Procurement Under Default Risk: Auctions or Lotteries?

Ottorino Chillemi * and Claudio Mezzetti **

December 15, 2009

Abstract

We study optimal procurement in the presence of default risk. Contractors differ in the penalty they suffer in case of default, which is private information. If the loss to the procurer from non-completion is high relative to the cost of completion, the optimal mechanism is to pay the winner enough so that the project is always completed and to assign the project by a fair lottery. The procurer can extract all contractors' surplus by charging them participation fees. Competition helps, because it allows the procurer to charge lower participation fees, an important advantage if contractors are liquidity constrained. When the loss to the procurer from non-completion is low relative to the cost of completion, the optimal probability of default is inefficiently low: projects that would be first-best efficient not to complete are completed. The project is assigned to the contractor with the highest probability of default; that is, the one with the lowest defaulting penalty.

KEYWORDS: procurement, auctions, abnormally low tenders, default risk.

JEL CLASSIFICATION NUMBERS: D44, D82, H57, L51.

* Department of Economics, University of Padua, Italy

** Department of Economics, University of Warwick, UK

1 Introduction

Public and private procurement is an important component of economic activity in most countries. According to the World Trade Organization, government procurement alone typically accounts for 10-15 percent of GDP.[1] An important feature of procurement is the specialized nature of the relationship between procurer and contractor. Once a contractor has been selected and execution of the project has begun, often the procurer has sunk specialized resources in the project and is not in a position to easily and costlessly replace the contractor. In such a situation, contractor default is a serious potential concern. The issue of bidder default has become widely known in the last 10-15 years, after several high-stake, high-profile, occurrences (e.g., the sale of the C-block spectrum licences by the FCC in 1996; see Zheng, 2001, Engel et al. 2006, and Board, 2007).

Bidder default is also a serious concern in low-stake procurement. In fact, as argued by Calveras et al. (2004), contractor default is possibly a more serious problem for small size projects. The construction industry is a particularly good example. Because most of the work is subcontracted, it is relatively easy to shut down and then open a new business under a different name. According to construction management professionals, a large number of USA construction firms stay in business for a short time. For example, Ganaway (2006) claims that only 43 per cent of U.S. construction firms remain in business after four years. The situation is not much different in other countries.

Procurers are aware of the risks of contractor default, and have put in place several contractual arrangements to ameliorate the problem. It is indeed likely that without such contractual arrangements we would observe a much larger number of defaults. A commonly used arrangement is penalties contingent on damages and, in the case of reverse auctions, performance bonds. Penalties and performance bonds, however, are not appropriate when the contractor's performance or cost are hard to measure by the procurer, when the contractor is a small firm with little to lose from default,

---

[1] See http://www.wto.org/english/tratop_e/gproc_e/gproc_e.htm

or when the legal costs of enforcing the contract are high relative to the stakes. This is often the case, for example, for small public procurement projects and in countries (e.g., Italy) in which the contracts are often administered by small municipalities that do not have the technical and financial resources to perform thorough ex-post monitoring. Another useful contractual arrangement is the use of third party guarantees, like letters of credit and surety bonds (in the USA, the Miller Act mandates 100% insurance cover on Federal contracts above $100,000). Such arrangements require well functioning banking and insurance markets, and are much less common outside of the USA, especially when the procurer and contractor are relatively small entities (e.g., a small municipality and a small firm) and the scale of the project is also limited. In Italy, for example, the portion of a contract guaranteed by insurance is only around 10% (see Decarolis, 2009).

Procurement contracts, especially public procurement, are often awarded via competitive bidding. Indeed, in many countries the law dictates the bidding procedure under which public procurement must take place. Limiting corruption of public officials and fostering efficient contract allocation are some of the well known advantages of competitive bidding. However, when default risk is an issue, competitive bidding has the drawback of encouraging the bidder most likely to default to bid low and hence win (e.g., see Spulber, 1990, and Zheng, 2001). Another approach that is used to address the contract default problem is to adopt bidding formats specifically designed to minimize default. One common feature of these bidding formats is to rule out bids that are perceived as excessively low. Winners that bid low, it is argued, have a bigger incentive not to perform the contractual task. For instance, the directory 2004/18/EC of the European Union for public works defines the notion of abnormally low tenders (hereafter ALT) and prescribes that an ALT can win the auction only if reliability is assessed in an audit conducted by the procurer. In Belgium, Greece, Italy, Portugal, Romania, Spain and Switzerland, among other countries, a tender is defined an ALT if it falls below the mean of the distribution of tenders by more than a certain percentage value, which is endogenously determined in some cases and exogenously given in others. In Belgium, Italy, Switzerland and Taiwan, ALT are automatically excluded from the set of valid bids. However, procedures that exclude bids automatically have been recently opposed by the EU Commission, because of their anticompetitive flavor, and now they can be used in the EU only for awarding contracts of limited amount (e.g., up to 1 million euros in the case of Italy) – this exemption is explicitly justified by the high costs of testing bidder reliability in the case of small projects.

The properties of bidding procedures that exclude ALT have not been much studied in the economics literature. Exceptions are the informal discussion in Engel et al. (2006, p.339), according to whom cutting ALT "will lead to lower (or zero) bankruptcy rates but at a very high price", and the empirical analysis in Decarolis (2009). As we will argue in the discussion of the literature in Section 7, most of the literature on auctions with the risk of default has focused on standard auctions under different assumptions about the information structure, bidders' default risk, and available ameliorating contractual arrangements. In this paper, we pose instead the following natural questions. When default risk is of paramount importance and penalties contingent on damages, performance bonds, or third party guarantees are not feasible, what is the optimal bidding procedure? When is the elimination of ALT beneficial to the procurer? Are there other arrangements that are useful in minimizing default risk? Some of the answers we provide may at first appear surprising, but are easily explained once the fundamental trade-off facing the procurer is understood.

To answer our questions, we model procurement as a mechanism design problem, assuming that payments or penalties to bidders cannot depend either on successful project completion or on the realized cost of completing the project, which is only observed by the winning bidder after bidding has taken place.[2] In order to focus on default risk, we postulate that bidders differ from one another depending on the penalty that they will pay in case of default. We think of the defaulting penalty as

---

[2] See Ramchurn et al. (2009), for a model in which project failure does not imply default. They show that by rewarding all bidders in case of success and penalizing them in case of failure, efficient task allocation can be achieved even in the case of multidimensional private information. In their model it is important that no firm ever goes out of business.

the loss of tangible and intangible assets, including reputation, the cost of closing and reopening business, etc., that follows from a default, and view it as private information of the contractor. If the contractor discovers that the cost of completing the project exceeds the penalty from non-completion, he will default. It may seem natural to think that the contractor will attempt to renegotiate the terms of the contract and only if renegotiation fails, he will default. However, when the procurer cannot observe the size of any claimed cost overruns, the contractor will have an incentive to always claim that they have been substantial. In such a situation, there is no scope for meaningful renegotiation; either the procurer is prepared to pay enough so as to cover any possible cost overrun, or she sets a limit over which she will not go and let the contractor default if the cost overruns are indeed very high. In this paper we will focus on such a situation, which we view as common in small size projects involving small firms and small procurers.

We distinguish two cases. In the first case, the loss to the procurer from non-completion is high relative to the cost of completion. This seems especially appropriate for contracts of limited amount, when the value of smooth project completion and avoiding delays can be expected to be higher than the cost of the project. In this case, we find that the optimal procedure is to pay the winner enough so that the project is always completed and, quite surprisingly, to assign the project by a fair lottery. The procurer can extract all contractors' surplus by charging them well designed participation fees, or, equivalently, by asking them to submit a participation deposit that will be refunded to the losers. Thus, contrary to common belief (e.g., Engel et al., 2006), the price of guaranteeing project completion is not necessarily high. We also show that, in spite of the optimal mechanism being a lottery, competition helps, because it makes it optimal for the procurer to charge low participation fees, a potentially important advantage if contractors are liquidity constrained.[3] These

---

[3] Participation fees and deposits are common in procurement. For example, in Italy the participation fees are between 20 and 100 euros, and the deposit is typically 2% of the bid and is to be refunded 30 days after the adjudication of the winner (no interest is applied). As we pointed out, small participation fees are optimal when there are many bidders, a common occurence. In Decarolis' (2009) sample of 929 auctions for construction projects held by italian municipalities, there are on

findings shed some light on the rules designed to prevent contractor default. Indeed, as is immediately seen, all the procedures which cut ALT have Nash equilibria in which all the bids are equal, just as it occurs in the optimal procedure, and hence they may be justified when the procurer overriding objective is project completion.[4]

All contractors submitting the same bid is not just a theoretical possibility. Consider the auction to build a new police station in the Sicilian municipality of Palma di Montechiaro, held in February 2008. The project was worth a base price of 2,332,539.62 euros and 82 contractors submitted legally valid bids (28 other bids were declared invalid). Bids consisted of percentage reductions over the base price. The auction rules required first to eliminate the 10% biggest price reductions (or ALT) and the 40% lowest price reductions, and then to pick as winner the bid closest to the average of the remaining bids. In this auction there were exactly 24 bids closest to the average, all submitting a percentage reduction of 7.3151%! The actual winner was determined by a lottery draw.[5]

The second case is when the loss to the procurer from non-completion is low relative to the cost of completion. In this case, first-best efficiency requires that the winner defaults if the cost of completion turns out to be too high. We show that the optimal probability of default is inefficiently low: projects that would be first-best efficient not to complete are completed. This distortion is larger the larger the cost

---

average 56 bidders for an average contract value of 373,187 euros.

[4] We should stress that we do not advocate the generalized use of lotteries as allocation mechanisms. Our paper uncovers settings were they make sense and may be optimal, but it is well known that they are highly inefficient in other settings. For example, Milgrom (2004) notes several drawbacks of the practice of assigning radio spectrum rights by lottery that prevailed in the USA between 1982 and 1993. In particular, since lottery winners could resell their licenses, speculators participated (and won) in large numbers. Milgrom argues that substantial economic costs were incurred because of the genuine wireless operators having to negotiate with the speculators. He also claims that the small size of the licenses contributed to the geographic fragmentation of the cellular industry in the USA.

[5] Data for this auction is available from the authors upon request. Far from being atypical, the outcome of this auction seems to be the norm in Sicily (Decarolis, 2009). While equal bids in standard auctions evoke the possibily of collusion, we should again stress that they are a Nash equilibrium in an average bid auction in which ALT are cut.

of default for the bidder. Interestingly, the project is assigned to the bidder with the highest default probability, who is also the bidder paying the lowest penalty in case of default. While standard auctions would generate the same project allocation than an optimal mechanism, the optimal mechanism is not a standard auction; generalized participation fees or loser-refundable deposits, that depend on the bidder's type, must be paid by all bidders taking part in the optimal mechanism. Thus, standard auctions perform poorly not because they tend to award the contract to the least reliable supplier but because they allow bidders to appropriate larger than necessary information rents.

The paper proceeds as follows. Section 2 presents the model, while Section 3 introduces the procurer's problem. Section 4 studies the optimal procurement mechanism. The equilibrium of a second-price auction in our set up is studied in Section 5. Section 6 discusses the role of competition when contractors are liquidity constrained. Section 7 discusses related literature and concludes.

2 The Model

The following is common knowledge. A procuring principal needs a task to be performed, at a cost, by one agent. There are N risk-neutral agents, indexed by $i \in I = \{1, ..., N\}$. Agent i learns the cost of performing the task only when he is about to start, or during completion of, the task, after he has won the project. (We may think of there being cost overruns, or cost savings over the expected cost.) The cost of performing the task for agent i is drawn from a distribution $F(c_i)$, which is absolutely continuous with support $[c^-, c^+]$ and density $f(c_i) = F'(c_i)$.[6] The assumption that the cost of performing the task is only discovered when the winner is about to start the task allows us to focus on the properties of different allocation mechanisms, when the risk of non performance is the main concern of the procuring

---

[6] Since the cost is only observed after the project has been assigned, it does not matter whether all agents face the same cost, or each agent's cost is drawn independently from the same distribution F.

principal, and the driving force of agents' behavior. Each agent i loses value (incurs a penalty) equal to k_i if he wins the project and defaults not performing the task. We can think of k_i as the value of tangible and intangible (e.g., reputation,) assets that are lost by defaulting. The value k_i is private information of agent i and hence represents his type; for the other agents k_i is the realization of an absolutely continuous distribution G with support $K = [k^-, k^+]$ and density $g = G'$. Let $p_i - t_i$ be the transfer to agent i when he is assigned the project and $-t_i$ the transfer when he is not assigned the project. We can think of p_i as the price paid by the procurer to the winner and t_i as a "generalized" participation fee. While in a standard auction participation fees are independent of the bidders' types, in a general mechanism both p_i and t_i are allowed to depend on the type profile (k_i, k_{-i}). Note that t_i could be negative, in which case agent i is paid to participate in the procurement mechanism. Thus, agent i's payoff is $p_i - t_i - c_i + k_i$ if he wins at price p_i and completes the contract, it is $-t_i$ if he wins and defaults, and it is $-t_i + k_i$ if he does not win. We will impose a participation, or individual rationality, constraint: an agent will accept to participate in the procurement mechanism only if his total expected payoff is not below his asset value k_i. In our model default is an option; if he wins, agent i discovers the cost and completes the contract if and only if $p_i + k_i \geq c_i$. The generalized participation fee t_i is paid before observing the completion cost and it is sunk at that point. As we shall see, we can always replace the participation fee t_i with a participation deposit τ_i that is refunded to the losers; if π_i is the probability of i winning, it suffices to set $\tau_i \pi_i = t_i$.

It remains to specify the procurer's objective function. If the project is completed without default, we assume that the procurer obtains a benefit V which is higher than the expected completion cost, $V > E[c]$. Hence, the procurer's payoff is $V - p_i + t_i$ from the winning agent i and t_j from each losing agent j. If winner i defaults, it is convenient to write the procurer's payoff from the winner as $V - d(c_i, k_i) + t_i$; $d(c_i, k_i)$ is the loss of non-completion, which may depend on the cost of completing the project c_i and the asset value k_i of the defaulting agent. The procurer's payoff from a losing agent j is still t_j. Our formulation does not rule out, but does not require, that the project is completed when the winner defaults and that some assets are appropriated from the defaulting agent. For example, if the project is not completed, the procurer cannot size any assets from the defaulting agent, and she incurs a fixed loss L, then $d = V + L$. On the other hand, if by defaulting the agent loses assets that can be fully appropriated by the procurer, who may complete the project at a cost $c_i + L$, then $d(c_i, k_i) = L + c_i - k_i$.[7] Our results hold for the general formulation, which includes cases in which the project may only be partially completed and the procurer may only appropriate some assets of the defaulting agent.

As we shall see, the size of the loss function d will play an important role in our results. We make the following assumption.

Assumption A.1. For all c and k_i it is:

$$\frac{\partial d(c, k_i)}{\partial c} \leq 1.$$

It seems reasonable that the loss of non-completion to the procurer be less sensitive to the completion cost than the completion cost itself. A.1 is satisfied in the special cases $d(c_i, k_i) = V + L$ and $d(c_i, k_i) = L + c_i - k_i$.

In a standard auction the participation fee t_i is a constant that does not depend on the type profile, while the price p_i and probability of winning the project π_i are non-constant functions of the type profile. In a lottery, on the other hand, both p_i and π_i are constants. We are interested in the following questions. What is the optimal procurement mechanism? Do standard auctions perform well in our setting? Do lotteries perform well?

3 The Procurement Problem

In this section we state formally the procurer's problem and derive some implications of incentive compatibility. The main departure from standard techniques is that revenue equivalence does not hold in our setting. In our model, the payment to the

---

[7] We only allow the procurer possibly to obtain some information about the cost c_i if the winning bidder i defaults. This is consistent with our assumption that the procurer cannot use contracts that are contingent on the realized cost.

winning agent determines whether the winner defaults, and thus payments are not fully determined by the allocation rule and the payoff of the worst-off agent.

By the revelation principle, there is no loss of generality in considering only direct mechanisms. Denote with $K_{-i} = [k^-, k^+]^{N-1}$ the set of types of agent i's opponents with generic element k_{-i} and let $g_{-i}(k_{-i}) = \Pi_{j \in I, j \neq i} g(k_j)$ be the associated density function. The probability that the project is assigned to agent i is $\pi_i(k_i, k_{-i})$, the payment he gets when he wins the contract and completes the project is $p_i(k_i, k_{-i})$, and finally $t_i(k_i, k_{-i})$ denotes the generalized participation fee, all as functions of the reported types.

Agent i's expected payoff when his type is k_i and he reports z, while the other agents report their true types, is

$$\begin{aligned} U_i(z; k_i) &= \int_{K_{-i}} \Big\{ \textstyle\int_{c^-}^{p_i(z,k_{-i})+k_i} \left[p_i(z, k_{-i}) + k_i - c\right] f(c) dc \pi_i(z, k_{-i}) \\ &\quad + k_i \left[1 - \pi_i(z, k_{-i})\right] - t_i(z, k_{-i}) \Big\} g_{-i}(k_{-i}) dk_{-i} \end{aligned} \tag{1}$$

Note that $f(c) = 0$ for $c > c^+$ and that the probability of default is an endogenous variable in the model. By raising the payment to the winner, the procurer may reduce the probability of default; $\min\{p_i(z, k_{-i}) + k_i, c^+\}$ is the highest cost level at which the project is completed. Note also that by setting $t_i(k_i, k_{-i}) = \tau_i(k_i, k_{-i})\pi_i(k_i, k_{-i})$ we could think of τ_i as the deposit that agent i's must post to participate and that is refunded if he does not win the project. In what follows we will mostly use the participation fee interpretation, but also occasionally remind the reader of the alternative, posted deposit, interpretation.

Since k_i is the outside option payoff if agent i does not take part in the mechanism, define $U_i^N(z; k_i) = U_i(z; k_i) - k_i$ as the net utility gain over the outside option. Given that in equilibrium each agent must report truthfully, $U_i^N(k_i) = U_i^N(k_i; k_i)$ is type k_i of agent i's net utility gain.

The procurer's expected payoff from agent i under truthtelling is:

$$\begin{aligned} W_i = \int_K \int_{K_{-i}} \Big\{ &\left[V - \int_{c^-}^{p_i(k_i,k_{-i})+k_i} p_i(k_i, k_{-i}) f(c) dc - \int_{p_i(k_i,k_{-i})+k_i}^{c^+} d\left(c, k_i\right) f(c) dc \right] \pi_i(k_i, k_{-i}) \\ &+ t_i(k_i, k_{-i}) \Big\} g_{-i}(k_{-i}) dk_{-i} g(k_i) dk_i \end{aligned}$$

Using (1) and the definition of $U_i^N(k_i)$, the procurer's payoff from agent i can be rewritten as

$$W_i = \int_K \int_{K_{-i}} \left\{ \left[V - E\left[c\right] - \int_{p_i(k_i,k_{-i})+k_i}^{c^+} \left[d(c,k_i) - c + k_i\right] f(c)dc \right] \pi_i(k_i,k_{-i}) \right. \\ \left. -U_i^N(k_i) \right\} g_{-i}(k_{-i})dk_{-i}g(k_i)dk_i \tag{2}$$

The expression in square brackets is the expected social surplus when agent i wins and the state is k_i, k_{-i}; the expression in braces is the procurer's surplus from agent i in state k_i, k_{-i}.

The procurer's program is to maximize $W = \sum_{i=1}^N W_i$ subject to the constraints that (1) it is an equilibrium for the agents to report their true types; (2) all agents make more than their outside option payoff, i.e., $U_i^N(k_i) \geq 0$ for all types k_i. It can be safely assumed that all types participate. The procurer can always set at zero both the probability of winning and transfers for types $k_i > k^T$. Such types are then indifferent to participation: $U_i^N(k_i; k_i) = 0$ for $k_i \geq k^T$. Since π_i is a probability, the constraints (3) $\sum_{i=1}^N \pi_i(\cdot) \leq 1$ and $\pi_i(\cdot) \geq 0$ must also hold.

The standard approach to solve for an optimal mechanism uses revenue equivalence; that is, it uses the fact that in the standard problem the payments to the agents (and hence the procurer's payoff) are determined once one fixes the payoff of the worst-off agent and the probability of winning by each agent. We need to modify this approach here, because the payment to the winner determines whether the winner defaults, and also affects the procurer's payoff through that channel. What will be true in our model is that the participation fees are determined, once one fixes the payoff to the worst-off agent, the probabilities of winning and the payment to the winner.

Consider equation (1); the incentive compatibility constraint and an envelope theorem argument yield:

$$\frac{dU_i^N(k_i)}{dk_i} = \left.\frac{\partial U_i^N(z,k_i)}{\partial k_i}\right|_{z=k_i} = -\int_{K_{-i}} \left[1 - F\left(p_i(k_i,k_{-i}) + k_i\right)\right] \pi_i(k_i,k_{-i}) g_{-i}(k_{-i}) dk_{-i} \tag{3}$$

Equation (3) is a first order condition on agent i's maximization problem. We will proceed by ignoring the second order condition; we will check whether it is satisfied once we have found a candidate solution of the procurer's problem.

Since by (3) agent i's equilibrium expected utility gain over the outside option is decreasing in k_i, the individual rationality constraint is satisfied as long as it is satisfied for the highest type. Then, we can write the individual rationality constraint as follows:

$$U_i^N(k^+) \geq 0 \tag{4}$$

Using (3) and integrating $\int_{k^-}^{k^+} U_i^N(k_i)g(k_i)dk_i$ by parts, the procurer's total payoff can be written as

$$W = -\sum_{i=1}^{N} U_i^N(k^+) + \sum_{i=1}^{N} \int_{k^-}^{k^+} \cdots \int_{k^-}^{k^+} \Big\{ \Big[V - E\left[c\right] \tag{5}$$
$$- \int_{p_i(k_i,k_{-i})+k_i}^{c^+} \left(d(c,k_i) - c + k_i + \frac{G(k_i)}{g(k_i)} \right) f(c)dc \ \Big] \pi_i(k_i,k_{-i}) \Big\} g(k_1) \cdots g(k_N)dk_1 \cdots dk_N$$

The procurer program is to maximize W as defined in (5), subject to the constraint that π_i be a probability and $U_i^N(k^+) \geq 0$.

This is a calculus of variation problem, where $p_i(k_i,k_{-i})$ and $\pi_i(k_i,k_{-i})$ are the choice variables. We can solve it point-wise. Define the functions

$$H_i\left(p_i,\pi_i,k_i\right) = \left[V - E\left[c\right] - \int_{p_i(k_i,k_{-i})+k_i}^{c^+} \left[d(c,k_i) - c + k_i + \frac{G(k_i)}{g(k_i)} \right] f(c)dc \right] \pi_i(k_i,k_{-i})$$

and let the Hamiltonian function be $H\left(\{p_i,\pi_i,k_i\}_{i=1}^N\right) = \sum_{i=1}^N H_i\left(p_i,\pi_i,k_i\right)$. The solutions p_i^*,π_i^* must satisfy the following conditions:

$$p_i^*,\pi_i^* \in \arg\max H\left(\{p_i,\pi_i,k_i\}_{i=1}^N\right) \tag{6}$$

with the transversality conditions

$$U_i^N(k^+) = 0 \tag{7}$$

$$H_i\left(p_i^*,\pi_i^*,k^+\right) \geq 0 \tag{8}$$

These first order conditions will guide us in solving the problem. Note that differentiating the Hamiltonian function with respect to p_i when $p_i + k_i < c^+$ gives:

$$\frac{\partial H}{\partial p_i} = \left[d(p_i(k_i, k_{-i}) + k_i, k_i) - p_i(k_i, k_{-i}) + \frac{G(k_i)}{g(k_i)} \right] f(p_i(k_i, k_{-i}) + k_i) \pi_i(k_i, k_{-i}) \quad (9)$$

Assumption A.1 guarantees that $\frac{\partial^2 H}{\partial^2 p_i} \leq 0$ when $\frac{\partial H}{\partial p_i} = 0$, and hence when H is maximized by an interior value of p_i (i.e., $p_i \in (c^- - k_i, c^+ - k_i)$), such a value is the unique solution to $d(p_i + k_i, k_i) - p_i + \frac{G(k_i)}{g(k_i)} = 0$.

4 The Optimal Mechanism

As a benchmark, consider the case in which k_i is publicly known (i.e., there are no incentive constraints). By (2), the sum of the procurer and winning agent's payoffs is:

$$V - E[c] - \int_{p_i(k_i, k_{-i}) + k_i}^{c^+} [d(c, k_i) - c + k_i] f(c) dc.$$

The term $p_i(k_i, k_{-i}) + k_i$ is the cut-off cost above which the project is not completed. First-best efficiency requires that the project not be completed whenever the expression $d(c, k_i) + k_i - c$ is negative. Assumption A.1 guarantees that the expression is decreasing in c, and hence it is indeed optimal to follow a cut-off policy. Let $c^B(k_i)$ be the cost below which it would be first-best efficient to have the project completed. There are three possible cases: 1) If for all c it is $d(c, k_i) + k_i - c < 0$, then $c^B(k_i) = c^-$. 2) If for all c it is $d(c, k_i) + k_i - c > 0$, then $c^B(k_i) = c^+$. 3) In all other cases $c^B(k_i)$ is the solution to $d(c, k_i) + k_i - c = 0$.

The form of the first-best cut-off rule suggests that it is useful to distinguish between two cases, depending on the size of the procurer's loss d and the highest possible completion cost c^+.

Definition D.1. The non-completion loss is high if for all k_i and all c, $d(c, k_i) + k_i \geq c$.

The non-completion loss is high when, for any given value of c, the sum of the non-completion loss of the procurer and the defaulting penalty of the winning agent is higher than the completion cost c. Under Assumption A.1 this reduces to $d(c^+, k_i) +$

$k_i \geq c^+$. In this case, it would be socially optimal for the winner to always complete the project, rather than default. The non-completion loss is always high in the special case $d(c, k_i) = L + c - k_i$ (provided $L \geq 0$). In the special case $d = V + L$, the non completion loss is high if $V + L + k^- > c^+$.

Definition D.2. There is a low non-completion loss if for some positive measure set of types k_i, it is $d(c^+, k_i) + k_i < c^+$.

When the non-completion loss is low, it is not socially optimal to complete the project in the worst-case scenario of a cost c^+.

4.1 High Non-Completion Loss

As the next proposition shows, in the case of high non-completion loss it is optimal for the procurer to pay the winner enough so that default never takes place. Interestingly, a simple way to do so is by assigning the project using a fair lottery. Constant participation fees are collected from each agent in a way that reduces every agent's information rent to zero. Thus, the procurer is able to obtain the first best outcome and extract all surplus by randomly assigning the project.

Proposition 1 *Suppose A.1 and D.1 hold (there is high non-completion loss to the procurer). Then it is an optimal policy to assign the project using a fair lottery. More precisely, an optimal procurement mechanism satisfies the following conditions:*

$$\pi_i(k_i, k_{-i}) = \frac{1}{N} \tag{10}$$

$$p_i(k_i, k_{-i}) = c^+ - k^- \tag{11}$$

$$t_i(k_i, k_{-i}) = \frac{c^+ - E[c] - k^-}{N} \tag{12}$$

$$U_i^N(k_i) = 0 \textit{ for all } k_i \tag{13}$$

Proof By assumption, when $p_i + k_i < c^+$ it is $\frac{\partial H}{\partial p_i} > 0$. On the other hand, if (11) holds, then $\frac{\partial H}{\partial p_i} = 0$ and, by Assumption A.1, H is maximized. Furthermore, (12) and (10) together imply that no type of agents obtains any information rent: (13) holds. The procurer's expected payoff W_i is the same from each agent, and hence

(10) is also optimal. Finally, since p_i and π_i are constants the second order condition of the agent's reporting problem holds, by Lemma 1 in the Appendix. □

When the loss from non-completion is high, the procurer finds it profitable to have the project completed with probability one; that is, irrespective of the agent's defaulting penalty k_i. To accomplish this, the procurer chooses a sufficiently high payment to the winner, which does not depend on the winner's type, $p_i(k_i, k_{-i})+k_i \geq c^+$ for all k_i. The procurer can extract all the surplus from each agent by charging a participation fee equal to the ex-ante expected payoff from the mechanism.[8] In order to maintain symmetry among agents and charge them the same participation fee, a fair lottery is used to assign the project. The total outlays of the procurer are $E\left[c\right]$ and the expected utility gain from participation is zero for all agents; the procurer is able to guarantee that the contract is performed and just pays the expected cost of the task. Thus, an increase in the number N of agents bidding for the project does not help the procurer; he is able to implement the first-best outcome and to keep the agent's informational rent to zero irrespective of N. While it may seem surprising at first, it is quite natural that the optimal mechanism be a fair lottery in the case of high loss of non-completion, because the procurer's optimal outcome is that the project be completed, irrespective of the size of the agent's defaulting penalty. Instead of charging all participating agents a fee $t_i = \frac{c^+ - E[c] - k^-}{N}$, the procurer could equivalently ask each agent to post a participation deposit $c^+ - E[c] - k^-$ which will be refunded to all losers of the lottery.

A fair lottery is the simplest, but not the only optimal mechanism; the procurer could obtain the same payoff by assigning the project to the agent with the lowest asset value k_i. This could be accomplished by setting a price $p_i = c^+ - k_i$ and the following generalized participation fee

$$t_i(k_i, k_{-i}) = \left(c^+ - E[c] - k_i\right)\left[1 - G(k_i)\right]^{N-1}. \tag{14}$$

It is simple to see that (13) and the second order condition for truthful revelation hold in this alternative mechanism (the net expected payment is $E[c]\left[1 - G(k_i)\right]^{N-1}$).

---

[8] In fact, any $p_i > c^+ - k^-$ and $t_i = \frac{p_1 - E[c]}{N}$ is also optimal.

Note that this alternative mechanism is not a standard auction, since the participation fee depends on agent i's type k_i; low asset value agents pay a higher generalized participation fee. In the optimal, fair, lottery, on the other hand, t_i is independent of the agents' types; all agents pay the same participation fee. Thus, a standard auction may only be optimal if it generates an outcome that is equivalent to the fair lottery (e.g., all agents submit the same bid). In Section 5 we shall show that this may happen, provided we allow the procurer to set a lower bound on the bids that agents are able to submit.

Note also that in the alternative mechanism the participation fee of type k^- is independent of the number of agents, while, on the contrary, in the fair lottery it depends on N. As we shall see in Section 6, if there is an upper bound on the size of the participation fees that agents are able to pay, then the fair lottery performs better than the alternative mechanism. Furthermore, competition helps in that case, by reducing the fee that the procurer needs to charge each bidder to extract all the surplus.

It is interesting to observe that in the case of high loss of non-completion an optimal procurement mechanism (the fair lottery) may be implemented by an equilibrium of an auction in which ALT (abnormally low tenders) are ruled out and agents must pay the participation fee specified in (12). Consider a second price (or first price) auction in which the m lowest bids are ruled out, with a reserve (or maximum) price equal to $c^+ - k^-$, and in which the winner is the lowest bidder among those that are not ruled out (with a random draw deciding the winner in case of a tie). It's an equilibrium of this auction for all bidders to bid the reserve price.

Hence, it is possible that the practice of ruling out ALT is justified when the procurer's overriding concern is to guarantee that the project is completed. This would be true for projects of limited worst-case cost c^+, and where the value of completion and the loss of non performance are high for the public authority. When the worst-case cost is small, the cost variance among bidders and the gain of trying to select the lowest cost bidder are also likely to be small. Thus, the EU policy of allowing ALT to be ruled out for contracts of limited amount, may be justified.

4.2 Low Non-Completion Loss

A fair lottery and completion of the project irrespective of cost are not optimal when the procurer's non-completion loss is small. Instead, like in the alternative mechanism discussed in the last sub-section, the optimal solution is to assign the project to the agent with the lowest defaulting penalty. Furthermore, it is optimal to allow the winning agent to default if the completion cost turns out to be above a threshold level. As we will show below, the procurer will choose a different threshold cost than the first-best cut-off cost $c^B(k_i)$.

To avoid bunching and to guarantee that the optimal (incentive efficient) policy is indeed a cut-off rule, we will make the following regularity assumption.

Assumption A.2. For all c and k_i it is:

$$\frac{\partial\left(G\left(k_i\right)/g(k_i)\right)}{\partial k_i} \geq \max\left\{-1, -\frac{\partial d\left(c,k_i\right)}{\partial c} - \frac{\partial d\left(c,k_i\right)}{\partial k_i}\right\}.$$

Note that the familiar condition of log-concavity of the type distribution would require that G/g be increasing in k_i. This is exactly what A.2 says in the special cases $d\left(c,k_i\right) = V + L$ and $d\left(c,k_i\right) = L + c - k_i$. (Note, however, that if $d\left(c,k_i\right) = L + c - k_i$ then we are in the case of a high non-completion loss, since $d + k_i > c$.)

We may now define the cut-off function $c^*(k_i)$ that we will show to be optimal in the next proposition. 1) If for all c it is $d(c,k_i) + k_i - c + \frac{G(k_i)}{g(k_i)} < 0$, then $c^*(k_i) = c^-$. 2) If for all c it is $d(c,k_i) + k_i - c + \frac{G(k_i)}{g(k_i)} > 0$, then $c^*(k_i) = c^+$. 3) In all other cases $c^*(k_i)$ is the solution to $d(c,k_i) + k_i - c + \frac{G(k_i)}{g(k_i)} = 0$.

Define the interim expected probability of winning for type k_i of agent i as $\bar{\pi}_i(k_i) = \int_{K_{-i}} \pi_i(k_i, k_{-i}) g_{-i}(k_{-i}) dk_{-i}$.

Proposition 2 *Suppose A.1, A.2 and D.2 hold (there is low non-completion loss to the procurer). Then it is an optimal policy to assign the project to the agent with the lowest asset value. An optimal procurement mechanism satisfies the following conditions:*

$$p_i(k_i, k_{-i}) = p_i(k_i) = c^*(k_i) - k_i \tag{15}$$

$$\pi_i(k_i,k_{-i}) = \begin{cases} 1 & \textit{if} \quad k_i < \min_{j\neq i} k_j \ \textit{and} \ k_i \leq k^T \\ 0 & \textit{if} \quad k_i > \min_{j\neq i} k_j \ \textit{or} \ k_i > k^T \end{cases} \tag{16}$$

$$k^T = \max\left\{k_i : V - E\left[c\right] - \int_{p_i(k_i)+k_i}^{c^+} \left[d(c,k_i) - c + k_i\right] f(c)dc \geq 0\right\} \tag{17}$$

$$\begin{aligned} t_i(k_i,k_{-i}) &= \left[\int_{c^-}^{c^*(k_i)} \left(c^*(k_i) - c\right) f(c)dc - k_i\right] \bar{\pi}_i(k_i) \\ &\quad - \int_{k_i}^{k^+} \left[1 - F\left(c^*(k)\right)\right] \bar{\pi}_i(k)dk \end{aligned} \tag{18}$$

$$U_i^N(k_i) = \int_{k_i}^{k^+} \left[1 - F\left(c^*(k)\right)\right] \bar{\pi}_i(k)dk \tag{19}$$

Proof Condition (15) follows from $\frac{\partial H}{\partial p_i} = 0$; recall that H is locally concave in p_i by A.1. Note that p_i does not depend on k_{-i}. To show that it is optimal – i.e., it maximizes $H\left(\{p_i, \pi_i, k_i\}_{i=1}^N\right)$ – to assign the project to the agent with the lowest asset value, and hence that (16) holds, we need to show that the following expression is increasing in k_i :

$$\int_{p_i(k_i,k_{-i})+k_i}^{c^+} \left[d(c,k_i) - c + k_i + \frac{G(k_i)}{g(k_i)}\right] f(c)dc$$

Differentiating with respect to k_i and evaluating at the solution we obtain,

$$\int_{c^*(k_i)}^{c^+} \left[\frac{\partial d(c,k_i)}{\partial k_i} + 1 + \frac{d\left(G\left(k_i\right)/g(k_i)\right)}{dk_i}\right] f(c)dc\pi_i > 0,$$

where the inequality follows from Assumptions A.1 and A.2. Condition (17) excludes any agent that would generate a negative payoff to the procurer from winning the contest. Bidders obtain an information rent given by (19), which is derived by integrating (3) with types $k_i \geq k^T$ receiving zero rent. Condition (18) follows from (19) and the definition of U_i^N. Finally, by Assumption A.2, $p_i(k_i)$ is an increasing function of k_i. Since π_i is decreasing, by Lemma 1 in the Appendix the second order condition of the agent's reporting problem holds. □

If the loss from non-completion is low, then $c^*(k_i) < c^+$ for some k_i and it is optimal for the procurer to let the winner default if the cost turns out to be high. Furthermore, the cut-off cost $c^*(k_i)$ above which the project is not completed is an increasing function of the defaulting penalty k_i. This implies that agents with a lower defaulting penalty are more likely to default. In spite of this, the project is assigned to the agent with the lowest defaulting penalty.[9] To understand why it is the agent with the lowest defaulting penalty that wins, observe that a default by an agent with a higher defaulting penalty entails a higher potential social loss. In the absence of incentive reasons (i.e., with no private information), it would be socially efficient to assign the project to the agent with the lowest defaulting penalty.

Recall that $c^*(k_i)$ is the highest cost at which type k_i completes the project. By (15) and A.1, the agent with the lowest defaulting penalty only completes the project when completion is socially efficient, $c^*(k^-) = c^B(k^-)$. On the other hand, agents with higher defaulting penalties complete the project even for some cost realization under which it would be socially efficient to default; more precisely, if $c^- < c^B(k_i) < c^+$ then $c^*(k_i) > c^B(k_i)$. This distortion from efficiency is due to the usual reason: to reduce the information rent of the agents. If the procurer's loss of non-completion is low, then every type $k_i < k_i^T$ earns a positive information rent. In our model, it is the agent with the lowest defaulting penalty that receives the highest information rent. What is different from the usual mechanism design or principal-agent problem is that the distortion from efficiency is an upward as opposed to a downward distortion. The project is completed more often than what first-best efficiency would dictate. To see why this is optimal, observe from (3) that the slope of agent i's net utility gain from the mechanism is increasing (i.e., smaller in absolute value) in the cut-off cost $c^*(k_i) = p_i(k_i) + k_i$. Since the agent with the highest possible defaulting penalty gets zero net utility gain, it follows that the utility gain, or information rent, of agent i is decreasing in the cut-off cost $c^*(k_i)$. By increasing the cut-off cost above the first-best

---

[9] Note that Proposition 2 applies irrespectively of the value of the non-completion loss. In the case of high non-completion loss, it describes the alternative mechanism discussed in the second part of Section 4.1.

level, the procurer is able to decrease agent i's information rent.

When his loss from non-completion is low, the procurer benefits from an increase in the number of competing bidders N. As N grows large, the expected defaulting penalty of the winning agent decreases and the information rents of agents also decrease.

Note that the generalized participation fee t_i varies with agent i's defaulting penalty k_i. Hence, no standard auction with constant participation fees is optimal in the case of low loss of non-completion; that is, when $c^*(k_i) < c^+$ for some positive mass set of types. To see this point more clearly, we now study the simplest of standard auctions, the second-price auction.

5 The Second-Price Auction

In the second-price procurement auction the contract is awarded to the bidder who has submitted the lowest price and the price the winner is paid equals the second lowest bid. We assume that a bidder that defaults is not replaced by any other bidder. We will allow the procurer to set a participation fee (that does not depend on k_i). Such a fee does not affect bidding of the participating bidders; its size only determines which bidder types decide to participate. More importantly, the analysis so far as shown that the procurer may want to impose a bound on how low bids can be, in order to reduce the probability of default by the winning bidder. This minimum bid acts as a reverse reserve price; in a standard procurement auction a reserve price would be an upper (as opposed to lower) bound on bids. We allow the procurer to set a minimum bid, or reverse reserve price, and denote it with r. Bids below r are interpreted as the bidder declining to participate in the auction. A minimum bid r plays a role analogous to the practice of eliminating ALT, abnormally low tenders, which, as we explained in the introduction, is often used in public procurement. We restrict attention to symmetric Bayesian equilibria and look for an equilibrium bidding function $B : k_i \rightarrow B(k_i)$. Standard derivations allows us to prove the following proposition.

Proposition 3 *The equilibrium bidding function of a participating bidder in the second-price auction with reverse reserve price* r *is*

$$B(k_i) = \max\{r, \beta(k_i)\}$$

where $\beta(k_i)$ *is the solution to*

$$\beta(k_i) = E\left[c|c < \beta(k_i) + k_i\right] + k_i \frac{1 - F\left(\beta(k_i) + k_i\right)}{F\left(\beta(k_i) + k_i\right)}$$

Proof See the Appendix. $\square$

Note that, if $k_i \geq c^+ - E[c]$, then $\beta(k_i) = E[c]$. To understand the formula for the bidding function, suppose for a moment that $r = 0$ (i.e., there is no minimum bid). Recall that in a second price auction without risk of default, the equilibrium bid is the expected cost of the bidder; if the price were equal to the winner's bid (i.e., if the winner's bid is in a tie with the price-setter's bid), the winner would make zero profit. Proposition 3 shows that in the presence of default risk, if the price were equal to his bid, the winner would also make zero expected profit. To see this, observe that when $p = \beta(k_i)$ the expected payment is $\beta(k_i)F\left(\beta(k_i) + k_i\right)$, while the expected cost is $E\left[c|c < \beta(k_i) + k_i\right] F\left(\beta(k_i) + k_i\right) + k_i\left[1 - F\left(\beta(k_i) + k_i\right)\right]$. The first component of the expected cost is the cost of completion times the probability of completion; the second component is the cost of default times the probability of default. If $\beta(k_i) < r$, then it is optimal to bid r, otherwise the optimal bid is $\beta(k_i)$. (In all cases, if the expected payoff from participating does not cover the participation fee, the bidder does not participate.)

It is easily shown that equilibrium bids are *strictly* increasing in the value of the defaulting penalty k_i for k_i such that $B(k_i) > r$; intuitively, a bidder's expected cost increases with the value of his loss in case of default. Pooling instead emerges for types k_i such that $B(k_i) = r$. By setting a (constant) participation fee, the procurer will determine the cut-off asset value k^T above which bidders decide not to participate in the auction.

It is noteworthy that the equilibrium bidding function is independent of both the number of bidders and the distribution G of the defaulting penalty. It is also clear that in the case of high loss of non-completion the procurer may reduce the second-price auction to the optimal lottery by setting a minimum bid $r = \max\{c^+ - k^-, E[c]\}$ and a participation fee $t_i = \frac{r - E[c]}{N}$. We now present a simple example, which will help clarifying the optimality property of the second price auction.

5.1 An Example

Let c_i be uniformly distributed in the unit interval. Then, $\beta(k_i) = \frac{\beta(k_i)+k_i}{2} + \frac{k_i[1-\beta(k_i)-k_i]}{\beta(k_i)+k_i}$ for $k_i < 1/2$, which simplifies to $\beta(k_i) = (2k_i)^{1/2} - k_i$, and $\beta(k_i) = \frac{1}{2}$ for $k_i \geq \frac{1}{2}$.

Consider the special case $d(c, k_i) = V + L$. For concreteness take $V = 5/9$ and $L = 1/9$. Suppose k_i is also uniformly distributed in the interval $[k^-, k^+]$. It is instructive to distinguish two different cases.

Case 1: $k^- \geq 1/3$. In this case the loss of non-completion is high, since $d + k_i = 2/3 + k_i \geq c^+ = 1$ for all k_i. The bidding function is $B_i = \max\left\{r, (2k_i)^{1/2} - k_i\right\}$ for $k_i \leq 1/2$ and $B_i = \max\{r, 1/2\}$ for $k_i > 1/2$. By setting $r = 1 - k^-$, the procurer can reduce the auction to a lottery among the participating bidders. However, if $r > 1/2$ (i.e., $k^- < 1/2$) the auction is not optimal, since it leaves surplus to the winning bidder; adding a participation fee $t_1 = (1/2 - k^-)/N$ to the auction would implement the optimal mechanism. Furthermore, note that if $k^- \geq 1/2$ a minimum bid is not needed to implement the optimal lottery, $r = 0$ leads to all bidders participating and bidding the expected cost 1/2.

Case 2: $k^- < 1/3$. In this case the loss of non-completion is low. By (15), the optimal cut-off cost above which a winning bidder of type k_i must default is $2/3 + 2k_i - k^-$, which is less than $c^+ = 1$ for low values of k_i. In the optimal mechanism there is a positive probability that the project is not completed. No choice of the minimum bid r can guarantee that the equilibrium outcome of the second-price auction coincides with the outcome in the optimal mechanism. First, if the minimum bid is not irrelevant, then an interval of low types will bid r and hence the winner will result from a random draw and not necessarily be the type with the lowest k_i. Second, the price paid by the winner does not depend on the winner's type; it only depends either on the type of the bidder with the second lowest bid, or the minimum bid r. Third, in any standard auction the participation fee must be independent from a bidder's type, while we know from (18) that in the optimal mechanism the participation fee of each bidder must depend on his type k_i. A second-price auction (more generally, a standard auction) cannot be optimal when the procurer's non-completion loss is low, but the policy of setting an appropriate minimum bid r (which is similar in spirit to eliminating ALT) typically increases the procurer's payoff by reducing default.

6 Liquidity Constraints and the Value of Competition

In our analysis, we have imposed no bounds on the participation fee t_i that an agent must pay. In this section, we will relax this assumption. In some circumstances agents may be liquidity constrained; that is, they may be unable to pay participation fees above a given threshold. It is thus useful to consider what changes when adding this complication to the model. Rather than a full analysis of this case, we will focus on the new insights that emerge when agents are subject to liquidity constraints. Two new lessons can be learned. First, competition in the form of additional bidders helps the procurer even in the case of high non-completion loss. As we saw in Section 4, without liquidity constraints competition does not help when the procurer wants the project to be completed for certain. If there are liquidity constraints, on the other hand, the larger the number of competing agents, the more the procurer is able to reduce the size of the participation fee. Hence, the easier it is to satisfy agents' liquidity constraints; competition helps. Second, with liquidity constraints and a high non-completion loss, the optimal lottery mechanism performs better than the alternative optimal mechanism in which the project is assigned to the bidder with the lowest defaulting penalty. This is because for some agent types, the participation fees in the alternative mechanism are higher than in the lottery.

We model liquidity constraints by assuming that the participation fee that an agent pays must be bound above by some amount h; that is, it must be $t_i(k_i, k_{-i}) \leq h$, where h is common knowledge.

Our first result is that for a sufficiently large number of competing agents the lottery described in Proposition 1 satisfies the liquidity constraint, and hence remains optimal.

Proposition 4 *Suppose A.1 and D.1 hold (there is high non-completion loss to the procurer). Suppose the liquidity constraint $t_i(k_i, k_{-i}) \leq h$ must hold. Then the following condition guarantees that it is an optimal policy to assign the project using a fair lottery and that the optimal procurement mechanism is the one described in Proposition 1:*

$$N \geq \frac{c^+ - E[c] - k^-}{h} \tag{20}$$

Proof Condition (12) gives the formula for t_i, from which (20) immediately follows.

□

If $c^+ - E[c] - k^- \leq 0$, then agents are paid to participate in the lottery and the constraint will be slack. If $c^+ - E[c] - k^- > 0$, on the other hand, the constraint may be violated. Interestingly, the proposition shows that even in this case, if competition is sufficiently intense (i.e., N is high), then the fair lottery remains an optimal mechanism. The intuition is simple. When $c^+ - E[c] - k^- > 0$, on average winning the lottery and completing the project raises an agent's payoff. To bring the agent's total payoff in line with his outside option, the procurer charges a positive participation fee. The larger the number of participants in the lottery, the smaller the chance of winning, and hence the smaller the expected benefit from participating. As a result, the participation fee that the procurer charges will have to be reduced. Indeed, in the limit, as N goes to infinity, the participation fee goes to zero. For sufficiently large N the constraint will be satisfied.

Our second result is that the lottery performs better than the alternative mechanism in which the project is assigned to the bidder with the lowest defaulting penalty.

Proposition 5 *Suppose A.1 and D.1 hold (there is high non-completion loss to* *the procurer). Suppose the liquidity constraint* $t_i(k_i, k_{-i}) \leq h$ *must hold. Then the following condition guarantees that the unique optimal policy is to assign the project using a fair lottery with the procurement mechanism described in Proposition 1:*

$$N \geq \frac{c^+ - E[c] - k^-}{h} > 1 \tag{21}$$

Proof By (14), the participation fee in the alternative mechanism is highest for type k^-, for whom it equals $(c^+ - E[c] - k^-)$. It follows that the alternative mechanism does not satisfy the liquidity constraint for $(c^+ - E[c] - k^-) > h$. On the other hand, by Proposition 4, the condition $N \geq \frac{c^+ - E[c] - k^-}{h}$ guarantees that the lottery satisfies the liquidity constraint. □

In the mechanism in which the project is assigned to the agent with the lowest defaulting penalty, the participation fees are more dispersed and as a result it is harder to satisfy the liquidity constraints. Furthermore, in such a mechanism the lowest type agent's participation fee is independent of the number of bidders, and hence competition does not help satisfying the liquidity constraint.

7 Conclusions

The first main point of departure of our paper from the literature on default risk in auctions is that we look at optimal mechanisms, rather than specific auction formats. The second point of departure is that we are interested in instances, like small scale construction projects, in which uncertainty about the risk of default is more important than cost uncertainty, and in which contractual arrangements like penalties for default, performance bonds, surety bonds, etc. are not feasible. We model uncertainty about risk of default fairly generally; it is due to the loss of utility to the contractor following default, which is not known by the procurer.

Spulber (1990) was the first to note that auctions may provide incentives for contractors to default, when there are cost overruns. His paper focuses on the first-price auction, and shows that using expectation damages as the penalty for default restores efficiency. Other papers have looked at arrangements that insure the auctioneer against the risk of default. In Waehrer (1995), the winning bidder is required to post a deposit that is lost in case of default. He finds that the seller's payoff is decreasing in the level of the deposit. Calveras et al. (2004) show how the introduction of third party guarantees may eliminate defaults in a second-price procurement auction. In their model, the contractor enters into an agreement with a surety company which guarantees project completion to the procurer in case of contractor default.

A few papers have looked at the implication of budget constraints on the probability of default. Zheng (2001) studies the first-price auction for an item whose common value is discovered after the auction. In his model, the winning bidder may borrow in order to pay above his budget, which is private information. Zheng shows that, when the interest rate is low the winner is the bidder with the lowest budget (and hence the greatest probability of default), while when the interest rate is large the highest-budget bidder wins. Rhodes-Kropf and Viswanathan (2005) extend Zheng's analysis; their focus is on how different ways of financing bids affect bidding behavior. Zheng (2009) shows that, if implemented, the 2008 U.S. Treasury plan of auctioning toxic assets might have induced poor bidders to outbid rich bidders, and then to default on the government loans in case of unsalvageable assets. In Parlane (2003), bidders have a publicly known asset value that they lose when defaulting; she shows that the expected price is higher in the first-price than in the second-price auction. Board (2007) considers a similar setting and finds that the seller prefers the second-price auction when the cost of bankruptcy is low and the first-price auction when it is high.

We are only aware of two papers that, like us, take a mechanism design approach. Wan and Beil (2009) consider a very different setting than us. In their model production costs are private information, while the probability of default is learned after the contest. Furthermore, the procurer can test a bidder's risk of default both before and after the contest. They find that, if the contract is assigned, it is assigned to the lowest cost supplier among those that pass the test. Burguet et al. (2009) are the closest to us.[10] They study procurement mechanisms with bidders that have

---

[10] We only became aware of Burguet et.al. (2009) after writing the first draft of this paper.

limited liability, and assume that the procurer appropriates the winner's budget and completes the project at a fixed loss in case of default (this is as the special case of our model in which $d(c_i, k_i) = L + c_i - k_i$). Unlike us, they impose the restriction that only the winner receives any (positive or negative) transfers from the procurer. They do not solve for the optimal mechanism, but instead provide some properties that all incentive compatible mechanisms must satisfy.

While the economics literature on default risk has focused on standard auctions, non-standard auctions like average bid auctions, or auctions that cut abnormally low tenders, are common in practice, especially for low-stake projects.[11] One of our main insights is that they may be the "right" mechanism in settings that fit the assumptions of our model; that is, when contractual arrangements that insure the procurer against the risk of default are not feasible, and the procurer's loss of non completing the project is high. Indeed, in these settings auctions that cut ALT and assign the project to the bid closest to the average of the remaining bids are optimal, if complemented with the imposition of participation fees or participation deposits that are refunded to the losers. While participation fees and deposits are common in procurement auctions, they tend to be small. Another insight of our work is that as long as there are many competing bidders, participation fees need not be large. Thus, a potentially useful policy implication of our paper is that procurers charge larger participation fees the smaller the number of competing contractors in the market.

We have also shown that, when the procurer's non-completion loss is low, the optimal procurement procedure should generate less defaults than it is socially efficient, with the distortion being larger for contractors having a high defaulting penalty. Standard auctions are not optimal in this case either, even though a standard auction with a lower limit on bids goes in the right direction of distorting down the probability of default. Thus, another insight of our paper is that more complex negotiating procedures may be appropriate when the procurer only incurs a moderate loss in case of contractor default.

---

[11] Even in the USA the average bid auction with elimination of ALT is not unheard of; it is for example used by the Florida Department of Transportation (Decarolis, 2009).

Appendix

In this appendix, first we prove a lemma that deals with the second order conditions of the agents' reporting problem. Then we provide a proof of Proposition 3.

Lemma 1 *Consider the mechanism described by the functions* $p_i(k_i, k_{-i})$, $t_i(k_i, k_{-i})$, $\pi_i(k_i, k_{-i})$ *for all* i. *Suppose that (a)* $p_i(k_i, k_{-i}) = p_i(k_i)$ *(i.e.,* p_i *does not depend on the types* k_{-i}*); (b)* $p_i(k_i)$ *is increasing in* k_i *and differentiable; (c)* $\bar{\pi}_i(k_i) = \int_{K_{-i}} \pi_i(k_i, k_{-i}) g_{-i}(k_{-i}) dk_{-i}$ *exists and is decreasing in* k_i. *If this mechanism satisfies the first order condition of the agent's reporting problem, then it also satisfies the second order condition and hence it is incentive compatible.*

Proof Consider the first order condition of agent i reporting problem:

$$\left.\frac{\partial U_i(z, k_i)}{\partial z}\right|_{z=k_i} = 0.$$

Differentiating it totally yields

$$\left.\frac{\partial^2 U_i(z, k_i)}{\partial z \partial k_i}\right|_{z=k_i} + \left.\frac{\partial^2 U_i(z, k_i)}{\partial z^2}\right|_{z=k_i} = 0.$$

Since

$$\frac{\partial U_i^N(z, k_i)}{\partial k_i} = -\int_{K_{-i}} \left[1 - F\left(p_i(z, k_{-i}) + k_i\right)\right] \pi_i(z, k_{-i}) g_{-i}(k_{-i}) dk_{-i},$$

under the hypotheses of the lemma, we can write the second order condition as:

$$\begin{aligned} -\left.\frac{\partial^2 U_i(z, k_i)}{\partial z^2}\right|_{z=k_i} = \left.\frac{\partial^2 U_i(z, k_i)}{\partial z \partial k_i}\right|_{z=k_i} = \left.\frac{\partial^2 U_i^N(z, k_i)}{\partial z \partial k_i}\right|_{z=k_i} = \\ -\left[1 - F\left(p_i(k_i) + k_i\right)\right] \frac{d\bar{\pi}_i(k_i)}{dk_i} \\ + f\left(p_i(k_i) + k_i\right) \frac{dp_i(k_i)}{dk_i} \bar{\pi}_i(k_i) \geq 0. \end{aligned}$$

□

Proof of Proposition 3 Let $\beta(k_i)$ be the bidding function under the provisional assumption that the bidding function is strictly increasing everywhere. Letting

$Q(k) = 1 - [1 - G(k)]^{N-1}$, and $Q'(k)$ be its derivative, we can write bidder i's problem of determining the optimal value of his bid b as:

$$\max_{b} \int_{\beta^{-1}(b)}^{k^{+}} \left\{ \int_{c^{-}}^{\beta(k)+k_i} [\beta(k) - c + k_i] f(c) dc \right\} Q'(k) dk + k_i Q(\beta^{-1}(b)). \tag{22}$$

The FOC for problem (22) is

$$-\frac{d\beta^{-1}(b)}{db} \left\{ \int_{c^{-}}^{b+\beta^{-1}(b)} [b - c + k_i] f(c) dc Q'(\beta^{-1}(b)) - k_i Q'(\beta^{-1}(b)) \right\} = 0.$$

Then, using the Nash equilibrium condition $b = \beta(k_i)$ yields

$$\int_{c^{-}}^{\beta(k_i)+k_i} [\beta(k_i) - c + k_i] f(c) dc - k_i = 0 \tag{23}$$

or

$$\beta(k_i) = E[c|c < \beta(k_i) + k_i] + k_i \frac{1 - F(\beta(k_i) + k_i)}{F(\beta(k_i) + k_i)}. \tag{24}$$

Equation (24) defines the equilibrium bidding function, provided $\beta(k_i)$ is strictly increasing and $\beta(k_i) \geq r$. To see that $\beta(k_i)$ is increasing, note that if we differentiate (23) with respect to k_i we obtain

$$\int_{c^{-}}^{\beta(k_i)+k_i} \left[\beta'(k_i) + 1\right] f(c) dc - 1 = 0$$

and hence

$$\beta'(k_i) = \frac{1 - F(\beta(k_i) + k_i)}{F(\beta(k_i) + k_i)} > 0 \qquad \text{for } \beta(k_i) + k_i < c^{+}$$

Now, observe that using (24), for types k_i such that $E[c] + k_i \leq c^{+}$ it will be $\beta(k_i) \leq c^{+} - k_i$. These types then will bid according to $B(k_i) = \max\{r, \beta(k_i)\}$. Each bidder $k_i \geq c^{+} - E[c]$ will complete the contract with certainty and therefore must bid no less than the expected cost; furthermore, he will certainly lose the auction if he asks for more than the expected cost. Note that for such bidders $\beta(k_i) = E[c]$. Hence, bidding according to $B(k_i)$ is also an equilibrium for these types. $\square$

References

[1] Albano, G., M. Bianchi and G. Spagnolo (2006), Bid Average Methods in Procurement, *Rivista di Politica Economica*, 1-2, 41-62.

[2] Board, S. (2007), Bidding into the Red: A Model of Post-Auction Bankruptcy. *Journal of Finance*, 62, 2695-2723.

[3] Burguet, R., J.J. Ganuza and E. Hauk (2009), Limited Liability and Mechanism Design in Procurement, Institut d'Anàlisi Econòmica, working paper.

[4] Calveras, A., J.J. Ganuza, and E. Hauk (2004), Wild Bids. Gambling for Resurrection in Procurement Contracts, *Journal of Regulatory Economics*, 26, 41-68.

[5] Decarolis, F. (2009), When the Highest Bidder Loses the Auction: Theory and Evidence from Public Procurement, *mimeo*, University of Wisconsin.

[6] Engel, A.R, J.J. Ganuza, E. Hauk and A. Wambach (2006), Managing Risky Bids, in (Dimitri N. and al., eds.), *Handbook of Procurement*, Cambridge University Press, 322-343.

[7] Ganaway, N.B. (2006), *A Guide to Contracting for Business Success*, Elsevier.

[8] Milgrom. P. (2004), *Putting Auction Theory to Work*, Cambridge University Press.

[9] Parlane, S. (2003), Procurement Contracts under Limited Liability, *The Economic and Social Review*, 34, 1-21.

[10] Ramchurn, S.D, C. Mezzetti, A. Giovannucci, J.A. Rodriguez-Aguilar, R.K. Dash, and N.J. Jennings (2009), Trust-Based Mechanisms for Robust and Efficient Task Allocation in the Presence of Execution Uncertainty, *Journal of Artificial Intelligence Research*, 35, 119-159.

[11] Rhodes-Kropf, M. and S. Viswanathan (2005), Financing Auction Bids, *Rand Journal of Economics*, Winter, 36, 4, 789-815.

[12] Spulber, D.F. (1990), Auctions and Contract Enforcement, *Journal of Law, Economics and Organization*, 6, 2, 325-344.

[13] Waehrer, K. (1995), A Model of Auction Contracts with Liquidated Damages, *Journal of Economic Theory*, 67, 2, 531-555.

[14] Wan, Z. and D. R. Beil (2009), RFQ Auctions with Supplier Qualification Screening, *Operations Research,* 57(4), 934-949.

[15] Zheng, C.Z. (2001), High Bids and Broke Winners, *Journal of Economic Theory*, 100(1),129-171.

[16] Zheng, C.Z. (2009), The Default-Prone U.S. Toxic Asset Auction Plan, *B.E. Journal of Economic Analysis & Policy*, 38(1), 41-72.