Euler equation of each household holds

[11] See the appendix for a detail description in the updating procedure

withuot restrictions. Domeij and Floden show that constrained househols have a distorted Euler equation and that their labor elasticity should be close to zero, or could be negative. This is the case of this model[12]. Allthough, the Frisch labor elasticity is greater than one, some agents of this economy will exhibit no response in hours to changes in wages, when they are financially constrained. Furthermore, it is clear that the models misses other important margins such as the extensive margin of labor or career-lenght decisions that contribute to have a higher elasticity of labor at the aggregate level.

For the calibration of the penalty function parameter, I follow the approach of Preston and Roca (2007) and set a parameter value that guarantees that no agent violates the constraint. The value of ϕ, which governs the cost of being close to the borrowing constraint is chosen to be 0.1, in the middle range of the values used by Kim, Kollmann and Kim (2010). I experiment with other parameter values but the main results are robust to these changes.

Regarding the stochastic processes of both the aggregate productivity shock and the idiosyncratic shocks, I follow Kydland and Prescott (1982) for the former, and use the empirical findings by Storesletten, Telmer and Yaron (2004) for the latter. The persistence parameter for the two processes is set to 0.95. The variance of idiosyncratic shocks is choose to be 0.12 and to reach a peak of 0.21 when the economy get hits by a large negative shock (See the Appendix for details). The next table summarizes all the parameter values use in the benchmark calibration.

Parameter	Description	Value
β	Discount Factor	0.98
δ	Depreciation Rate	0.025
A	To Target h=1/3	2.46
γ	Risk Aversion	1
η	Elasticity Labor Supply	1.5
ϕ	Penalty Function	0.1
ρ_{ε}	Agent-specific shock persistence	0.95
σ_{ε}	Agent-specific shock stdv	0.12
ρ_z	Aggregate shock persistence	0.95
σ_z	Aggregate shock stdv	0.007

5.3 Impulse-Response Functions

I compute the impulse-response of the model for a one-standard deviation negative shock of aggregate productivity. I will compare the model with the standard representative RBC model in

[12] I estimate the Frisch Elasticity of Labor Supply using a simulation of the model at the household level. The findings are in line with the main argument of Domeij and Floden (2006); household level estimates are downward bias.

order to highlight the role of heterogeneity for the dynamics of aggregate variables. I use the same parameter values for the RBC and also solve a second-order approximation.

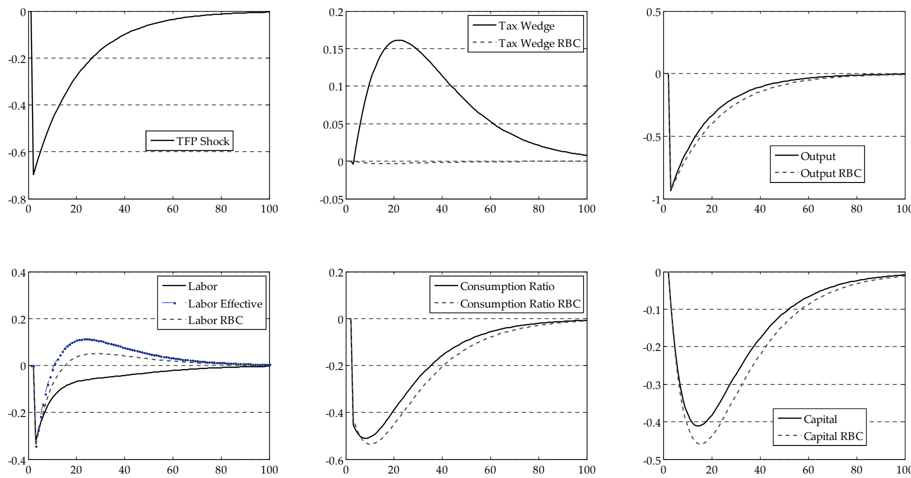

Figure 1. Impulse Response Functions

The dynamics of aggregate variables in the two model are close enough with the exception of the response in aggregate hours. The Heterogeneous Agent (HA) model delivers a counter-cyclical tax wedge, absent in the standard model, which manifests in a long and protected recovery of labor, relative to benchmark RBC. As I mentioned in Section 4, the "Tax Labor Wedge" (TLW) is result of the aggregation problem and it is defined as the one minus the ratio of the Marginal Rate of Substitution between Consumption and Hours Worked and the Marginal Product of Labor, all defined for aggregate variables:

$$\text{Tax Labor Wedge} = 1 - \frac{MRS}{MPL}$$

The difference in dynamics of aggregate hours is explained by the various labor supply elasticities induced by the distribution of productivity, the wealth distribution and the borrowing constraint. Upon shock, aggregate hours in the HA model fall less than in the benchmark RBC, as more people hit the borrowing constraint. Constrained households work the same, despite lower wages, in order to smooth consumption. This effect cushions the drop in aggregate hours worked at the moment of the shock. The recovery, however, is longer and weaker for the HA model as unconstrained households, who have accumulated wealth because of precautionary motives have a weaker income effect. People that are not at the constraint accumulate more assets than they would do in a complete insurance model, as if they were more risk averse. This leads to a stronger substitution effect and a weaker income effect that keeps labor subdued for a longer period of time. Therefore, relative to benchmark model, labor remains below trend until output fully recovers.

In general, the RBC exhibits higher volatility so all aggregate quantities respond stronger to the negative shock. This is mainly because this economy has no constraints.

The next table presents statistics regarding the second moments of the cyclical components[13] for aggregate variables of the two models and the data[14]

Table I. Second Moments

	Data	RBC	HA Model
σ_Y	1.34	1.34	1.22
σ_C/σ_Y	0.57	0.41	0.54
σ_K/σ_Y	0.10	0.29	0.29
σ_L/σ_Y	0.98	0.52	0.32
σ_H/σ_Y	-	-	0.37
$corr\,(C,Y)$	0.99	0.69	0.95
$corr\,(K,Y)$	0.63	0.35	0.46
$corr\,(L,Y)$	0.74	0.93	0.98
$corr\,(H,Y)$	-	-	0.90
$corr\,(TLW,L)$	-0.89	0.00	-0.66
$corr\,(TLW,Y)$	-0.65	0.00	-0.50

The second moments confirm the results from the impulse-response functions. Overall, the HA model exhibits lower output volatility, as a result of the borrowing constraint. In terms of relative volatilities the two models deliver similar numbers. Physical capital has the same volatility relative to output, and for both models labor is much less volatile than in the data[15]. Our benchmark parameterization has a Frisch –Elasticity for labor supply relative small compare to other macro-models. This explains the failure of the two models in reproducing the high volatility of hours worked observed in the data. I also report the volatility of aggregate effective units of la-

[13] All series are detrended using the HP filter with a parameter of 1,600

[14] Quaterly data for the US economy during the period 1950:Q1 to 2009:Q2. See Appendix for the description of the data used

[15] The raw of volatiliy of labor in the HA model is higher than in the RBC, as it can be seen from the impulse response functions. The fact that in relative terms is lower is explained by the use of the HP filter. The HP filter reduces the volatility of output of the RBC and it distorts the relative volatility numbers.

bor. Notice that this variable is more volatility than hours worked because of unresponsiveness of low-productive households who in effective units weight less for the aggregate. The model does a better job in terms of consumption volatility, relative to output. Aggregate consumption is more volatility in the HA model, given the restrictions of insurance for some households, whose consumption processes track more closely their income processes. At the aggregate level, this causes less consumption smoothing over the cycle. The correlation between consumption and output is 0.95, closer to the 0.99 observed in the data, improving upon the 0.69 predicted by the RBC. Other correlations are roughly similar between the two models. A key result is that the HA model predicts a labor wedge that is counter-cyclical, with a correlation between the tax wedge and hours worked and output of -0.66 and -0.5, respectively.

6 A Direct Measure of the Aggregation Wedge

Thanks to the assumption of log-normality, we can find an expression for the aggregation wedge, without assuming a particular financial structure. In this section, I use this expression to test the hypothesis that the aggregation wedge is important for reconciling the model with the data. Without relying on any assumptions about the risk sharing opportunities in the economy, the aggregation wedge can be expressed as:

$$\tau_t = \frac{1}{2}\left[(1-\eta)\, V_t\left(\varepsilon\right) - \gamma\left(1+\eta\gamma\right) V_t\left(c^i\right) + 2\eta\gamma cov\left(\varepsilon^i, c^i\right)\right]$$

This expression is a function of the cross-sectional variance of wages, the cross-sectional variance of consumption, and the covariance between wages and consumption at the household level and only relies on the distributional assumption of the individual productivity shocks. Recall that both the Complete Markets and the Financial Autarky aggregation wedges were functions only of the cross-sectional variance of wages. I will use this expression to calculate the aggregation tax wedge using household survey data for both wages and consumption and test whether this aggregation wedge is related to the Labor Wedge estimated from aggregate data. For this calculation I will make use of the same parameter values discussed in the calibration section. Before discussing the results, let me describe the data used for this purpose.

6.1 Empirical Exercise- Data

6.1.1 Real Wages

For the cross-sectional variance of wages I use the Current Population Survey (CPS) data. This data is annual and starts in 1967. I will use data at the household level of wages of males (head of the household of families in which there are either one or two adults age 25-60). The main reference for data is Heathcote et al (2010).

I will use the raw data but also do the same exercise after controlling for demographic variables following Heathcote et al. (2010). The control variables for these demographics are: 3 race dummies (white-white, non-white-non-white, mixed race). 2 sex dummies (male-female, male-male). 4 education dummies (college-collge, college-noncollege, noncollege-college, noncollege-noncollege), average years of education for all adults, quadratic age for the head, quadratic age for the non-head, and the number of household members below 25 and above 60[16].

6.1.2 Consumption

The most comprehensive data set for consumption at the household level is the Consumption Expenditure Survey (CEX), available since 1980. I will calculate from here, the cross-sectional variance of non-durable consumption and the correlation between consumption and wages for each household on the survey. Calculations are made for the household sample and consumption don't include expenditures in services and durables.

Figure 2 present the time series of the cross-sectional variances of wages and consumption. The most evident pattern is the sustain increase in the level of inequality both before an after controlling for the demographic variables. There is also an increase in consumption inequality, which has sharpen in the last few years.

As Figure 2 shows, in addition to the upward trend, the series show some cyclical changes which confirm that recessions are associated to periods of increasing inequality. For the years associated with depressed economy activity, 1982, 1991 and 2001,the CPS data for wages show accelerations in the growth rate of wage inequality. The consumption series exhibit lower cyclical variations, in what seems a reflection that there is some risk sharing in the economy, but still shows some spikes in inequality around the 1982-1983 recession, the 1991 downturn and the post-2001 period. Using these series and the correlation between wages and consumption and the household level from the CEX survey, I calculate the expression for the aggregation wedge for the US since 1980. I compare it to the "Labor Wedge" as measured using aggregate data and show that the two are related[17].

[16]For the long-run analysis the two series mostly differ in the level. Cyclical changes are quite the same.

[17]I explain in the Appendix how I compute the labor wedge from aggregate series.

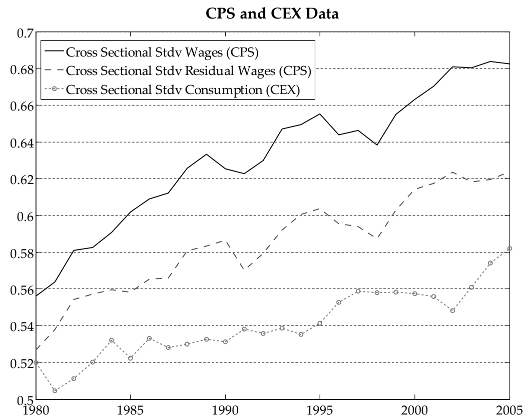

Figure 2. Cross Sectional Standard Deviation of
Wages and Consumption

Figure 3 presents the Aggregation Wedge and the implied "Labor Wedge" from Aggregate Data. The two have been normalized to have the same starting value in 1980. Both series show a sustain decline in the tax wedge as if labor taxes dropped during the entire period. For the aggregation wedge, the fall is mostly explained by the sustain increase in labor income inequality.

The right panel of Figure 4 shows the relationship between the two series at the business cycle frequency, after de-trending using the Hodrick-Prescott filter[18]. The two series are positively correlated (for this particular calibration the correlation is 0.3). Moreover, the Aggregation Wedge is negatively correlated with the cyclical component of output (with a correlation of -0.2) and with aggregate labor (-0.3), exhibiting the same counter-cyclical behavior as the tax labor wedge observed in the data. I find this result to support the theory that the observed wedge at the aggregate level can be explained up to a degree by an aggregation issue.

[18]This graph is based on a HP-parameter of 100. I also experiment with 6.25 and some intermediate values. Results are robust to the various especifications

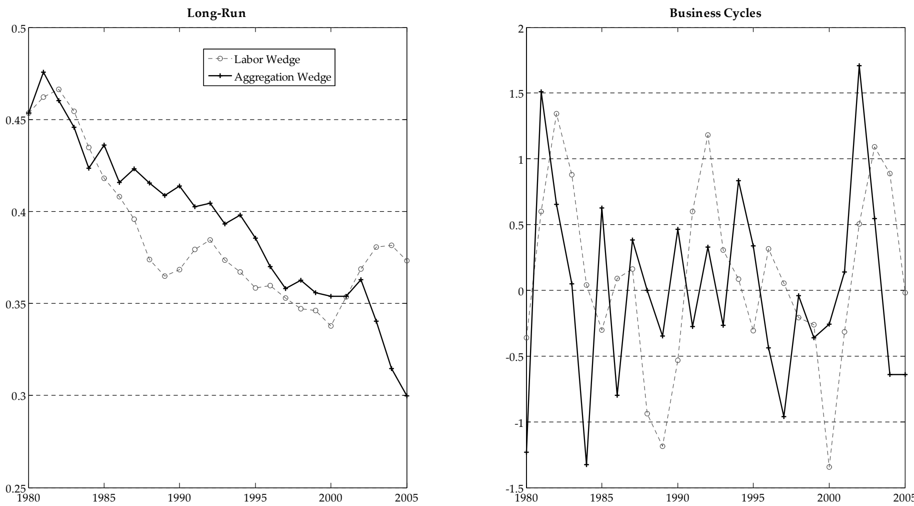

Figure 3. The Labor Wedge and the Aggregation Wedge (Long Run and Business Cycles)

7 Conclusions

In this paper I show that there is an aggregation problem regarding the labor supply decision of households that are subject to idiosyncratic labor productivity shocks. The extra term arising from the aggregation problem can be mapped into labor taxes or the disutility of working parameter of the representative agent. I show under which conditions, this extra term can explain the systematic deviation between the Marginal Rate of Substitution between consumption and leisure and the Marginal Product of Labor, through the lens of the neoclassical growth model. I show that a symple model with a borrowing constraint can replicate the counter-cyclical properties of the labor wedge and deliver novel dynamics for aggregate hours worked. The key factor behind this finding is the various labor supply responses induced by the household specific productivity and the wealth distribution. The model exhibit a jobless recovery as if labor taxes were higher during a recession. This is explained by the weaker income effect of households that have accumulated wealth for precautionary motives.

Finally, I test the predictions of the model by calculating directly the deviation between the MRS and the wage implied by the aggregation condition, using household micro-level data for the US. I find this calculation to be positively correlated with the "Labor Wedge" implied by the aggregate data.

An important assumption here used is that the labor-leisure condition at the household level holds without distortions. In fact, I assume that all the dispersion in wages is explained by differences in productivity. Arguably this is not the case. However, the same framework here proposed can shed some light on the aggregate implications of distortions at the household level. Most of the analysis on the aggregation wedge extends to a case where instead idiosyncratic productivity shocks, ε^i, there are idiosyncratic distortions: $\left(1-\tau^i\right)$ [19]. Future research should try to idenfity a mapping between distortions at the micro and macro levels.

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[19] The aggregation wedge for this case will be:

$$\left[\sum_i \left(\frac{1-t_t^i}{\hat{c}_t^{i\gamma}}\right)^{\eta}\right]^{\frac{1}{\eta}}$$

The challange to this type of specification will be how to measure and idenfity the household-level distortions.

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8 Appendix

8.1 Documenting the Labor Wedge

The basic efficient condition of most macroeconomic models with endogenous labor supply states that households will supply labor up to the point that the MRS is equal to the prevailing wage. The analogous condition on the production side stays that, when operating at optimal level, firms should hire workers until the marginal product of labor is equal to wages. In a frictionless environment, the MRS should be equal to the MPL. Previous papers have confirmed that this condition fails to hold at the aggregate level and there is a wedge between these two. Here, I decompose this wedge into two parts just to test how much of it is coming from distortions on the household side and how much can be attributed to distortions on the production side. Under the assumption of separable preferences of the form:

$$\log\left(C_t\right) - A\frac{H_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}$$

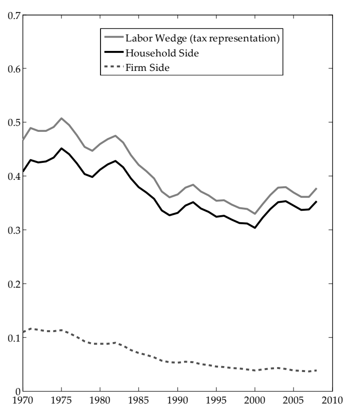
and assuming a constant returns to scale technology, the efficient condition can be written as:

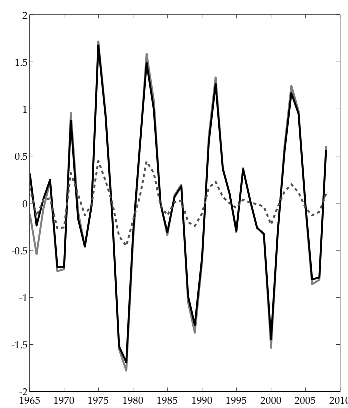
$$MRS_t = \frac{AH_t^{\frac{1}{\eta}}}{C_t} = W_t = \left(1-\alpha\right)\frac{Y_t}{H_t} = MPL_t$$

The ratio of the MRS and the MPL can be expressed as:

$$LW_t = (1 - \tau_t) = \frac{MRS_t}{MPL_t} = \frac{MRS_t}{W_t} \frac{W_t}{MPL_t} = \frac{\left(1 - \tau_t^{HH}\right)}{\left(1 + \tau_t^{FF}\right)}$$

where τ_t^{HH} represents the distortion to households and τ_t^{FF} the distortions to the production side. Most representative agent models deliver the prediction that MRS should be equal to wages ($\tau_t^{HH} = 0$). Labor market imperfections map into distortions between wages and the marginal product of labor. Using aggregate data, I calculate the total labor wedge in its tax representation, τ_t, and τ_t^{HH} as well as τ_t^{HH}. For this calculation I need aggregate data on the consumption-output ratio, hours worked, and wages. The next graph shows the result for both the long-run and the business cycle. I use the same parameter values than in the benchmark calibration of the paper ($\eta = 1.5$ and $\alpha = 1/3$). The panel of the left shows the decomposition of the labor wedge (as a tax) in levels, while the panel of the right shows the cyclycal components.





The picture shows that most of the labor wedge, both the level and the cyclical fluctuations, is explained by a distortion to the optimal labor supply decision of the representative household.

8.2 Data

For calculating the Labor Wedge I follow Shimer (2009, 2010) and use annual data on aggregate consumption, output and hours worked for the 1965-2005 period. These aggregate series are also used for calculating the RBC statistics but at quaterly frequency and for the 1950:Q1- 2009:Q1 period. The source is the NIPA Accounts:

(http://www.bea.gov/national/nipaweb/SelectTable.asp?Selected=Y).

Table 1.1.5 GDP and Expenditure Side Components measure at current dollars (billions of dollars) and seasonally adjusted and Table 1.1.4 Price Indices. I use the Price Indices to deflate each specific expenditure component of the GDP. Consumption of durables is classified as investment. I don't' use the chain-weighted GDP numbers, which provide disaggregate numbers for each category only after 1995. For the decomposition of the labor wedge on the household and firm side, I use data on wages from the CES (CES0500000008) from the BLS, available since 1964. I also did the calculation using the sama dataset I use at the micro-level, the CPS, for the average wage of males (head of households). Conclusions don't change when using this alternative measure. The series for hours worked is from Cociuba, Prescott and Ueberfeldt (2009). http://sites.google.com/site/alexanderueberfeldt/research. This includes total hours worked of the noninstitutional population with ages between 16 and 64, quartely since 1959.

For the household level data, the main reference is Heathcote et al (2010). Their paper and the dataset can be access at: http://ideas.repec.org/c/red/ccodes/09-214.html

8.3 Solution Method

One of the problems that the described solution has to circumvent is the fact that in order to a get law of motion for the cross sectional moments one should have an infinite number of moments, whenever the approximation is higher than a first-order. For instance, in order to get a direct expression for $\sum_i a_t^{i2}$ if one takes the square of a_t^i, the term a_t^{i4} will appear, and a separate law of motion for $M_{a_t^4}$ will be required. In order to avoid this problem, I follow Den Haan and Ocaktan (2010) which proposes an approximation for $M_{a_t^2}$ based on the same arguments of the approximation for K_t. The approximation is described by

$$M_{a_t^2} = F_{M_{a^2}}\left(Z_t, K_t, M_{aet}, M_{a_t^2}, \Pi^1\right)$$

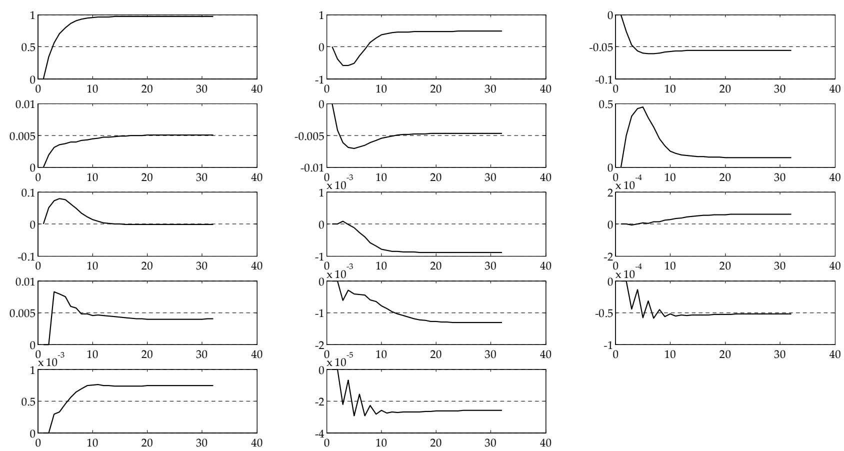
In addition, in order to find the law of motion for the covariance term between assets and idiosyncratic shocks one can start by finding at the household level the coefficients for $a_{t+1}^i\tilde{\varepsilon}_{t+1}^i$, where $\tilde{\varepsilon}_{t+1}^i = \log\left(\varepsilon_{t+1}^i\right)$. Then using the fact that $\tilde{\varepsilon}_{t+1}^i = \rho_\varepsilon\tilde{\varepsilon}_t^i + \epsilon_{\varepsilon t+1}$, it is possible to re-writte $a_{t+1}^i\tilde{\varepsilon}_{t+1}^i$ as

$$a_{t+1}^i\left(\rho_\varepsilon\tilde{\varepsilon}_t^i + \epsilon_\varepsilon\right) = \rho_\varepsilon a_{t+1}^i\tilde{\varepsilon}_t^i + a_{t+1}^i\epsilon_\varepsilon$$

Then, by constructing a law of motion for $a_{t+1}^i\tilde{\varepsilon}_t^i$, one can back up the law of motion for $a_{t+1}^i\tilde{\varepsilon}_{t+1}^i$. For this purpose I re-write the model in terms of $\tilde{\varepsilon}_{t.}^i$ The other law of motions can be calculated directly from aggregation. The next graph presents the iteration and convergence of the coefficients related to the law of motion of aggregate capital. Recall that the equation that describes the law of motion for aggregate capital is

$$
\begin{aligned}
K_{t+1} \quad = \quad & \pi_{0,K} + \pi_{1,K} K_t + \pi_{2,K} Z_t + \pi_{3,K} M_{aet} + \pi_{4,K} M_{a^2t} \\
& + \pi_{5,K} K_t^2 + \pi_{6,K} Z_t^2 + \pi_{7,K} K_t Z_t + \pi_{8,K} K_t M_{ae,t} + \pi_{9,K} K_t M_{a^2t} \\
& + \pi_{10,K} Z_t M_{aet} + \pi_{11,K} Z_t M_{a^2t} + \pi_{12,K} M_{aet} M_{a^2} + \pi_{13,K} M_{aet}^2 + \pi_{14,K} M_{a^2t}^2
\end{aligned}
$$

in the graph each coefficient is presented in a separate panel.



The convergence is relatively fast. The speed of convergence is governed by λ as follows:

$$\Pi^j = \lambda \Theta^{j-1} + (1 - \lambda)\, \Pi^{j-1}$$

I set $\lambda = 0.35$. The coefficients convergence after 30 iterations, for most paramaterizations.

8.4 The role of the borrowing constraint

For the baseline calibration b is set to zero, which is consistent with the natual borrowing constraint of the model (given that I abstract for other sources of income as an unemployment insurance). In order to assess the role of the borrowing constraint I also solved the model with using $b = -0.5 * K_{ss}$. In this case, the borrowing constraint is much less tight and households can borrow to smooth consumption up to half of the steady-state capital stock. Moreover, relaxing the borrowing constraint reduces incentives to acumulate precautionary wealth. The next figure presentes the IRF for the labor tax wedge and hours worked after a negative shock. Relative to the benchmark

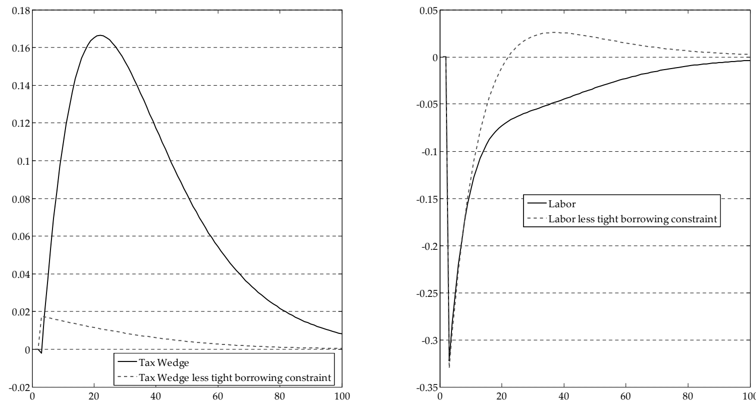


Figure 1: Model with a less tight borrowing constraint

specification, the model still delivers a counter-cyclical labor wedge but the effect is much weaker.

Relaxing the borrowing constraint reduces also the protracted respond of labor. This confirms that the borrowing constraint plays an important role for the novel results of the model.

8.5 Cyclical Changes in volatilty

In order to bring cyclical changes in the volatility of household-specific shocks, I re-define the stochastic process of idiosyncratic shocks as follows:

$$\log\left(\varepsilon_t\right) = \rho_\varepsilon \log\left(\varepsilon_{t-1}\right) + Z_t^\lambda \epsilon_\varepsilon$$

where λ governs the changes in the variance of the process. If $\lambda = 0$, the variance of idiosyncratic shocks is constant over the cycle. I set λ to match the fact that during a deep recession (defined as two-standard deviations below trend) the standard deviation of idiosyncratic shocks doubles. $\lambda = \frac{\log 2}{\log(0.986)}$ =-49.16.